Joint Economic Lot Sizing at Two Levels of Supply Chain in Food and Beverage Industry

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Abstract

Order lot sizing is one of the important mechanisms that must be used by a supply chain to survive in today's business competition, especially in food beverage industries with reviews of their wide market coverage. And the problem is that generally the order lot size between manufacturers and distributors has not been integrated yet. This study discussed how to get their lot size using the Joint Economic Lot Size (JELS) for two levels of supply chains in order to reduce total costs. Using the JELS coordination model algorithm, the total cost of integration is Rp 58,271,000- with a savings of Rp 15,519,000, where the greatest value of savings occurs to the manufacturer, in the amount of Rp 14,991,000. The result shows that using the JELS mechanism, 60.81% cost saving in manufacturer and 21.03% cost saving in all levels could be Obtained.

Keywords
Order Lot Size, Two Level Supply Chains, Joint Economic Lot Size.

1. Introduction
The integrated inventory management between manufacturers and distributors are part of the supply chain management. In determining the lot size inventory management, the manufacturers and distributors can collaborate to minimize the total cost of inventory combined. PT HMS is a food and beverage company that has a problem in inventory. The problem caused by the lot size determination ordering between manufacturers and distributors is not integrated. Determination of an integrated ordering lot size is commonly known as the Joint Economic Lot Size (JELS).

Goyal (1977) in 1976, has developed a method of Joint Economic Lot Size (JELS) by finding a supplier-manufacturer ordering interval to minimize the total cost of inventory combined to demand fixed and deterministic. In 1989, Goyal and Gupta (1989) JELS is concluded that the early stages of the achievement of coordination in the supply chain (SC). Furthermore, several studies about JELS developed for conditions that are not the same as the size of the production batch manufacturers are integer multiples of the lot size of the ordering (Goyal 1989), lot different shipping (Goyal et al. 2000), lot shipping the same, and the different (Goyal 2000). Besides, research on the two-level supply chain coordination with probabilistic demand has been developed by Anshori et.al, (2017). On the same topic, research about JELS has been developed by Sarakhsi et al. (2015) to introduce a scatter search algorithm and Nelder-Mead to optimize JELS problems.

Based on the description above, this study compares the application of JELS models that have been developed by Anshori et al. (2017) with the real conditions in PT. HMS.

2. Methodology
The secondary data has obtained from PT. HMS. While the settlement method used in this study is a model of coordination that has been developed by Anshori et al. (2017);
Notation
The notation used in the development of the model, are:

b is a distributor
v is the manufacturer
i is the number of items/components of the product, where i = 1,2, ... k

j is the number of buyers, where j = 1,2, ... n
k is the number of units of the components of the product
μ_j is the number of product demand buyers j (units/year)
σ_j is the standard deviation of demand for the product buyer to-j (units/year)
A_{bj} is an ordering fee per cycle buyer j ($)
A_v is the setup cost per cycle manufacturers ($)
a_{vi} is an ordering fee i-th component, where i = 1,2, ..., k
h_{bj} is the cost of storage of products in the buyer j ($)
h_v is the manufacturer's products in the storage costs per year ($ / unit/year)
h_{vi} is the cost of storage in the i-th component manufacturers per year ($ / unit/year)
S is a shortage fee ($ / unit / year)
T_j is the ordering cycle time buyers j (years)
$Imax_j$ is the maximum inventory in each cycle (units)
ITL_j is the remaining inventory in each cycle (units)
QTL_j is the number of orders made by the buyer-j in each cycle (unit)
λ_{vj} is the number of buyers ordering delivery

Total cost in buyer
The total cost of the purchaser (TCB) is formulated as follows:

$$TC_b = \sum C_{b,j} = \sum_{j=1}^{n}\left[\frac{A_{b,j}}{T_j} + h_{b,j}\left[\frac{\mu_j T_j + Z_j \sigma_j \sqrt{T_j}}{2}\right] + \frac{S_j}{T_j} ES_j\right] \quad (1)$$

Where:

$$ES = -ss\left(1 - F_s\left(ss/\sigma\right)\right) + \sigma f_s\left(ss/\sigma\right)$$

Optimal ordering cycle in each of buyers following the EOQ model, in which:

$$T_j^* = \sqrt{\frac{2A_{b,j}}{h_{b,j}\mu_j}} \quad (2)$$

To find the number of orders made by the buyer-j in each cycle (unit) (QTL) by calculating a maximum supply (Imax) minus inventory at time T (ITL) and added with the expectation of shortage that occurred (ES). This formulation can be expressed as $Q_{Tl} = I_{\max} - I_{Tl} + ES$ where $l = T, 2T, ..., xT$ So that the number of ordering for each buyer j is done by using the formula $Q_{Tl,j} = I_{\max,j} - I_{Tl,j} + ES_j$ or it could be described as the following formula:

$$Q_{Tl,j} = \mu_j T_j + Z_j \sigma_j - I_{Tl,j} + ES_j \quad (3)$$

Total costs in the manufacturer
The total cost of the manufacturer (TCV) is formulated as follows:

$$TC_v = \sum_{j=1}^{n}\left\{\frac{A_v + \sum_{i=1}^{k} a_{v,i}}{\lambda_{v,j} T_j} + \frac{h_v + \sum_{i=1}^{k} h_{v,i} u_i}{2}\left(\lambda_{v,j} - 1\right)\left(Q_{Tl,j}\right)\right\} \quad (4)$$

Total cost of supply chain
Total cost of supply chain can be expressed as :

$$TC_{chain}(\lambda_{v,j},T_j)=\sum_{j=1}^{n}\left[\frac{A_{b,j}}{T_j}+h_{b,j}\left[\frac{\mu_j T_j+Z_j\sigma_j\sqrt{T_j}}{2}\right]+\frac{S_j}{T_j}ES_j\right]\sum_{j=1}^{n}\left\{\frac{A_v+\sum_{i=1}^{k}a_{v,i}}{\lambda_{v,j}T_j}+\frac{h_v+\sum_{i=1}^{k}h_{v,i}u_i}{2}(\lambda_{v,j}-1)(Q_{Tl,j})\right\} \tag{5}$$

If the decision-making involving coordination, then the total cost of the combined formula can be rewritten as follows:

$$\overline{TC}_{chain}(\lambda,T)=\sum_{j=1}^{n}\left[\frac{A_{b,j}}{T}+h_{b,j}\left[\frac{\mu_j T+Z_j\sigma_j\sqrt{T}}{2}\right]+\frac{S_j}{T_j}ES_j\right]+\sum_{j=1}^{n}\left\{\frac{A_v+\sum_{i=1}^{k}a_{v,i}}{\lambda T}+\frac{h_v+\sum_{i=1}^{k}h_{v,i}u_i}{2}(\lambda-1)(Q_{Tl,j})\right\} \tag{6}$$

To find the optimal ordering cycle can be expressed by lowering the first $\overline{TC}_{chain}(\lambda,T)$ against T equal to zero. Finding the optimal cycle either model without coordination or with coordination can be seen in the attachment. The optimal ordering cycle (T*) can be formulated as follows:

$$T^*=\sqrt{\frac{2\sum_{j=1}^{n}\left(\lambda A_{b,j}+A_v+\sum_{i=1}^{k}a_{v,i}\right)}{\lambda\left(\sum_{j=1}^{n}\mu_j h_{b,j}+\left(h_v+\sum_{i=1}^{k}h_{v,i}u_i\right)(\lambda-1)\left(\sum_{j=1}^{n}\mu_j\right)\right)}} \tag{7}$$

Finding algorithm solution model

Finding solutions for T * and λ * value and QTL can minimize the total cost of the supply chain by using an algorithm. The algorithm to complete the research model is formulated as follows:

Algorithm For Models Without JELS

The steps to calculate the model without coordination can be formulated as follows:

1 Each buyer determines the optimal ordering cycle by using formula 2.
2 Rated T^*_j is used to calculate Imax, Safety Stock, and expected Shortage of each buyer.
3 Calculate the amount of the order made by the buyer in each cycle (QTL_j), by first looking for the ultimate value of inventory (ITL, j) of each buyer. QTL value, j sought by using formula 3.
4 For value T^*_j, Imax, and expected shortage are still used to find the value λv, j optimized by using the formula 5. If $TC_v(\lambda_{v,j}^*,T_j^*)\leq TC_v(\lambda_{v,j}^*-1,T_j^*)$ repeat the calculation for the value $\lambda^*_{v,j}=\lambda^*_{v,j}+1$ and compare again TCV value, but otherwise proceed to step 5.
5 Score $TC_v(\lambda_{v,j},T_j^*)$ the smallest occurred at the time Tj* and $\lambda^*_{v,j}$.
6 Accumulate all the total cost at the buyer and the manufacturer to get the value $TC_{chain}(\lambda^*_{v,j},T^*_j)$ of the formula 5 for a model without coordination.

Algorithm To Model JELS

While the steps to complete the model with coordination can be formulated as follows:

1. Set λ = 1 to obtain the optimal ordering cycle value (T*) by using formula 7.
2. Rated T* is used to calculate Imax, Safety Stock, and expected Shortage of each buyer.
3. Calculate the amount of the order made by the buyer in each cycle (QTL), by first looking for the ultimate value of inventory (ITL) each buyer. QTL value sought by using formula 3.
4. For T* value, Imax, and expected shortage fixed, find the optimal λ value. If $\overline{TC}_{chain}(\lambda^*,T^*)\leq\overline{TC}_{chain}(\lambda^*-1,T^*)$ repeat the calculation for the value λ* = λ* + 1 and compare more value $\overline{TC}_{chain}$, But otherwise proceed to step 5.
5. Score $\overline{TC}_{chain}(\lambda^*,T^*)$ the smallest occurred at time T* and λ*.

3. Results and Discussion

The following are the cost parameters obtained from the distributors and the manufactures with each of the known cost details. Existing costs on the distributor's side include ordering costs (A), storage costs (h), as well as shortage costs (S). This data is obtained from each distributor, as well as for demand data. Details of the costs for each distributor can be seen in table 1.

Table 1. Parameters in the distributor.

Distributor (j)	**Ordering Cost ($A_{b,j}$) (Rp/Order)**	**Holding Cost (h_{bj}) (Rp/unit)**	**Shortage Cost (S_j) (Rp/unit)**	**Demand**	
				μ_j	σ_j
1	3,000,000.-	155,000.-	242,000.-	670	364
2	1,800,000.-	133,000.-	242,000.-	5516	1666

Costs for the manufacturer include setup costs (A_v = Rp. 2.0732 million / month), and cost savings (h_v = Rp. 107 220 / unit / month). Each product requires 5 items to making up the product (i = 1,2, .. 5). The cost of ordering each item at the same product is (a_{vi} = Rp. 357 400), while the needs of each item forming product (u_i) and costs save item ($h_{v,i}$) can be seen in Table 2 below:

Table 2. Unit constituent products and storage costs.

Raw material	**u_i / unit (kg)**	**cost savings ($h_{v,i}$) (Rp./unit/bulan)**
x1	5	22,500
x2	5	15,000
x3	2	13,500
x4	1	360
x5	1	2,250
Total	**12**	**53 610**

The result using the model and steps to resolve the described methodology is shown in Table 3.

Table. 3. Comparison of results with and without the JELS model.

Attribute	**Without JELS**		**With JELS**	
Distributor j	1	2	1	2
T^*_j (year)	0.34	0.099	0.203	0.203
λ^*_j	2	1	1	1
I_{max} (unit)	159	374	98	374
C_B (Rp)	15,820,000.-	33,320,000.-	15,290,000.-	33,320,000.-
C_v (Rp)	5,332,000.-	19,320,000.-	9,661,000.-	
TC(T*,λ*) (Rp)	73,790,000.-		58,271,000.-	

From table 3 above it can be seen that there are differences in the total cost of the results of calculations between the models without JELS and by using the JELS model. Differences are also seen in the order cycle of each distributor. The optimum order cycle without JELS for buyer 1 is 0.34 years (4 months), and buyer 2 is 0.099 years (1 month). This means that the shipment made by the manufacturer to buyer 1 is 3 times, and buyer 2 is 12 times. Whereas if the JELS model is used, the cycle for buyers 1 and 2 is the same, amounting to 0.204 years (bi-monthly) with optimal delivery of 6 times each. Based on table 3, it can also be seen that there is a total cost saving on the overall level, where the savings are Rp 73.79 million – Rp 58.27 million = Rp 15.52 million. The biggest savings occurred on the manufacturer, amounting to Rp 24.652 million - Rp 9.661 million = Rp 14.991 million. In other words, by using the JELS model there are reductions in costs by 14,991,000/24,652,000 or by 60.81% for manufacturers, as well as reductions in costs by 15,519,000/73,790,000 or by 21.03% for the entire level.

4. Conclusion

This research has succeeded in proving that the two-level JELS model yields a smaller overall total cost than the model without JELS. The calculation results show that there is a substantial reduction in costs for the manufacturer and for the entire level. This inventory model could still be developed for different problems. In this research, a lead time is assumed to equal to zero. For the next research could be applied in non-zero lead time, or considering the assumption that damage could happen in delivery.

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