

Lot-Sizing Single Item Model with Stochastic Demand, Multiple Suppliers, Backlogging and Quantity Discounts

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Abstract

One of the activities that can be done to control costs in production activities is by optimizing the level of supply and demand. Dynamic Lot Sizing Model is one method to get a minimum inventory level which in essence can reach the minimum costs that must be incurred by the company. This model seeks to eliminate the assumption that fixed demand level is calculated the same for each period that is commonly used in the EOQ model. This paper considers dynamic lot sizing, stochastic demand, by selecting one supplier, each supplier has quantity discount system and also transportation cost. In this case, backlogs are permitted, and the standard normal loss function used to represent the standardized unit's number of shortage function. By having an objective function and some constraints where the variables can be integer and biner, then solving using MINLP is a good choice for this formulation model, with commercial optimization software. Study case performed in manufacturing, and model can help to determine quantity order to purchase, with specific supplier and for each period. The goal to be achieved is to obtain the minimum costs from the purchase cost, order cost, backlogging cost, handling cost and transportation cost.

Keywords

Dynamic Lot Sizing, Stochastic Demand, Backlogs and Inventory

1. Introduction

Inventory management is very important for a company to achieve competitive costs and to obtain a decent profit in the market. It has become a popular topic both in academia and in real practice for decades. As the production environment becomes more complex, various types of mathematical models have been developed, such as linear programming, non-linear programming, mixed integer programming, geometric programming, gradient-based nonlinear programming, dynamic programming and also another method such as heuristic tools try to solve the problems more effective and get the result, especially when the problems become non-deterministic.

Having good production planning through effective inventory management is important for companies to stay competitive in the market. With single item, and many periods are one of the most common and basic problems and are often dealt with in the literature. There are various extensions of the model to consider various problems in the real environment.

Inventory storage and backlogging costs incurred to store purchased products and do not meet product needs when needed, this is a concern. Therefore, finding the optimal set of suppliers and the number of goods to be obtained in each time period, can maintain inventory and reduce costs related to company suppliers.

2. Literature Review

In the inventory management literature, the problem of dynamic lot size has received a lot of attention especially when a set of planning periods is considered. In 1958, Wagner and Whitin introduced the first dynamic programming algorithm to solve the dynamic lot-sizing problem for only one type of product and at one supplier. The next researcher tries to find a solution with other algorithms to improve efficiency empirically. The researcher tried several algorithms, starting from mixed integer programming, multi-objective programming, genetic algorithms, heuristic dynamic programming or others, to get solutions with more efficient solutions (Wagner and Whitin 1958; Evans 1985; Federgruen and Tzur 1991).

Research also develops by considering the backlog and quantity discounts. Other researchers mention that this quantity discount has two kinds, namely the quantity discount is based on all products, or the quantity discount is given on

increasing the number of products ordered (Hu and Munson 2002; Ghaniabadi and Mazinani 2017; Kang and Lee 2013; Lee et al. 2013; Mazdeh et al. 2015; Alfares and Turnadi 2015; Absi et al. 2011). Related with supplier selection some researchers studied to determine the amount of material should order in each period without using quantity discount (Aissaoui et al. 2007; Mendoza and Ventura 2012). Quantity discounts are a common and effective practice for suppliers to promote their products, and buyers can buy products at lower unit prices when the number of orders exceeds a certain amount. In addition, with suppliers with more than one number, it provides an opportunity for companies to buy the same material from different sources. Ordering costs for each purchase can be in different forms, including fixed, increased, or decreased. And the company can determine which suppliers can reach the minimum cost (Parsa et al. 2013; Chang et al. 2006; Chaudry et al. 1993).

Likewise, with the costs that must be incurred, the booking fee itself consists of fixed costs and additional costs. Fixed costs do not depend on lot size, and additional order fees depend on the specific lot size. Reduction of ordering costs is positively related to ordering frequency, i.e. the higher-order frequency from suppliers, the higher the reduction in ordering costs from these suppliers by Bai and Xu (Bai and Xu 2011). It also appears in the reality that demand is changing in nature and influenced by many uncertainties. The uncertainty that can occur is due to time factors, product quality, or returns that do not reach the expected quantity. Some previous studies using deterministic demand turned to research aimed at stochastic demand (Kang and Lee 2013; Tempelmeier and Hilger 2015).

There are two studies that discuss the same case and data source but have different characteristics. From the research conducted by Kang and Lee (2013) using the MIP (*Mixed Integer Programming*) and HDP (*Heuristic Dynamic Programming*) methods, researchers calculate how many products must be ordered and from which suppliers are selected so as to get a minimum total cost. In his case considering the service level. The settlement method chosen previously was using the dynamic lot size method, this is based on previous research by Kang and Lee that mentions results that are close to the optimal value with faster time consumption (Kang and Lee 2013). This makes the dynamic lot size method can help solve optimization problems on a large scale and faster time consumption. This happens especially if the conditions faced with a long-time span, faced with a number of suppliers that are quite a lot, and the number of levels of quantity discounts. Other facts that mention the method chosen by using this method are also conveyed by (Tarim et al 2011; Vargas 2009).

Based on the literature that has been studied previously, the models created have not yet developed stochastic models, quantity discounts and single or more suppliers are considered at the same time, and this is part of the previous researcher's suggestion that the development of the models that have been worked on can be included in other parameters, where demand can be stochastic, then this is used as a reason for research carried out by developing two new models by combining them, taking into account parameters related to costs incurred to meet these. Another thing that is very important and different from previous research is that the system allows for a backlog, this is included in the parameters under consideration because stochastic demand allows this condition to occur.

In the reality of business, the number of demand for goods experiences uncertainty which can be caused by factors related to the quality of goods, such as when production takes place defects that require additional basic products, or the uncertainty of the number of goods that come due to the grace period which is uncertain, this is the reason why the demand chosen for the object of research is stochastic demand. And another thing the company will be faced with a number of suppliers that compete with each other by offering prices and discounts on a number of quantities of goods ordered.

Considering the two previous researchers by Ghaniabadi (2017) and Lee (2013), it is necessary to develop a new model to complete the previous model in solving the problem of lot size determination, namely by using the dynamic lot size method, with many supplier parameters, there is backlogging and application of quantity discounts for one type of product. With this new model, we expected the module will achieve the minimum total cost and can develop a model that has been made previously in dealing with the reality of the problems that occur and the parameters mentioned earlier become an important part in making improvements.

2.1. Cost Related.

The performance criteria of the inventory system will be evaluated based on the smallest *Total Inventory Costs*, where the decision variables will include when an order must be made and how many volumes of orders each time an order will be placed. Both of these variables have met with the demand rate in a minimal cost level. Central point in the inventory

control system is the formation of an appropriate model and can explain the relationship between the variables. The decision in the form of an optimal answer from the inventory model is a major problem in inventory management.

2.1.1. Order cost

Equation (1) is ordering cost equal calculates the ordering cost for the system, where o_i is the ordering cost per time from supplier i and F_{it} represents whether a quantity is purchased from supplier i in period t .

$$Cost\ order = O = \sum_{t=1}^T \sum_{i=1}^I O_i \times F_{it} \quad (1)$$

2.1.2. Purchase Cost and Discount Quantity

The purchase cost is obtained by equation (2), where $P(Q_{it})$ is the unit purchase cost based on the discount schedule with the order quantity Q_{it} , and F_{it} represents whether a quantity is purchased from supplier i in period t . Equations (3), (4) and (5), for quantity discounts selection. Constraint (5) defines that k price break selected only one and this applied from supplier i in period t , so number of purchase can decide. Equation (6) also defines U_{itk} can 1 or 0 to determine k as a price break and applied to the quantity purchase for supplier i in period t . Equation (7) defines F_{it} can be 1 if number of quantity should be purchased or 0 if quantity should not purchase is not made from supplier i in period t .

$$Purchase\ cost = P = \sum_{t=1}^T \sum_{i=1}^I (P(Q_{it}) \times Q_{it} \times F_{it}) \quad (2)$$

$$P(Q_{it}) = \sum_{k=1}^K p_{ik} \times U_{itk}, \text{ for all } i, t \quad (3)$$

$$q_{ik-1} + M \times (U_{itk} - 1) \leq Q_{it} < q_{ik} + M \times (1 - U_{itk}), \text{ for all } i, t, k \quad (4)$$

$$\sum_{k=1}^K U_{itk} = 1, \text{ for all } i, t \quad (5)$$

$$U_{itk} \in \{0,1\}, \text{ for all } i, t, k \quad (6)$$

$$F_{itk} \in \{0,1\}, \text{ for all } i, t, k \quad (7)$$

2.1.3. Piecewise linear function.

In addition, Chang (2006) did calculation and the equation is linearized. Quantity purchase can be happening in multiples period for one or more supplier. Equation (8) is formula to assume that equation is liner, and that variable as a quantity purchase should be integer, and Equation (9) β_{itn} can be 1 or 0, number of purchase can be made if value of $\beta_{itn}=1$ and quantity purchase did not made if $\beta_{itn}=0$ and purchase can be made for supplier i in period t . Equation (10) is formula to defines purchase quantity can be made from supplier i in period t .

$$Q_{it} = \sum_{n=1}^N 2^{v_{it}-1} \beta_{itn} \text{ for all } i, t \quad (8)$$

$$itn \in \{0,1\} \text{ for all } i, t, n \quad (9)$$

$$Q_{it} = 2^0 \beta_{it1} + 2^1 \beta_{it2} + 2^2 \beta_{it3} + \dots + 2^N \beta_{itN}, \text{ for all } i, t \quad (10)$$

This linearization approach can be formulated by variable $\beta_{(it1)}$. Variable $\beta_{(it1)}$ is linearized by the following inequality constraints: $(\beta_{(it1)} - 1)M + \psi \leq \tau \leq \psi$ and $0 \leq \tau \leq \beta_{it1}M$, where M is a big value, τ is a continuous variable, β_{it1} is a binary variable and is an integer variable. Two cases available are

- If $\beta_{(it1)}=0$, then $-M + \psi \leq \tau \leq \psi$ from (1), $0 \leq \tau \leq 0$ from (2). Thus, $\tau = 0$.
- (2) If $\beta_{(it1)}=1$, then $\psi \leq \tau \leq \psi$ from (1), $0 \leq \tau \leq M$ from (2). Thus, $\tau = \psi$.

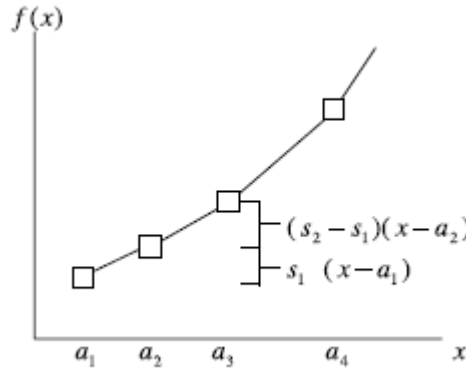


Figure 1. Piecewise linear functions $f(x)$ graphic.

Chang (2002) did calculation using piecewise linear functions, piecewise linear function of x as $f(x)$, can see in Figure 1, where $a_i, i = 1, 2, \dots, n$, are the break points of $f(x)$, and $s_i, i = 1, 2, \dots, n$, are the slopes of line segments between a_i and a_{i+1} . This piecewise linear function can be expressed as the sum of absolute terms:

$$f(x) = a_1 + s_1(x - a_1) + \sum_{i=2}^{n-1} \frac{s_i - s_{i-1}}{2} (|x - a_i| + x - a_i) \quad (11)$$

where $|x|$ is the absolute value of x . This proposition can be examined as follows:

If $x = a_1$, then $f(x) = f(a_1)$

If $x \leq a_2$, then

$$f(x) = f(a_1) + \frac{f(a_2) - f(a_1)}{a_2 - a_1} (x - a_1) \quad (12)$$

$$= f(a_1) + s_1(x - a_1) \quad (13)$$

If $x \leq a_3$, then

$$f(x) = f(a_1) + s_1(x - a_1) + s_2(x - a_2) \quad (14)$$

$$= f(a_1) + s_1(x - a_1) + \frac{s_2 - s_1}{2} (|x - a_2| + x - a_2) \quad (15)$$

Consider $f(x)$ in equation (11),

If $s_i > s_{i-1}$, then $f(x)$ is convex within the interval $a_{i-1} \leq x \leq a_{i+1}$

If $s_i < s_{i-1}$, then $f(x)$ is concave within the interval $a_{i-1} \leq x \leq a_{i+1}$

2.1.4. Standard Normal Loss Function

The approach for formulation (15), we can use standard normal loss function this formula refer to (Cachon and Terwiesch 2003). This value from normal distribution and this formula for $L(z)$ is:

$$L(z_t) = \text{Normdist}(z_t, 0, 1, 0) - z_t * (1 - \text{Normsdist}(z_t)) \quad (16)$$

2.1.5. Backlog Cost

Equation (23) formula for the backlog cost. This calculation is summation from period $i = 1$ to period t for backlog cost per unit (s) multiply by pool standard deviation (σ_t) and multiply by unit standardized number for backlog function $L(Z_t)$.

$$\int_{y_t}^{+\infty} (d_t - Y_t) f(d_t) dd_t = \sigma_t \times L(z_t), \text{ for all } t \quad (17)$$

$$\text{Set } z = \frac{d-E(d)}{\sigma}$$

$$d = z\sigma + E(d) \quad (18)$$

$$dd = \sigma dz \quad (19)$$

$$f(d) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{((z\sigma)+E(d)-E(d))^2}{2\sigma^2}} \quad (20)$$

$$f(d = z\sigma + E(d)) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{((z\sigma)+E(d)-E(d))^2}{2\sigma^2}} = \frac{1}{\sigma} \phi(z) \quad (21)$$

$$\begin{aligned} \int_{y_t}^{+\infty} (d_t - Y_t) f(d_t) dd_t &= \int_{z_t = \frac{Y_t - E(d_t)}{\sigma_t}}^{+\infty} (z_t \sigma_t + E(d_t) - Y_t) \phi(z_t) dz_t \\ &= \sigma_t \int_{z_t = \frac{Y_t - E(d_t)}{\sigma_t}}^{+\infty} \left(z_t - \frac{Y_t - E(d_t)}{\sigma_t} \right) \phi(z_t) dz_t = \sigma_t \int_{z_t = \frac{X_{t+1}}{\sigma_t}}^{+\infty} \left(z_t - \frac{X_{t+1}}{\sigma_t} \right) \phi(z_t) dz_t \\ &= \sigma_t \times L(z_t) \text{ for all } t \end{aligned} \quad (22)$$

$$\text{Backlog Cost} = S = \sum_{t=1}^T s \times \sigma_t \times L(z_t) \quad (23)$$

2.1.6. Holding Cost

The holding cost in period t calculated from quantity stock in end of period t multiply with the holding cost per unit times. To get stock in period t , we should consider pool standard deviation and standardised number of unit's function. The holding cost for a planning horizon is the summation of the holding cost for each period, as in Equation (27).

$$\int_{-\infty}^{Y_t} (Y_t - d_t) f(d_t) dd_t = X_{t+1} + \sigma_t \times L(z_t), \text{ for all } t \quad (24)$$

$$\int_{-\infty}^{Y_t} (Y_t - d_t) f(d_t) dd_t + \int_{Y_t}^{+\infty} (Y_t - d_t) f(d_t) dd_t = Y_t - E(d_t), \text{ for all } t \quad (25)$$

$$\begin{aligned} \int_{-\infty}^{Y_t} (Y_t - d_t) f(d_t) dd_t &= Y_t - E(d_t) - \int_{Y_t}^{+\infty} (Y_t - d_t) f(d_t) dd_t \\ &= Y_t - E(d_t) + \int_{Y_t}^{+\infty} (d_t - Y_t) f(d_t) dd_t = X_{t+1} + \sigma_t \times L(z_t), \text{ for all } t \end{aligned} \quad (26)$$

$$\text{Holding Cost} = H = \sum_{t=1}^T h \times (X_{t+1} + \sigma_t \times L(z_t)) \quad (27)$$

2.1.7. Transportation Cost.

Equation (28) define as a calculating transportation. This result coming from sum of transportation cost per vehicle multiply by A_{it} as a number of vehicle, from period $t = 1$ to period t . Number of vehicle calculated from the smallest integer greater than or equal to Q_{it}/b_i , for each supplier and period, in symbol we can state $\lceil Q_{it}/b_i \rceil$.

$$\text{Transportation Cost} = TR = \sum_{t=1}^T \sum_{i=1}^I ts_i \times \lceil Q_{it}/b_i \rceil = \sum_{t=1}^T ts_t \times A_{it} \quad (28)$$

3. Assumptions and definitions

The assumptions are summarized as follows:

- This case only for single item.
- One order can be set in each period from each supplier.
- There is a limited time horizon, which consists of time periods T .
- The planning horizon is finite and known. In the planning horizon, there are T periods, and the duration of each period is the same.

- Independent demand of each period has a normal distribution with a constant coefficient of variant (θ) = 1/3.
- Holding costs are expensed at the end of the inventory period.
- Inventory holding cost for each unit is known and constant, independent of the price of each unit.
- Initial inventory level (X_1) is zero.
- The expected ending inventory level in period t (the expected beginning inventory level in period $t + 1$) is the safety stock level in period t .
- Not considered for supplier capacity.
- Unit price depend on the order quantity, considered to table unit discounts for each vendor.
- Discount schemes calculated using the incremental discount system.
- Lead time for replenishment is known duration an order quantity will delivered at once in the beginning of period.

Indices:

t	Index planning period ($t = 1, 2, \dots, T$)
i	Supplier ($i = 1, 2, \dots, T$)
n	Calculating the quantity purchased in integer number ($n = 1, 2, \dots, T$)
k	Price break ($k = 1, 2, \dots, T$)x

Parameters:

$E(d_t)$	Demand expectation in period t
$\hat{\sigma}_t$	Standard deviation of demand in period t
σ_t	Pool Standard deviation of demand in period t
h	Inventory holding cost per unit per period
s	Shortage cost, per unit per period
z_α	Standard normal value of service level α
$L(z_\alpha)$	Standardized number of units short with service level α
p_{ik}	Unit purchase cost from supplier i with price break k
q_{ik}	The upper bound quantity of supplier i with price break k

Decision Variables:

F_t	Binary variable, set equal to 1 if a purchase is made from supplier i in period t , and 0 if no purchase is made from supplier i in period t .
Q_{it}	Purchase quantity from supplier i in period t
β_{itn}	Binary variable for calculating the purchase quantity from supplier i in period t
$P(Q_{it})$	Purchase cost for one unit based on the discount schedule of supplier i with order quantity Q_{it} in period t
F_{it}	Binary variable, set equal to 1 if a purchase is made from supplier i in period t , and 0 if no purchase is made from supplier i in period t
X_t	Expected beginning inventory level in period t
U_{itk}	Binary variable, set equal to 1 if a certain quantity is purchased, and 0 if no purchase is made, from supplier i with price break k in period t
z_t	Standard normal value of stock level in period t
$L(z_t)$	Standardized number of units short of stock level in period t .
β_{itn}	Binary variable for calculating the purchase quantity from supplier i in period t .

4. Model Development and Formulation

According to the assumption and definition mentions in previous section, we can state the objective is in Equation (29), and details of constraint is (30) - (44). Details of equation as below:

$$\begin{aligned} \text{Min } TC = \sum_{t=1}^T & \left[\sum_{i=1}^I o_i \times F_{it} + \sum_{i=1}^I P(Q_t) \times Q_{it} \times F_{it} + s \times L(z_t) \times \sigma_t \right. \\ & \left. + h \times (X_{t+1} + L(z_t) \times \sigma_t) + \sum_{i=1}^I ts_i \times [Q_{it}/b_i] \right] \end{aligned} \quad (29)$$

Constraints:

$$X_{t+1} = Y_t - E(d_t) \text{ for all } t \quad (30)$$

$$Y_t = X_t + \sum_{i=1}^I Q_{it} \times F_{it} \text{ for all } t \quad (31)$$

$$Q_{it} \leq M \times F_{it} \text{ for all } t \quad (32)$$

$$Q_{it} = \sum_{n=1}^N 2^{n_{it}-1} \beta_{itn} \text{ for all } i, t \quad (33)$$

$$z_t = \frac{Y_t - E(d_t)}{\sigma_t} \text{ for all } t \quad (34)$$

$$z_t \geq z_\alpha \text{ for all } t \quad (35)$$

$$\hat{\sigma}_t = \theta \times E(d_t) \text{ for all } t \quad (36)$$

$$\sigma_t = \sqrt{\sum_{t'=1}^t \hat{\sigma}_{t'}^2} \text{ for all } t \quad (37)$$

$$L(z_t) = \text{Normdist}(z_t, 0, 1, 0) - z_t * (1 - \text{Normsdist}(z_t)) \quad (38)$$

$$P(Q_{it}) = \sum_{k=1}^K p_{ik} \times U_{itk} \text{ for all } i, t \quad (39)$$

$$q_{ik-1} + M \times (U_{itk} - 1) \leq Q_{it} < q_{ik} + M \times (1 - U_{itk}) \text{ for all } i, t, k \quad (40)$$

$$\sum_{k=1}^K U_{itk} = 1 \text{ for all } i, t \quad (41)$$

$$F_{it} \in \{0, 1\} \text{ for all } i, t \quad (42)$$

$$\beta_{itn} \in \{0, 1\} \text{ for all } i, t, n \quad (43)$$

$$U_{itk} \in \{0, 1\} \text{ for all } i, t, k \quad (44)$$

Equation (30) and (31) is a function to balance the amount of inventory in the warehouse. Equation (32) has a value of M which is a very large positive number to ensure that the number of orders is sufficient to meet the number of requests in the period t . Assuming a large M value can be greater or equal to the total sum of requests in period 1 to period T . $L(z_t)$ the number of standard units that is shorter than stock level quantity in end of period t can be formulated as an absolute number explain in Equation (38). Equation (39) – (41) these function to determine the quantity of purchased by the company which is between the upper and lower limit of the k price level given by the supplier for each supplier and in each period. Equation (42) – (44) shows the type of each variable. Value of F_{it} , β_{itn} and U_{itk} is biner. F_{it} determine whether purchased or not from suppliers i in period t . β_{itn} can 1 or 0 to calculate the number of purchases from supplier i in period t . U_{itk} can 0 or 1 to determine the price break and applied for quantity purchase for supplier i in period t .

5. Study Case

The case study taken is related to aircraft parts assembly manufacturers in Bandung, where the company buys components from other suppliers to assembly on the parent parts then send it to the customer. To meet optimum policy, the organization consider having multiple suppliers, and each supplier apply quantity discounts, safety stock, also customer will give penalty if company did not meet with his requirement. Currently, the company has one supplier, and starting work together with different suppliers. The purpose of this model is to achieve total costs minimum and to decide the optimal number of purchases from each supplier in each period. This planning horizon for 1 year, and period of the planning in month. Each has a different supplier ordering cost. In addition, a unit holding cost is determined per period that includes handling costs, storage costs and capital costs. Those cost are known and fixed.

Table 1. Data demand for 12 months.

Period	1	2	3	4	5	6	7	8	9	10	11	12
Expected Demand	2,546	4,153	3,965	4,375	3,872	3,202	1,696	1,693	1,120	1,812	2,213	1,552
Standard Deviation	849	1,384	1,322	1,458	1,291	1,067	565	564	373	604	738	517

Table 2. Supplier A quantity discount table.

Price break	Quantity of purchase	Price per unit (USD)
1	< 2.500	9.7857
2	≥ 2.500	6.6054
3	≥ 5.000	6.4772
4	≥ 10.000	5.5802

Table 3. Supplier B quantity discount table.

Price break	Quantity of purchase	Price per unit (USD)
1	< 2.500	9.8090
2	≥ 2.500	6.7568
3	≥ 9.000	5.3589

Table 4. Supplier C quantity discount table.

Price break	Quantity of purchase	Price per unit (USD)
1	< 2.000	9.7857
2	≥ 2.000	6.6986
3	≥ 5.000	6.4073
4	≥ 10.000	5.5918

In the interview session with department related, to produce this part organization arrange planning horizon for one year or in 12 periods. The order cost of supplier A (o_1) is set to be \$230, supplier B (o_2) is set to be \$200 and supplier C (o_3) is set to be \$220, which handling cost, storage cost and capital expense, set to be \$0.15. Demand in every period is thought to be normally distributed with $E(D_t)$ and a coefficient of variety (θ) of 0.333. Table 1 displays the expected of demand $E(D_t)$ and standard deviation of demand σ_t in every period t. Table 2–4 displays the quantity discount plan for each supplier under various purchase amounts. Table 5 shows the data case.

Table 5. Data case.

Period	1	2	3	4	5	6	7	8	9	10	11	12
Cumulative Demand	2,546	6,699	10,664	15,039	18,911	22,113	23,809	25,502	26,622	28,434	30,647	32,199
Pool Std Deviation	849	1624	2,094	2,551	2,859	3,052	3,104	3,155	3,177	3,234	3,317	3,357
Cumulative Purchase	3,943	9,370	14,109	19,235	23,614	27,134	28,915	30,692	31,848	33,754	36,103	37,721

The solutions of the MIP model are shown in Tables 6, respectively. This calculation has been done using LINGO to get the result. The model provides the optimal solution, this process takes only 10 seconds. In the MIP model, the result that

purchase should be done with quantity 5,037 units from supplier A in period 1, in period 2 from supplier B with 16,383 units, and in period 3 from supplier A with 16,302 units. And the optimal solution as a total cost is \$243,153.

Table 6. Solution of case.

Period	1	2	3	4	5	6	7	8	9	10	11	12
X_t		2,491	14,721	27,058	22,683	18,811	15,609	13,913	12,220	11,100	9,288	7,075
Y_t	5,037	18,874	31,023	27,058	22,683	18,811	15,609	13,913	12,220	11,100	9,288	7,075
Q_{1t}	5,037		16,302									
Q_{2t}		16,383										
Q_{3t}												
Z_t	2.93	9.06	12.92	8.89	6.58	5.11	4.48	3.87	3.49	2.87	2.13	1.65
$L(Z_t)$							0.76e-6	0.12e-4	0.60e-4	0.60e-3	0.59e-2	0.21e-1

6. Analysis

From the results obtained in table 6, it shows that ordering in the first period to meet needs until period 2, this causes no need for a backlog in that period. This also happened in period 3, where orders were made to fulfil the next three periods, and so on. Although the objective function states that there is a holding cost and backlog cost, the variables related to quantity will indirectly contradict, where if there is a stock, it means that there is no backlog for each period. Thus, the results obtained indicate that there is no quantity backlog for each period.

The outcomes of sensitivity analysis are appeared in Table 7. This analysis performed dependent on the optimal solution. When the order cost from every supplier (o_i), inventory holding cost (h), backlog cost (s) or transportation cost (ts) is increase or decrease 50%, the total cost in the framework is changed. For this study case, with decrease of holding cost by half, the all-out costs will diminish by 7.06% to 225,991 dollars. From three parameters above, we got that holding cost is a significant parameter since that variable can prompt a bigger change in all out expense.

Table 7. Sensitivity analysis for study case.

Parameter	Pct. changes	Order Cost	Purchase Cost	Holding Cost	Backlog Cost	Transport	Total Cost	Total Cost Change
Order cost	+50	1,275	210,264	16,503	2,200	6,000	236,242	-2.84%
	-50	315	204,362	24,934	1,840	5,850	237,301	-2.40%
Holding cost	+50	650	206,993	26,291	2,060	5,850	241,844	-0.54%
	-50	660	208,089	9,662	1,880	5,700	225,991	-7.06%
Backlog cost	+50	800	205,645	21,638	2,760	5,850	236,693	-2.65%
	-50	620	205,129	23,274	920	5,850	235,793	-3.03%
Transportation cost	+50	650	207,230	17,539	2,040	9,000	236,459	-2.75%
	-50	830	205,346	18,763	1,860	3,000	229,799	-5.49%

It is significant impact if we expand for the number of quantity discounts price break (k) for each supplier, include with add more suppliers (i), more periods (t) and those will make difficult size and may cause the issue to become NP-hard and restrictive computerized.

7. Conclusion

Proposes of this paper is develop a MIP model to decide the stochastic part estimating renewal for various periods consider over amount limits, different suppliers and service levels. The contextual investigation shows the common sense of the model in accomplishing the best fulfilment under different objectives, which are limiting absolute expense and fixing the service level. At the point when a recharging issue isn't unreasonably confounded, for instance with hardly any periods, suppliers and amount price breaks, a MIP model can be effectively explained by scientific programming, for example, LINGO. Nonetheless, when there are numerous periods, a few suppliers and various amount markdown breaks, the difficult will become NP-hard and restrictive computerized. As far as we could possibly know, a MIP model, which

considers amount limits, various suppliers and administration level to at the same time limit all out expense and utilize the piecewise straight capacity to speak to the normalized number of unit deficiency work, is non-existent.

In addition, the investigation gave in this examination is exceptionally valuable for buyer in planning a recharging approach to manage the stochastic part estimating issue which has the attributes of safety stock, numerous suppliers and distinctive discount schedules from suppliers.

The proposed MIP model can be custom fitted and applied to other inventory management issues. For future research, we can consider an increasingly complete case for flexibly supply chain. A model that considers backorders, delay purchases, lost sales, variable lead time, and diverse need of requests can likewise be established.

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