Planning Hydroelectric Power Distribution Under Uncertain Supply

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Abstract

This paper presents the power distribution planning problem for integrated storage dam and run-off-river (ROR) hydroelectric power plant (HEPP) projects under supply uncertainty. The problem is formulated as a two-stage stochastic program, and numerical comparisons of the stochastic solution, expected value solution, and wait and see solutions are made. The solutions provide the economic dispatch of generators and optimal plan that the power system operators use to coordinate, control, and monitor the hydroelectric power generation and distribution systems.

Keywords
Generation Dispatch, Run-Off-River, Storage Dams, Stochastic Program

1. Introduction

Integrated power systems (IPS) usually consist of electricity generation power plants (PP), transmission lines (TL), power substations, and distribution lines (DL) that need to be efficiently coordinated to deliver power to the end users (Parra et al., 2017; Wood et al., 2013). A schematic diagram of such a power system with fourteen bus is shown in Figure 1 (taken from IEEE 14-bus test system and updated for the purpose of this paper). The operational characteristics of power systems depend on the inherent and external factors such as power delivery capacity, demand location, and weather conditions. In addition, as the number of power system components increases, the complexity of the problem that the power systems operators (PSO) deal with to effectively deliver power to the end users also increases.

![Figure 1. Integrated power system consisting of hydroelectric power projects](image-url)

The power supply uncertainty is a great challenge to reliably deliver power to the end users. There are certain suggestions that are proposed to handle the uncertainties such as the use of storage devices to improve the power supply reliability (Ibrahim et al., 2008). However, the capabilities of storage technologies have not been well...
developed to store energy in bulk to shave peak demands (Luo et al., 2015). On the other hand, the PSO must ensure that the supply matches the demand instantaneously. This necessitates the proper planning of the power supply sources before integrating them into the existing power system, under limited power storage capacity. Otherwise, serious supply reliability issues may arise and cause the whole power system shutdown due to the unbalanced energy supply and demand (Correa-Florez et al., 2016). Proper planning involves the integration of variable energy generators (VEGs) such as solar, wind, and ROR generators with non-variable energy generators. These non-variable supply sources could be storage hydro, geothermal, or nuclear power plants that supply electricity when VEGs are not available.

The power system planning with VEGs interconnected is well studied in the literature in the recent years (Deng & Lv, 2020). Bensalem et al. (2012) looked at a deterministic optimal power management strategy of hydroelectric power plants to maximize potential energy stored in the reservoir. Saravanan et al. (2013) and Suresh & Kumarappan (2013) addressed a hydroelectric power units scheduling problem to minimize the total cost of energy generation where they considered stochastic generation under uncertain demand. Huang (2001) reviewed an ant colony system (ACS) based optimization approach for the enhancement of hydro-thermal generation scheduling. The ACS algorithm provides a solution with minimum thermal production cost while maximizing energy output from the hydro power plants and maximizing power system reliability. Similarly, Doganis & Sarimveis (2014) presented a convex mixed integer quadratic programming model and Jiekang et al. (2008) described a hybrid approach to meet the optimal plan of hydro-thermal units. El-Khattam et al. (2005), Hegazy et al. (2014), and Xia et al. (2010) proposed adaptive genetic algorithm to integrate distributed generators (DG) (like ROR hydro, solar, and wind) with the grid so that the size of generators, operational cost, and power loss cost would be optimum. These studies present system constraints such as power balance, power network capacity, and substation capacity with demand and generation uncertainties that are also included in this paper. The variabilities exist within DG because usually ROR hydro, solar, and wind power plants are integrated into the power grids without battery storage systems. The hydro-thermal scheduling presented in (Bhattacharya et al., 2019; Doganis & Sarimveis, 2014; Huang, 2001; Jiekang et al., 2008) would reduce operational costs of thermal power plants and increases the reliability, however, integration of renewables with storage hydro would provide more clean energy solution. Hydropower system has been the major source of energy for many countries in the world and climate change is affecting the power supply from HEP due to change in hydrological cycles. There is a need of evaluating the impacts of variability of HEP to facilitate the decision making process for the PSO in energy planning (Guerra et al., 2019; Liu et al., 2016).

In this paper, the ROR generators, and relatively stable hydro power supply sources, the storage dams, are integrated to guarantee reliable power delivery. The power supply from ROR projects is treated as an uncertain supply source because of the fact that water flows in the river, and hence power production capacity is largely determined by the uncertain weather conditions. It is also assumed that the storage dams as relatively stable power supply sources because dams store water and the flow of water is regulated so that power from the storage projects dispatch can follow a pre-planned schedule. We provide analyses of expectation of expected value (EEV), stochastic solution (SS), and wait and see (WS) techniques that the PSOs can utilize to plan the power generation and distribution system.

The remaining portion of the paper is organized as follows. The problem formulation is presented in section 2. Numerical experiments are provided in section 3 followed by results and discussion in section 4, and conclusion and recommendations in section 5.

2. Modeling the Problem

In this section the mathematical model of the hydroelectric power planning problem is presented. The objective of the problem is to minimize the total energy production, transmission, and distribution costs over the planning horizon $T$; where $T$ (one year) is divided into six seasons.

Let,

\[
\begin{align*}
C_{gi} & \quad \text{Cost of energy generation in dollar per MW hr.} \\
C_{ti} & \quad \text{Cost of energy transmission in line } j \text{ in dollar per MW hr from power plan to substation.} \\
C_{sj} & \quad \text{Operation and maintenance cost at substation } j \text{ in dollar per MW hr.} \\
C_{zj} & \quad \text{Cost of energy transmission in between substations line } j \& p \text{ in dollar per MW hr.}
\end{align*}
\]
\[
\begin{align*}
\text{Minimize } Z &= T \times \left[ \sum_{i=1}^{n_1} \sum_{s=1}^{s} P_{ts} \left( \sum_{j=1}^{m} C_{li}(x_{ijs}) + \sum_{j=1}^{m} C_{li}(y_{ijs}) + \sum_{j=1}^{m} \sum_{p=1}^{m-1} C_{li} \left( \sum_{i=1}^{n_1} \sum_{j=1}^{m} C_{lj}(x_{ij}) \right) \right) \right] \\
\sum_{i=1}^{n_1} \sum_{s=1}^{s} P_{ts} \left[ \sum_{j=1}^{m} C_{li} \left( \sum_{i=1}^{n_1} \sum_{j=1}^{m} C_{lj}(x_{ij}) \right) \right] + \\
\sum_{i=1}^{n_1} \sum_{s=1}^{s} P_{ts} \left[ \sum_{j=1}^{m} \sum_{p=1}^{m-1} \sum_{s=1}^{s} \sum_{p=1}^{m-1} C_{lj} \left( \sum_{i=1}^{n_1} \sum_{j=1}^{m} C_{lj}(x_{ij}) \right) \right] + \\
\sum_{i=1}^{n_1} \sum_{s=1}^{s} P_{ts} \left[ \sum_{j=1}^{m} \sum_{p=1}^{m-1} \sum_{s=1}^{s} \sum_{p=1}^{m-1} C_{lj} \left( \sum_{i=1}^{n_1} \sum_{j=1}^{m} C_{lj}(x_{ij}) \right) \right]
\end{align*}
\]

Subject to:

\[
\begin{align*}
x_{it} &\leq S_i \quad \forall i, t (2a) \\
y_{ijs} &\leq f_{its} \times S_i \quad \forall i, t, s (2b) \\
\sum_{t=1}^{n_1} (x_{ij} - x_{it}) &= 0 \quad \forall i, t (3a) \\
\sum_{j=1}^{m} (y_{ijs} - y_{it}) &= 0 \quad \forall i, t, s (3b) \\
x_{ij} &\leq T_{Lij} \quad \forall i, j \times t \quad (4a) \\
y_{ijs} &\leq T_{Lij} \quad \forall i, j, t, s \quad (4b) \\
x_{ij} + y_{ijs} &\leq Z_{jp} \quad \forall j, p, t, s \quad (4c) \\
d_{jkt} &\leq DL_{jk} \quad \forall j, k, t, s \quad (4d) \\
\sum_{j=1}^{m} x_{ij} + \sum_{j=1}^{m} y_{ijs} + \sum_{p=1}^{m-1} (x_{ij} + y_{ijs}) \leq SS_j \quad \forall j, t, s \quad (5) \\
\sum_{j=1}^{m} x_{ij} + \sum_{j=1}^{m} y_{ijs} + \sum_{p=1}^{m-1} (x_{ij} + y_{ijs}) - \sum_{p=1}^{m-1} (x_{ij} + y_{ijs}) - \sum_{k=1}^{m} d_{jkt} = 0 \quad \forall k, t, s \quad (6) \\
\sum_{j=1}^{m} d_{jkt} \geq D_k \quad \forall k, t, s \quad (7) \\
x_{it}, y_{ijs}, x_{ij}, y_{ijs}, x_{ij}, y_{ijs}, d_{jkt} \geq 0 \quad \forall i, j, k, t, s \quad (8)
\end{align*}
\]

Where each of the terms of equation (1) is defined as:

\[
\begin{align*}
\sum_{i=1}^{n_1} C_{gi}(x_{it}) &= \text{Total generation cost of an HEPP under a scenario.} \\
\sum_{i=1}^{n_2} C_{gi}(y_{ijs}) &= \text{Total generation cost of all ROR for a season under a scenario.}
\end{align*}
\]
\[
\begin{align*}
\sum_{i=1}^{m} \sum_{j=1}^{n} Ctl_{ij}(x_{ijt}) & \quad \text{Total cost of power transmission from storage HEPP to substation for a season.} \\
\sum_{i=1}^{m} \sum_{j=1}^{n} Ctl_{ij} \cdot y_{ijts} & \quad \text{Total cost of power transmission from ROR PP to substation for a season and a scenario.} \\
\sum_{i=1}^{m} \sum_{j=1}^{n-1} Cz_{ij} \cdot (x_{jpt} + y_{jpts}) & \quad \text{Total cost of power transmission between inter-substation lines for a season and a scenario.} \\
\sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} \cdot (x_{ijt} + y_{ijts}) & \quad \text{Total power loss cost at a substation for a season and a scenario.} \\
\sum_{j=1}^{n} \sum_{k=1}^{n} C_{pk} \cdot d_{ijts} & \quad \text{Total cost of power distribution for a season and a scenario.}
\end{align*}
\]

In addition, equations (2a) and (2b) represent that the total power generation from a power plant during time \( t \) of a season \( s \) is limited by the generator’s capacity. The equation (2a) represents storage power plant; equation (2b) represents the ROR power plants capacity which depends on the fraction of total capacity available during the time \( t \) of the season \( s \). The generated power from the power plant needs to be transmitted as given by equation (3a) and (3b). There is no storage at the power plants. As given in equations (4a) - (4d) power transmitted over transmission lines is limited by the line capacity. Equation (4a) indicates that power transmitted from storage PP to substation is limited by the transmission line capacity connecting the power plants and the substations. There will be power transmission between two substations which is limited by the substation line capacity, and the power transmitted in distribution line is limited by the distribution line capacity as given in equations (4c) and (4d), respectively. The substations transformer limits the total power that can be connected to it as indicated by equation (5). The power that is transmitted to the substation needs to be delivered to the loads without any storage at the substation as given by equation (6). Finally, every demand should be met as indicated in equation (7).

3. Numerical Experiments
Scenario-based approach is used to address the problem of variability in the ROR power generation. Each set of the input data has random scenarios, where the number of scenarios is chosen based on the sample average approximation approach. It is assumed that the data set follows a uniform distribution during each season. Any unmet load due to the decreased generation is assumed to be outsourced to any privately owned energy producers within the proximity; or power is purchased from neighboring state or countries through cross border transmission lines. To discourage the importing energy from external sources under normal operations, a large penalty is imposed on the outsourcing agent. The computational tools used are Gurobi optimizer and Python programming language.

3.1 Model Input Data
The electrical power from HEPP (\( P \) or \( S_i \) in the objective function) is estimated from the flow rate and water head (Department of the Army Washington, 1995). Table 1 lists the power plants capacity obtained from a power utility’s database. It also lists the existing substations and their capacities as well as average hourly demand at each demand center. The line capacity depends on voltage level and line length. The substation capacity depends on the voltage level of incoming and outgoing TL and power carrying capacity of the DL lines as well as the number of customers (energy users or loads) that needs to be served in that region by the substation transformer bank.

<table>
<thead>
<tr>
<th>PP</th>
<th>Installed Capacity in MW</th>
<th>PP Type</th>
<th>Substation</th>
<th>Substation Capacity MVA</th>
<th>Demand Centers</th>
<th>Average Demand MW</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP-1</td>
<td>144</td>
<td>ROR</td>
<td>SS-1</td>
<td>57.5</td>
<td>D-1</td>
<td>25.0</td>
</tr>
<tr>
<td>PP-2</td>
<td>139</td>
<td>ROR</td>
<td>SS-2</td>
<td>142.6</td>
<td>D-2</td>
<td>41.0</td>
</tr>
<tr>
<td>PP-3</td>
<td>60</td>
<td>Storage</td>
<td>SS-3</td>
<td>30.0</td>
<td>D-3</td>
<td>51.5</td>
</tr>
<tr>
<td>PP-4</td>
<td>30</td>
<td>Storage</td>
<td>SS-4</td>
<td>113.4</td>
<td>D-4</td>
<td>36.0</td>
</tr>
<tr>
<td>PP-5</td>
<td>24.8</td>
<td>ROR</td>
<td>SS-5</td>
<td>40.0</td>
<td>D-5</td>
<td>20.0</td>
</tr>
<tr>
<td>PP-6</td>
<td>82</td>
<td>ROR</td>
<td>SS-6</td>
<td>20.0</td>
<td>D-6</td>
<td>90.0</td>
</tr>
<tr>
<td>PP-7</td>
<td>14.1</td>
<td>ROR</td>
<td>SS-7</td>
<td>36.0</td>
<td>D-7</td>
<td>36.0</td>
</tr>
<tr>
<td>PP-8</td>
<td>17.55</td>
<td>ROR</td>
<td>SS-8</td>
<td>45.0</td>
<td>D-8</td>
<td>36.0</td>
</tr>
<tr>
<td>PP-9</td>
<td>60</td>
<td>ROR</td>
<td>SS-9</td>
<td>90.0</td>
<td>D-9</td>
<td>90.0</td>
</tr>
<tr>
<td>PP-10</td>
<td>45</td>
<td>ROR</td>
<td>SS-10</td>
<td>15.0</td>
<td>D-10</td>
<td>45.0</td>
</tr>
<tr>
<td>PP-11</td>
<td>200</td>
<td>Storage</td>
<td>SS-11</td>
<td>91.5</td>
<td>D-11</td>
<td>45.0</td>
</tr>
</tbody>
</table>
Variability is associated with the power generated from the ROR projects. The plant factor of each power plant is plotted in Figure 2. From Figure 2, maximum power generation occurs during months 1, 2, 3, and 4, while minimum power generation is during months 7, 8, 9, and 10. The actual month of minimum or maximum generation from ROR project is not the same for all fiscal years but time is approximately same for each year.

![Figure 2. Plant capacity factors of the ROR projects](image)

The energy generation decision variable is grouped into six intervals. Each interval represents a season during which the power plants generate power at certain percentage of its installed capacity as shown in the Figure 3. It should be noted that the use of ‘season’ in this paper does not mean the conventional seasons winter, summer, or spring. Rather, season is a time interval in a year over which the generation of power changes. The power generation is classified in to six distinct seasons. It is assumed that there are random scenarios of power generation by a power plant during each season, which are represented by a uniform probability distribution.

![Figure 3. Generation seasons in a year](image)

The input values of the costs are computed based on the models presented in (Joshi, 2016); $10 to $70 per MWhr for generation and $1 to $9 per MWhr for transmission and distribution lines based on power plant type which is consistent with the results provided in (DOE, 2020; Kirschen & Strbac, 2018). The planning horizon of 1 year \(T=8760\) hours is used for all the experiments.

4. **Results and Discussion**
The solution approaches performed in this paper are the expectation of expected value solution (EEV), stochastic solution (SS), and wait and see (WS) solutions as described below.

### 4.1 Expected Value Solution (EEV)

The results of the EEV solution for the power generated by each power plant in MW is given in Figure 4. Each bar represents average power generated per hour by the combination of all generators during each season. The power generated by a storage project (say PP-11) increases from season 1 to 6, it is due to decreasing capacity of ROR projects with water flow change and a constant demand that needs to be supplied at all times. This power generated by storage project PP-11 is calculated by taking the average of 5 random scenarios during season 6. The graphs for season-1 through 5 can be interpreted similarly. As the average of random scenarios is taken in EEV solution, the chances of getting inaccurate power generation is higher than using the random scenarios.

![Figure 4. Decision variable values from EEV approach](image)

### 4.2 Stochastic Solution (SS)

This paper approximates the uncertainties of future weather conditions (i.e. power output from ROR plants) using finite scenarios of the random variables. The random scenarios generated for each season have their corresponding probabilities as described in the mathematical model. The number of scenarios and the sample path for each scenario which are used in SS experiment are the same as that of the EEV solution approach. Each bar represents average power generated per hour by the combination of all generators during each season.

The power generated by storage project (say PP-11) increases from season 1 to 6, due to the decreasing capacity of ROR projects with water flow change. This power generated by storage project PP-11 is calculated randomly to best meet the demand which is more practical than first computing 5 different random samples then taking their average to get a random number as in EEV solution. The graphs for season-1 through 5 can be interpreted in the same way.

In Figure 5, each scenario of a season is a random number with some probability of occurrence. This random number represents the maximum capacity of ROR projects. When the ROR projects have low power generation (like in season 6) due to weather change the storage projects supply the rest of the power demand.

### 4.3 Wait and See Solution (WS)

The wait-and-see solution approach computes objective value (i.e. cost of energy supply in this paper) as if power generation by ROR projects is known prior to the complete power generation schedule is obtained. This is achieved
by assuming certain scenario of power generation by ROR plants during each season and then the need for power from the storage plants is computed to meet the demand.

Each bar represents average power generated per hour by the combination of all generators during each season. Five scenarios are selected and presented in Figures (6a) and (6b) to illustrate the use of the WS solution. In Figure (6a), the season 1-1 bar shows the power generated by the ROR projects in scenario 1 of season 1. The power generated by the ROR project during season-1 is realized certainly which equals to the random scenario-1 of season 1. The PSO now has perfect information of how much power generation will be obtained from the ROR projects to make reliable decisions to dispatch storage projects generators accordingly to meet the total demand.

![Figure 5. Power generation plan using SS approach](image)

The cost of energy generation obtained from the WS solution is specific to each scenario which is less than the cost obtained if the random scenarios are used in SS approach.

The per unit energy cost ($/MWhr$) is assumed to be constant for all seasons of the year. For each season, the product of per unit energy generation cost (in $/MWhr$) of a generator and energy generation by that generator (in MWhr) gives the power generation cost of a generator. The generation cost for all the generators is computed in the same way for all seasons and added together. Then, the sum is multiplied with the probability of the season to get the actual energy generation cost during that season. The same process is applied to get the energy transmission cost, power loss cost at substations, and distribution cost. The final sum of all the costs gives the total energy supply cost in period $T$. Table 2 is created to compare our objective values from the solution of mathematical model using the three different algorithms.

As seen in the Table 2, cost of energy supply increases as the river flow rate decreases. It is because of the fact that during some seasons of the year lower flow rate generates less power from cheaper ROR projects and the utility needs to operate expensive power plants at higher capacity in order to meet the demand. In short, by implementing generation schedule presented in this research the utility can reduce its cost by about 4.73% in average.

Expected value of perfect information (EVPI) is the absolute value of the difference between SS and WS i.e. in percentage EVPI would be $\text{EVPI} = (\text{SS-WS})/\text{SS} = 1.00\%$. The significance of EVPI is that it gives information about how much the decision maker should be willing to pay for the perfect information in the future.
Table 2. Objective values from SS, WS, and EEV solutions. Total cost in percentage of actual cost from the utility’s database.

<table>
<thead>
<tr>
<th>Season</th>
<th>SS</th>
<th>WS</th>
<th>EEV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (0.9-1.0)</td>
<td>88.08%</td>
<td>85.84%</td>
<td>88.08%</td>
</tr>
<tr>
<td>2(0.8-0.9)</td>
<td>89.09%</td>
<td>87.11%</td>
<td>89.09%</td>
</tr>
<tr>
<td>3(0.7-0.8)</td>
<td>94.48%</td>
<td>93.77%</td>
<td>94.52%</td>
</tr>
<tr>
<td>4(0.6-0.7)</td>
<td>96.93%</td>
<td>96.59%</td>
<td>96.94%</td>
</tr>
<tr>
<td>5(0.5-0.6)</td>
<td>101.64%</td>
<td>101.63%</td>
<td>101.65%</td>
</tr>
<tr>
<td>6(0.4-0.5)</td>
<td>101.37%</td>
<td>101.23%</td>
<td>101.37%</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td>95.27%</td>
<td>94.36%</td>
<td>95.28%</td>
</tr>
</tbody>
</table>

Value of stochastic solution (VSS)- is the difference in the objective function between the solutions of EEV and SS i.e. in percentage VSS would be VSS = (EV – SS)/EV = 0.011%. The significance of VSS is that it gives the possible cost reduction by considering all possible random scenarios.

5. Conclusion
The mathematical model developed in this paper provides a solution for power generation, transmission, and distribution over one-year period by the run-of-river and storage projects, and comparisons of the SS, EEV, and WS were made. The results from these solution approaches provide the economical dispatch of the ROR and storage power plants and optimal distribution plan to meet the power demand. The WS solution approach provides the least cost plan. The EEV solution is worse than the SS. The PSOs may invest in advanced technologies to reveal the uncertainties more accurately in the planning process, to operate at the WS operational cost. However, the trade-offs between using the SS and investing in new technologies to operate at WS solution may require rigorous new technology investment feasibility study.

The PSOs would interpret the results from these solution techniques as follows. The months in a year when the power plants can generate power from 90% to 100% of their installed capacity were grouped into the season-1 and so on. The results of the decision variables showed that the power generated by a ROR plant during season-1 is more than the power generated by the same ROR plant in other seasons. This means that the storage PP should be operating at minimum capacity and store some water during season-1 so that the stored water could be used to run the plants at
increased capacity during other seasons. If the local generation is not sufficient to meet the energy demand, then power should be purchased from out of state as described in Wollega et al. (2016).

References


Biographies

Mr. Govind Joshi is a doctoral candidate in Electrical Engineering department at Colorado School of Mines. He graduated with MS in Industrial and Systems Engineering from Colorado State University Pueblo in 2016. His interest areas of research include optimization of water and energy nexus. Mr. Joshi currently also works as a distributed energy resources engineer for Avangrid utilities in the east-coast United States.

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