# Robust Appointment Scheduling for Random Service Time Using Min-Max Optimization

**Tasmia Jannat Tumpa and Ahmed Azab**
Department of Mechanical, Automotive and Materials Engineering
University of Windsor
401 Sunset Ave, Windsor, ON N9B 3P4, Canada.
tumpa@uwindsor.ca azab@uwindsor.ca

**Mohammed Fazle Baki**
Odette School of Business
University of Windsor
401 Sunset Ave, Windsor, ON N9B 3P4, Canada.
fbaki@uwindsor.ca

## Abstract

Appointment Scheduling is an increasingly challenging problem for service-centers, healthcare, production and transportation sector. Challenges include meeting growing demand and high expectation of service level among the customers and ensuring an efficient service system which reduces the expenditure related to idle times and underutilization of the system. The problem becomes more complicated in the presence of processing time uncertainty, varying from customer to customer. In this regard, Appointment Scheduling can improve the system efficiency drastically and significant savings can be achieved. Specially, in developed countries where expenditures are huge for service centres, a good appointment schedule is crucial.

In this study, a Robust Appointment Scheduling model is developed using Min-max Optimization to provide appointment dates for jobs which will be processed sequentially with a highly utilized single processor. When a job finishes earlier than the next job's appointment date, the processor and the associated recourses remain idle until the next job's appointment time resulting in underutilization of the system. Whereas if a job takes more time than assigned, the system experiences overage cost due to the waiting of the next jobs. Therefore, an optimal appointment schedule is required considering the trade-off between the underutilization and overtime of both the processor and the jobs. Our objective is to minimize the worst cost of the appointment schedule by minimizing earliness and tardiness of both the jobs and the processor for any realization of the processing durations.

The proposed methodology requires less information regarding the uncertain parameters and can provide optimal solution while only considering the extreme values of the uncertain processing durations. Hence, scenario realization and computational effort for the robust model have reduced to a great extent without loss of optimality compared to other modelling approaches. The robust model does not require to assign probabilities to the future uncertain instances and is therefore applicable to any probability distribution of the uncertain parameters.

At first a Non-linear Robust Appointment Scheduling model is developed. To remove the nonlinearity, a Mixed Integer Linear Programming model is proposed. Since the proposed Robust Appointment Scheduling model is NP- hard, an Iterative Search Procedure is provided for solving the lager instances of the problem in polynomial time. The objective of the Iterative Search Procedure is to selectively choose the scenarios that incur worst cost and then optimizing among those scenarios to provide an appointment schedule that will work well for all other scenarios. For finding the worst case scenario, a Dynamic Programming model is proposed which allows to find the worst case scenario among all the scenarios in $O(n^2)$ time. Propositions that support the robust model are provided along with their theoretical proofs. Appointment scheduling of two case studies, a Dentist's clinic and VIA Rail Canada, are performed. Both case studies exhibit high performance of the proposed robust model in terms of cost savings and computational efforts. This work will contribute both to the literature related to uncertainty handling in decision making and to the industries, which aim to achieve an efficient service system.

## Keywords

Robust Appointment Schedule, Min-max Optimization, Dynamic Programming, Iterative Search Procedure

## Biographies

**Tasmia Jannat Tumpa** is a research assistant at the Production & Operations Management Research Lab, University of Windsor, Canada. She earned BSc in Industrial Engineering, Bangladesh University of Engineering and Technology (BUET), Bangladesh in 2017 and MSc in Industrial Engineering, University of Windsor, Canada in 2020. She has published journal papers and conference papers on supply chain management, healthcare, sustainable manufacturing and risk management. The journals where she has publications include Journal of Cleaner Production, Measurement and Journal of Retailing and Consumer Services. Her research interests includes operations research, operations management, non-traditional optimization, mathematical programming, sustainability, manufacturing, healthcare, data science and so on. She is the recipient of several national academic scholarships. She is the member of Ontario Society of Professional Engineers, Canada.

**Ahmed Azab** is Director of the Production & Operations Management Research Lab at the University of Windsor and Associate Professor at the Department of Mechanical, Automotive, and Materials Engineering, University of Windsor, Canada. He earned BSc in Machine Design and Production Engineering, Cairo University, Egypt, MSc in Mechanical Engineering, Cairo University, Egypt and PhD in Industrial & Manufacturing Systems Engineering, University of Windsor, Canada. He has been recipient and nominee for international and national research excellence awards. His research has been sponsored by National and Provincial granting agencies, which include NSERC, CFI, OCE, MITACS, FEDEV, as well as direct research funds provided by the local industry. He is one of three faculty members to receive a $1M CFI-LOF/MEDI-ORF grant supporting research infrastructure. He is partnering with national as well as international research collaborators from various research labs and academic institutions in Europe, the Middle East, and USA. He serves as a reviewer for a number of reputable international journals and national granting agencies; he also sits on the editorial board for a few international journals. His research interests include manufacturing, simulation, production planning and scheduling, mathematical programming and non-traditional optimization, algorithms/systems analysis, design and construction.

**Mohammed Fazle Baki** is Associate Dean and Professor in the Odette School of Business, University of Windsor, Canada. He graduated in Civil Engineering from Rajshahi University of Engineering and Technology (RUET), Bangladesh in 1987. He received MBA degrees from the University of Dhaka, Bangladesh in 1991 and the University of New Brunswick, Canada in 1995. He received a Ph.D. degree in Operations Management from the University of Waterloo, Canada in 1999. He teaches operations management, operations research, statistics, and quality control at the Odette School of Business. His research interest lies in the development and application of quantitative methods in business and industrial engineering. He is particularly interested in the combinatorial problems that arise in manufacturing, supply chain management, and healthcare management. He has published journal and conference papers on manufacturing, scheduling, healthcare, inventory management, traveling salesman problem and received awards for excellence in research. He has received research funds from Natural Sciences and Engineering Research Council (NSERC).