An Inventory Model for Growing Items with Quality Inspections and Permissible Shortages

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Abstract

This paper extends inventory models for growing items by considering quality inspections, permissible shortages with complete backordering, and holding cost during both the growth period and the consumption period. By backordering shortages, the firm can avoid the loss of sales by paying delay penalties to customers who wait for late items. After items reach certain weight preferred by the customers, they are fully inspected, and inferior-quality items are removed at the end of inspection period. The model determines the optimum cycle length and shortage level to minimize the total cost of the inventory system. This cost includes the purchasing, setup, inspection, feeding, holding, and shortage costs. A nonlinear programming model is formulated and an optimum solution algorithm is presented.

Keywords:

Inventory model, Optimization, Growing items, Permissible shortage, Quality inspection.

1. Introduction

The EOQ model is based on simplifying assumptions that are generally unrealistic, such as constant demand, constant costs, perfect quality, no shortages, and non-changing items. Many of these assumptions have been modified in order to represent real-life applications and various inventory models have been developed. Examples include inventory models with price-dependent demand (Mahata and De, 2016), variable holding cost (Alfares and Ghaithan, 2019), growing items (Nobil et al., 2019), and imperfect quality (Alfares and Attia, 2017). Inventory control of growing items are used for the cases in which the stored items can grow during the inventory replenishment cycle such as livestock, poultry and fish.

The paper proposes an EOQ inventory model of growing items, considering imperfect quality, permissible shortages & holding cost during both growth and consumption. Inspections are done to separate lower-quality items, which are disposed of as soon as the inspection period ends. Shortages are completely backordered, i.e. all unsatisfied demands are delivered late to affected customers. A nonlinear programming model is formulated and an effective algorithm is developed to determine the optimal cycle-length and shortage level. The objective is to minimize the total inventory system cost, including the costs of purchasing, setup, inspection, feeding, holding, and shortage.

Remaining sections are structured as follow. A brief survey of prior literature is provided in section 2. The proposed inventory problem is defined in section 3. The mathematical formulation for proposed inventory model is presented in section 4. An optimum solution algorithm is proposed in Section 5. The paper is then concluded in section 6.

2. Literature Review

This survey of previous literature focuses on recent inventory models for growing items, especially those models that include permissible shortages and quality aspects.

Hwang (1997, 1999) introduced the first inventory model on items that ameliorate (grow), meaning their values increase with time, e.g. livestock, poultry, and farm fish, etc. Rezaei (2014) introduced feeding costs in an EOQ/EPQ model for items that grow during inventory, aiming to maximize the total profit by determining the optimal order quantity and cycle length. Mahata and De (2016) derived an inventory model for ameliorating items taking into consideration price-dependent demand and partial trade credit, and proposed an algorithm to obtain the optimal pricing and inventory decisions. Zhang et al. (2016) extended Rezaei's (2014) model by considering an inventory system for growing items under environmental carbon constraints.

Nobil et al. (2018) extended Rezaei's model by relaxing the no-shortage assumption while approximating the item's growth using a linear function. Gharaei and Almehdawe (2019) developed a stochastic Economic Growing Quantity (EGQ) model considering the probability distributions of survival and mortality of the items. Sebatjane and Adetunji (2019) extended the model of Rezaei (2014) by considering both imperfect quality and growing items. Mondal et al. (2019) used a particle swarm optimization-based technique to solve crisp and fuzzy inventory models of ameliorating items, where amelioration and deterioration follow the Weibull distribution.

Malekitabar at al. (2019) proposed a supply chain inventory model consisting of one supplier and one retailer (farmer) incorporating item amelioration and deterioration. The model is applied to a case study for rainbow trout inventory control, where the objective is to maximize the profits of the supplier first and the retailer's second. Rai (2020) presented an inventory model for ameliorating/deteriorating items considering a supplier-manufacturer-retailer supply chain, where the supplier supplies newborn items to the manufacturer, who in turn grows them and then sells them to the retailer with trade credit opportunity. Shortages are permissible for the retailer and they are partially backlogged.

Next, an EOQ-type model for growing items is presented with three features that have not been combined in prior literature: permissible shortages, quality inspections, and holding cost in both the growth and the consumption periods.

3. Problem description

Figure 1 depicts the inventory level throughout a full cycle of duration T. When the growing period begins, the company orders y number of newborn items. At this point, each newborn item weighs w_0, so the total inventory weight is $Q_0 = yw_0$. During the growth period, items grow linearly until they reach a certain preferred weight w_1 at time t_1, hence the total inventory weight becomes $Q_1 = yw_1$. At time t_1, fully grown items are slaughtered and then they undergo an inspection process for quality assurance. A certain proportion of slaughtered items x, with probability distribution $f(x)$ and expected value E[x], is assumed not conforming to the quality standards.

When the inspection period t_2 ends, non-conforming items are grouped in a single batch and removed from inventory. At the same time, the shortage quantity from the previous cycle, B, is backordered, i.e. delivered late to the customers. The inventory level depletes during the consumption period according to the demand rate D until it reaches zero. Once the inventory is fully consumed, the remaining demands of the customers are backordered during shortage period t_3. As soon as period t_3 ends, the whole replenishment cycle ends, and the total shortage amount becomes equal to B.

Consecutive replenishment cycles overlap as illustrated in Figure 1. Particularly, newborn items purchased for the next cycle arrive t_1 time periods before the end of the current cycle. Hence, as soon as current cycle ends, the newborn items for the next inventory period have reached the required weight w_1 and are ready for consumption. The objective of the model is to minimize the total expected inventory cost to the company by optimizing order quantity y, the shortage level B, and the cycle time T. For this purpose, an optimization mathematical model is proposed and formulated in the following section.

4. Model Development

4.1. Cycle time and order size

As shown in Figure 1, the growth of newborn items is approximated by a linear function. Hence, the individual weight of newborn items increases from w_0 to w_1 at a constant rate of λ weight units per time unit, thus: $w_1 = w_0 + \lambda t$. Therefore, the growth period duration t_1 is calculated as:

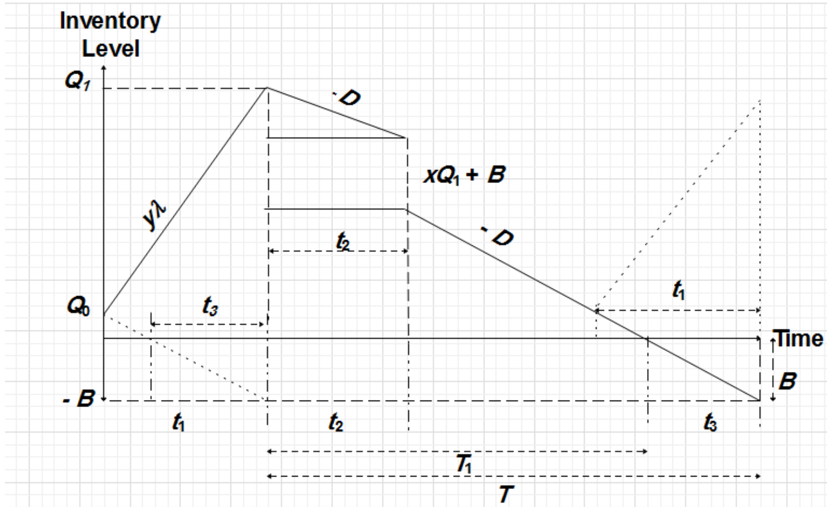

Figure 1. Growing items inventory with quality inspection and backordering

$$t_1 = \frac{w_1 - w_0}{\lambda} \tag{1}$$

Slaughtered items undergo an inspection process prior to being sold to the consumers. The order size, i.e. number of newborn items ordered per cycle, is equal to y, and hence the total weight of the slaughtered items is yw_1. Given the inspection rate r, the inspection period t_2 is determined as follows:

$$t_2 = \frac{yw_1}{r} \tag{2}$$

Given the shortage quantity in units per cycle B, and the demand rate for good quality items in units of weight per time unit D, the shortage period t_3 is computed as follows:

$$t_3 = \frac{B}{D} \tag{3}$$

At the beginning of consumption period T_1, the total inventory weight is yw_1. After disposing of poor-quality items in one batch, and satisfying the previous cycle shortage amount B, the net inventory level becomes $yw_1(1 - x) - B$. Since this amount is consumed at a demand rate D during T_1, the expected consumption period is then calculated by:

$$\mathrm{E}[T_1] = \frac{yw_1(1 - \mathrm{E}[x]) - B}{D} \tag{4}$$

By adding the consumption period T_1 and the shortage period t_3, the total expected cycle length $\mathrm{E}[T]$ is given as:

$$E[T] = \frac{yw_1(1 - \mathrm{E}[x]) - B}{D} + \frac{B}{D} = \frac{yw_1(1 - \mathrm{E}[x])}{D} \tag{5}$$

From Equation (5), the number of ordered newborn items is obtained as follow:

$$y = \frac{E[T]D}{w_1(1 - \mathrm{E}[x])} \tag{6}$$

4.2. Cost components

As each new cycle begins, y number of newborn items are purchased by the company at a cost of p per unit weight. Each of these newborn items weighs w_0, hence the purchasing cost per cycle is calculated as:

$$PC = pyw0 = \frac{pw_0 E[T]D}{w_1(1 - E[x])} \tag{7}$$

A fixed cost K occurs at the beginning of each cycle. Thus the setup cost per cycle is simply:

$$SC = K \tag{8}$$

Purchased items are fed throughout the growth period t_1 at a cost of c per unit weight per time unit. Food consumption by each item depends on its weight, which is approximated by linear growth function. Therefore, the total feeding cost FC is equal to c times the area under the curve in interval t_1, which is $t_1(Q_0 + Q_1)/2$. Using (1) and (6) gives:

$$FC = \frac{c(Q_0 + Q_1)(Q_1 - Q_0)}{2\lambda y} = \frac{cy(w_1^2 - w_0^2)}{2\lambda} = \frac{cE[T]D(w_1^2 - w_0^2)}{2\lambda w_1(1 - \mathrm{E}[x])} \tag{9}$$

An inspection process is carried out during period t_2. It costs the company z per unit weight of slaughtered items in order to inspect and separate the poor-quality items from the good ones. Hence, the total inspection cost IC is equal to zyw_1. Using equation (6), IC can be written as:

$$IC = \frac{zE[T]D}{(1 - \mathrm{E}[x])} \tag{10}$$

Inventory shortage occurs in period t_3. It costs company f per unit weight per time unit for the shortage. By the end of period t_3, the shortage reaches its maximum level B and it continues till the end of period t_2 of the next cycle. Hence, the backordering (shortage) cost is incurred in two periods: during period t_3 of the current cycle and during period t_2 of the next cycle. The shortage cost per cycle is thus obtained as:

$$BC = f\left(\frac{Bt_3}{2} + Bt_2\right) = f\left(\frac{B^2}{2D} + \frac{Byw_1}{r}\right) = f\left(\frac{B^2}{2D} + \frac{BE[T]D}{r(1 - \mathrm{E}[x])}\right) \tag{11}$$

The holding cost has two components: for live items in period t_1, and for slaughtered items in period T_1. In the growth period t_1, the unit holding cost of live items is h per weight unit per time unit, and the total holding cost is given by:

$$hC = \frac{hE[T]D(w_1^2 - w_0^2)}{2\lambda w_1(1 - \mathrm{E}[x])} \tag{12}$$

The unit holding cost associated with storing the slaughtered items is H per weight unit per time unit, and it is incurred during the consumption period T_1. From Figure 1, the total holding cost in period T_1 is given by:

$$E(HC) = H\left[\frac{(yw_1 - E[x]yw_1 - B)T_1}{2} + (E[x]yw_1 + B)t_2\right]$$

or

$$E[HC] = H\left[\frac{(E[T]D - B)^2}{2D} + \frac{E[T]D\,\{E(x)E[T]D + B(1 - E[x])\}}{r(1 - E[x])^2}\right] \tag{13}$$

By adding up the cost components from equation (7) to equation (13), the expected total cost per cycle is obtained as follows:

$$E[TC] = \frac{pw_0E[T]D}{w_1(1-\mathrm{E}[x])} + K + \frac{(c+h)E[T]D(w_1^2-w_0^2)}{2\lambda w_1(1-\mathrm{E}[x])} + \frac{zE[T]D}{1-\mathrm{E}[x]} + \frac{B^2(f+H)}{2D} + \frac{fBE[T]D}{r(1-\mathrm{E}[x])} + H\left[\frac{E[T]^2D}{2} - E[T]B + \frac{E[T]D\{E(x)E[T]D + B(1-E[x])\}}{r(1-E[x])^2}\right] \tag{14}$$

4.3. Nonlinear programming model

The proposed inventory model aims to minimize the expected total cost per time unit E[TCU] where E[TCU] is computed by dividing E[TC] by the expected cycle time T. Hence, the objective function of the model is given as:

$$Minimize\ \ \mathrm{E}[TCU] = \frac{\mathrm{E}[TC]}{\mathrm{E}[T]} = \frac{pw_0D}{w_1(1-\mathrm{E}[x])} + \frac{K}{\mathrm{E}[T]} + \frac{(c+h)D(w_1^2-w_0^2)}{2\lambda w_1(1-\mathrm{E}[x])} + \frac{zD}{1-\mathrm{E}[x]} + \frac{B^2(f+H)}{2\mathrm{E}[T]D} + \frac{fBD}{r(1-\mathrm{E}[x])} + H\left[\frac{\mathrm{E}[T]D}{2} - B + \frac{D\{\mathrm{E}(x)\mathrm{E}[T]D + B(1-\mathrm{E}[x])\}}{r(1-\mathrm{E}[x])^2}\right] \tag{15}$$

To guarantee the feasibility of proposed model, the above objective function is subjected to two constraints. The first constraint guarantees that all slaughtered items are available for consumption at the required time, which is at the beginning of next cycle. Moreover, to prepare the place according to proper hygienic conditions, a setup time of t_s time units is used before the start of each inventory cycle. Hence, the following constraint must be satisfied:

$$\mathrm{E}[T] \geq T_{min} = t_s + t_1 \tag{16}$$

Similarly, the second constraint avoids shortages during the inspection period t_2. The weight of all slaughtered items is equal to $Q_1 = yw_1$ at the beginning of each cycle. After excluding non-conforming items, the remaining good quality items $yw_1(1 - \mathrm{E}[x])$ should be enough to meet the demand during period t_2, which is Dt_2. Hence, we must satisfy:

$$yw_1(1-\mathrm{E}[x]) \geq Dt_2 = \frac{Dyw_1}{r}$$

or

$$x_{max} = 1 - \frac{D}{r} \geq \mathrm{E}[x] \tag{17}$$

There are decision variables for the nonlinear programming (NLP) model specified by equations (15)-(17), which are B and E[T]. Both variables must be two non-negative.

5. Solution procedure

5.1 Calculating the decision variables

To solve the above NLP model, we need to find the optimum expected cycle time $E[T]$ that minimizes the expected cost E[TCU] in equation (15). Setting the partial derivative of E[TCU] with respect to E[T] equal to zero, we obtain:

$$\mathrm{E}[T] = \sqrt{\frac{r(1-\mathrm{E}[x])^2[2KD + B^2(f+H)]}{HD^2r(1-\mathrm{E}[x])^2 + 2HD^3\mathrm{E}[x]}} \tag{18}$$

Similarly, setting the partial derivative of E[TCU] with respect to B equal to zero, the optimum value of B is found as:

$$B = \frac{\mathrm{E}[T]D\ \{Hr(1-\mathrm{E}[x]) - D(f+H)\}}{r(f+H)(1-\mathrm{E}[x])} \tag{19}$$

The following condition must be satisfied in order to get a non-negative value for B from equation (19):

$$Hr(1-\mathrm{E}[x]) \ \geq\ D(f+H) \tag{20}$$

If the above condition (20) is not satisfied from the given data values, then the shortage level $B = 0$ and equation (18) becomes:

$$\mathrm{E}[T] = \sqrt{\frac{2Kr(1-\mathrm{E}[x])^2}{HDr(1-\mathrm{E}[x])^2 + 2HD^2\mathrm{E}[x]}} \tag{21}$$

If condition (20) is satisfied, then both B and E[T] must be calculated. Using the optimum value of B from (19) and substituting it into equation (18) and going through several steps of simplification, the optimum E[T] is obtained:

$$\mathrm{E}[T] = \sqrt{\frac{2KDr^2(f+H)(1-\mathrm{E}[x])^2}{r(f+H)\{HD^2r(1-\mathrm{E}[x])^2 + 2HD^3\mathrm{E}[x]\} - D^2\{Hr(1-\mathrm{E}[x]) - D(f+H)\}^2}} \tag{22}$$

The following condition must be satisfied in order to obtain a real and a finite value for E[T] from above equation (22).

$$r(f+H)\,\{Hr(1-\mathrm{E}[x])^2 + 2HD\mathrm{E}[x]\} \ >\ \{Hr(1-\mathrm{E}[x]) - D(f+H)\}^2$$

Simplifying gives:

$$r^2Hf(1-\mathrm{E}[x])^2 > D(f+H)(Df + DH - 2rH) \tag{23}$$

To examine the relationship between conditions (20) and (23), let us assume that condition (20) holds. Since (1 – E[x]) < 1, then $Hr > Df + DH$, thus $2Hr > Df + DH$, and $(Df + DH - 2Hr) < 0$. Thus, if (20) holds, then the RHS of (23) is negative and (23) also holds. However, if (20) does not hold, then (23) may or may not hold. Consequently, it is sufficient to check condition (20) only. If (20) holds then (23) holds and we can use equations (19) and (22). If (20) does not hold, then both (19) and (22) are not applicable, as B is set equal to zero and (21) is used to find E[T].

5.2. Solution Algorithm

In order to obtain the optimum values for the expected cycle E[T] and the shortage level B for the proposed model, the following algorithm is proposed:

Step 1. Calculate E[x] and x_{max}.
If E[x] ≤ $x_{max,}$ the problem is feasible, and hence continue to step 2. Otherwise stop, the problem is infeasible.

Step 2. Calculate t_1 and T_{min} using equations (1) and (16).

Step 3. Calculate the two sides of (20) and check if this condition is satisfied.
If condition (20) is satisfied, then apply step 3(a), otherwise apply step 3(b).

3(a). Use equation (22) to calculate E[T].
If E[T] < T_{min}, set E[T] = Tmin.
Substitute the value of E[T] in (19) to calculate B.

3(b). Use (21) to calculate E[T].
If E[T] < $T_{\min}$, set E[T] = Tmin.
Set B = 0.

Step 4: Substitute the value of E[T] in equation (6) to calculate y.
Substitute the values of E[T] and B into equation (15) to calculate E[TCU].

6. Conclusions

This paper has presented an EOQ inventory model for growing items that considers imperfect quality, permissible shortages, and holding cost during both the growth and the consumption periods. A certain portion of fully-grown items is of lower quality and must be disposed of after inspection. Shortages are permitted and are completely backordered. Holding costs for both live items during the growth period and slaughtered items during the consumption period are considered in the model. The aim is to minimize the total costs of the inventory system, including purchasing, setup, feeding, inspection, shortage, and holding costs. A nonlinear programming model has been formulated to represent the inventory system, and an optimum solution algorithm has been developed.

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