The Shapley Value for Partially Defined Cooperative Games

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Abstract

In the classical approach to cooperative games, the worth of every coalition is assumed to be known. However, in real applications there may be situations in which the worth of some coalitions is unknown. The corresponding games are called partially defined games.

Partially defined cooperative games were first studied by Willson (1993). However, this author restricted attention to partially defined games in which if the worth of a particular coalition is known, then it is also known the worth of all the coalitions with the same cardinality. Moreover, Wilson (1993) proposed and characterized an extension of the Shapley value for partially defined games, which coincides with the ordinary Shapley value of a complete game. In this complete game coalitions whose worth were unknown are assigned a worth zero, that seems to be not well justified.

In this work we propose another extension of the Shapley value for general partially defined games by following the Harsanyi's approach by using dividends. That is, it is assumed that each coalition guarantees certain payments, called the Harsanyi dividends (Harsanyi, 1963), to its members. We assume that coalitions whose worth is not known assign a dividend equal to zero. The final payoff will be the sum of these dividends. We also study properties satisfied by this new value. And we characterize the proposed value using four axioms. Three of them are the well known axioms of carrier, additivity and positivity. The fourth one, called indispensable coalition axiom, is a weaker version of the anonymity axiom when referring to full defined games.

Further, we restrict attention to simple partially games in order to measure power with our new value when not all the possible coalitions are feasible. We study properties satisfied by this new power index.

Biography / Biographies

M. Josune Albizuri is Full Professor at the Faculty of Economics and Business of the University of the Basque Country, PhD in Mathematics with a thesis on Game Theory and head of the Department of Applied Economics IV. She has published a large number of articles indexed in the Journal Citation Report. She has collaborated with researchers from the Rovira and Virgil University and the University of Vigo in Spain, the School of Economics and Management from Tilburg University in Netherlands, the University of Southern Denmark, Maastricht University and Daito Bunka University in Japan. She has presented communications in international congresses, and seminars or invited presentations in institutions of Spain (Faculty of Economics of the University of Vigo, Institute of Mathematics of the University of Seville, Pablo de Olavide University in Seville, and Polytechnic University of Catalonia), and in international centers (Faculty of Economics and Business Administration in Tilburg, Tinbergen Institute Seminar: Institutions and Decision Analysis of Amsterdam, and University of Southern Denmark). She has been research member and main researcher in national projects obtained in competitive calls. She made research stays, among others, at Tilburg University (School of Economics and Management) and at the London School of Economics.

José M. Zarzuelo is Full Professor at the University of the Basque Country, PhD in Economics and head of the Department of Applied Economics IV for three periods. He has been Hans Christian Andersen Visiting Professor at Southern University of Denmark. He is co-author, in articles from the Journal Citation Report, of more than 30 researchers from various Spanish and international Universities: the Hebrew University in Jerusalem, Tilburg University in Netherlands, the University of Southern Denmark, Maastricht University, Daito Bunka University in Japan, etc. He gave invited presentations in all the former international institutions, in Paris 1 Panthéon Sorbonne University, the Free University of Amsterdam and in institutions of Spain (Faculty of Economics of the University of Vigo, the University of Seville, Pablo de Olavide University in Seville, School of Law and Social Sciences of the University of Carlos III in Madrid, Miguel Hernandez University in Alicante). He has been a member of the scientific committee and organizer of several national and international congresses, including the First World Congress of the Game Theory Society. He has been main researcher in a great number of projects and made numerous research stays. He is a Charter Member of the Game Theory Society.

Satoshi Masuya is an Associate Professor in the Department of Business Management at Daito Bunka University, Tokyo, Japan. He earned a Master in Management Engineering from Kansai University, Japan and PhD in Engineering Science from Osaka University, Japan. He has published articles indexed in the Journal Citation Report and conference papers. His research interest is Game Theory, in particular, cooperative game theory. He has made presentations at international conferences and has also organized international research meetings. He is a member of Game Theory Society.