Model of Volume of Transport Waste and Its Derivative Problems

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Abstract

Mathematics is a tool that can be used to solve everyday problems. One such problem is garbage (waste). The problem of determining the time when all waste can be transported is a mathematical modeling problem that can be modeled by arithmetic sequences. The purpose of this study is to obtain a mathematical model that can be used to determine the time such that waste can be transported entirely to the Final Disposal Site. In this modeling two types of data are used, namely data on the volume of waste and the volume of garbage transported in Purwokerto City from 2008 to 2017. The obtained model was stated to be very good because the volume of waste model and the volume of transported waste model had MAPE below 10%, respectively of 2.41% and 4.82%. With arithmetic sequences, it is predicted that in 2037 all rubbish in Purwokerto City can be transported to the Final Disposal Site. In addition, from the modeling process, three mathematical problems were obtained namely determining the time of occurrence of the break event point, modeling with averages of the differences, and analysis of the differences in arithmetic sequences.

Keywords:
Arithmetic sequences, break event point, mathematical problem, waste transported, waste volume.

1. Introduction

Mathematical modeling can be started from real problems, for example mathematical modeling to predict the relationship between the volume of waste produced and the carrying capacity of waste. Waste has become a problem in almost all cities, including regency and sub-district cities. The volume of waste produced by city residents will become a problem if there is no Final Disposal Site (FDS).

In generale, in the large and medium cities, the volume of waste generated is greater than the carrying capacity of waste. As a result, every day there is always residual waste that cannot be transported. The impact of not transporting all the waste makes the city slums and unhealthy.

Berm´Udez, Martin´On, and Noda (2014) collected several results about the development of high-order arithmetic called polynomial sequences and using arithmetic sequences on commutative groups. Hajdu (2017) investigates the existence of arithmetic sequences contained in Lucas sequences. Chinchodkar and Jadhav (2017) resenting effective transportation methods using transportation problems. The method offered previously is to compare transportation costs with daily transportation costs. Stecke (2005) explaining some industrial problems can be solved using a relatively simple mathematical model. Various types of industrial problems are discussed. Several models for overcoming industrial problems, such as inventory models, linear programming, network flow, decision analysis, queuing models, and simulations are discussed. Shaikh, Hussain, and Baig (2019) examined the transportation of solid waste to FDS locations. The reprocessing plant and the level of gas production at the disposal site are more supportive of living the concept. An environmental pollution program such as air pollution or noise control is added in this model.

Some reasons for the failure to transport all waste include (1) availability of waste transport vehicles to the landfill, and (2) the limited area of landfill available or remaining. If the available landfill area can still accommodate municipal trash, the lack of garbage transport vehicles can cause not all of the waste can be disposed of to the FDS.

From the above problems, it will be predicted that the time until all the rubbish residents of Purwokerto city can be disposed of in FDS. The data used is the volume of waste in one year, and the carrying capacity of vehicles each year. The prediction is done by modeling the volume of waste and the carrying capacity of waste to grow following the arithmetic sequence. By eliminating the real problem, a mathematical theory is obtained that can be used to solve problems that have characteristics similar to the problem of the volume of waste and the carrying capacity of waste. Furthermore, from mathematical modeling of the volume of waste and the carrying capacity of waste will be examined several mathematical problems that can be obtained.

2. Research Methodology

This study uses secondary data obtained from the Central Statistics Bureau of Banyumas Regency and the Office of the Environment of Banyumas Regency. The data is in the form of (1) the volume of waste and (2) the volume of waste transported in Purwokerto City from 2008 to 2017. Following are the research steps:

Stage 1: Arithmetic Progression
In the first step, we briefly review the Arithmetic progression or Arithmetic sequence. This sequence will be used as a tool to model the volume of waste and transported waste.

Stage 2: Modeling the Volume of Waste and Transported Waste
Based on the characteristics of the two types of data, the mathematical model that can be used to obtain predictions of waste volume and transported volume is Arithmetic progression.

Stage 3: Goodness of Fit Test
To obtain accuracy between prediction results and real data values, the mean absolute percentage error (MAPE) can be used which is the average of the overall percentage of error between the actual data and the predicted data. This error value is useful for evaluating the accuracy of predictions. The mean absolute percentage error formula is given in Equation (1)

$$\text{MAPE} = \frac{1}{n}\sum_{j=1}^{n}\left|\frac{X_j - F_j}{X_j}\right| \times 100\% \tag{1}$$

where X_j and F_j successively declare data values and data predictions $j-$th. The model is called valid if the MAPE value is less than 0.10.

Stage 4: The Time When All Waste Is Transported (Break Event Point)
Determine the time when the volume of waste and the volume of transport are the same size. Furthermore, from the data used it can be seen the time when the city of Purwokerto was free of waste, because all municipal trash can be transported to FDS. The timing is done by tabulation. In the next stage the new mathematical model is derived.

Stage 5: Break Event Point Mathematical Model with Arithmetic Progression
In this section, the time is modeled when all waste can be transported to FDS. Modeling is done using Arithmetic progression.

Stage 6: Mathematical Problems Derived from the Waste Volume Problem
From the real problem in the form of modeling the volume of waste and transported waste using Arithmetic progression, a mathematical problem is produced that is free from real phenomena. The solution obtained can be applied in many cases, not just a waste problem.

3. Discussion Results

This section will explain Arithmetic progression, mean absolute percentage error (MAPE), modeling the volume of waste and the volume of transported waste using Arithmetic progression, predicting the volume of waste and the volume of waste transported in subsequent years, determining the time when the volume of waste can be completely transported, the formulation of mathematical problems obtained from the modeling of the volume of waste and the volume of waste transported, and the derivative mathematical problems.

3.1 Arithmetic Progression

Arithmetic sequence is a sequence of numbers whose values are obtained from the previous tribe through the addition or subtraction with the difference or the difference between two consecutive terms. The difference or difference between two successive terms is denoted by b and look for using Equation (2):

$$b = U_n - U_{n-1} \quad (2)$$

where U_n declare term n -th from Arithmetic progression, and $n = 1,2,3,....$

Value b in Equation (1) can be positive or negative. If b positive, then the next term is obtained by addition b in the previous term. In this case it applies $U_n > U_{n-1}$ for every $n = 1,2,3,....$ If b negative, then the next term is obtained by subtraction b in the previous term. In this case it applies $U_n < U_{n-1}$ for every $n = 1,2,3,....$

Next, the term n -th in arithmetic sequence can be obtained from Equation (3)

$$U_n = a + (n-1)\cdot b \quad (3)$$

where $a = U_1$ states the first term of Arithmetic progression.

In addition to using Equation (2), finding differences can be done using two distant terms given in Equation (4)

$$b = \frac{U_n - U_m}{n - m} \quad (4)$$

For example given Arithmetic progression as follows: 11, 18, 25, 32, 39, 46, 53, 60, 67, …. The difference between the term on this progression is $b = 18 - 11 = 25 - 18 = ... = 7$. If Equation (4) is used, the difference from the sequence is

$$b = \frac{U_5 - U_1}{5-1} = \frac{39-11}{4} = \frac{28}{4} = 7$$

or

$$b = \frac{U_9 - U_3}{9-3} = \frac{67-25}{6} = \frac{42}{6} = 7$$

Proof for Equation (4) is as follows. From Equation (3) obtained

$$U_m = a + (m-1)\cdot b$$
$$U_n = a + (n-1)\cdot b$$

where m and n is an integer. Furthermore,

$$U_n - U_m = a + (n-1)\cdot b - a + (m-1)\cdot b$$
$$U_n - U_m = (a + nb - b) - (a + mb - b)$$
$$U_n - U_m = nb - mb$$
$$b = \frac{U_n - U_m}{n - m}$$

3.2 Modeling the Volume of Waste and Transported Waste

Table 1 presents data on the volume of waste and the volume of waste transported in Purwokerto City from 2008 to 2017.

Table 1. Volume of Waste and Transported Waste in the City of Purwokerto from 2008 to 2017

Year	Volume of Waste (m^3)	Waste Volume Difference (m^3)	Volume of Waste Transported (m^3)	Volume of Waste Transported Difference (m^3)
(1)	(2)	(3)	(4)	(5)
2008	630.82	-	315.41	-
2009	631.21	0.39	315.60	0.19
2010	644.96	13.75	322.48	6.88
2011	622.73	-22.23	311.36	-11.12
2012	642.58	19.85	321.29	9.93
2013	608.56	-34.02	389.48	68.19
2014	615.86	7.30	400.30	10.82
2015	623.06	7.20	404.99	4.69
2016	630.20	7.14	409.63	4.64
2017	657.21	27.01	439.70	30.07

Descriptive data analysis was carried out to obtain patterns of change in the volume of waste and the volume of waste transported per year. Data on the volume of waste per year (column 2) has an up and down pattern. Data on the volume of transported waste (column 4) also has a fluctuating pattern each year.

Up and down patterns in both types of data that have an upward trend can be modeled by differences between years (columns 3 and 5). The numbers in columns 3 and 5 are change with the numbers that can be made the same. Thus, modeling the difference between the volume of waste and the volume of waste transported can be seen as the difference in the Arithmetic progression. Although the differences can be made the same, the data description shows the differences that are positive and negative. This cause Arithmetic progression cannot be used to model the volume of waste and the volume of transported waste.

Assuming that the difference in volume of waste and the difference in volume of transported waste can be made the same, so that modeling with arithmetic sequences can be done, the two types of differences are calculated based on the first data and the last data. Thus the difference (volume) of waste is obtained and the difference (difference) of volume of waste transported is calculated by Equation (4) where m and n consecutively is the first and last term.

$$b_v = \frac{657.21 - 630.82}{2017 - 2008} = \frac{26.39}{9} = 2.93$$

$$b_t = \frac{439.70 - 315.41}{2017 - 2008} = \frac{124.29}{9} = 13.81$$

3.3 Predicted Waste and Transported Waste Volume

Modeling prediction of waste volume and volume of transported waste is done by Arithmetic progression. Difference for the volume of waste and the volume of transported waste consecutively is $b_v = 2.93$ and $b_t = 13.81$. The prediction of volume of waste and volume of transported waste carried out with the first term is real data from each volume. The predicted amount for each volume is calculated by Equation (3). Next, the predicted volume of waste is given in Table 2, where $b_v = 2.93$ dan $U_1 = 630.82$.

Table 2. Prediction of Waste Volume Data Using Arithmetic Progression

Year	Volume of Waste (m^3)	Volume of Waste Prediction (m^3)
2008	630.82	630.820
2009	631.21	633.752
2010	644.96	636.684
2011	622.73	639.616
2012	642.58	642.548
2013	608.56	645.480
2014	615.86	648.412
2015	623.06	651.344
2016	630.20	654.276
2017	657.21	657.208

Using Equation (1), the MAPE value for the garbage volume prediction data is $\text{MAPE}_v = 2.41\%$

The prediction of the volume of transported waste is given in Table 3, where $b_t = 13.81$ dan $U_1 = 315.41$.

Table 3. Prediction of Transported Volume Data Using Arithmetic Progression

Year	Transported Volume (m^3)	Transported Volume Prediction (m^3)
2008	315.41	315.41
2009	315.60	329.22
2010	322.48	343.03
2011	311.36	356.84
2012	321.29	370.65
2013	389.48	384.46
2014	400.30	398.27

2015	404.99	412.08
2016	409.63	425.89
2017	439.70	439.70

Using Equation (1), the MAPE value for the waste volume prediction data is $\text{MAPE}_t = 4.82\%$. A prediction result is said to be very good if it has a MAPE value of less than 10% and is said to be good if it has a MAPE value of less than 20%. Thus, modeling the volume of waste and the volume of transported waste can be done with arithmetic sequences.

3.4 Break Event Point Condition between Waste Volume and Transported Waste

Prediction of the volume of waste and the volume of garbage transported for 2018 and beyond is done with arithmetic progression. In the waste volume modeling is used a_t = 630.82 and $b_v = 2.93$ so with Equation (2) an arithmetic model for the volume of waste in Equation (5) is obtained:

$$U_{n_v} = 627.888 + 2.932 \cdot n_v \tag{5}$$

where $n_v = 11, 12, 13, \ldots.$

In the same way an arithmetic model for the volume of transported waste given in Equation (6)

$$U_{n_t} = 301.6 + 13.81 \cdot n_t \tag{6}$$

where $n_v = 11, 12, 13, \ldots.$

By using Equations (5) and (6), a prediction of waste volume and volume of transported waste is obtained respectively. The results are given in Table 4.

Table 4. The term for Waste Volume and Transport Volume

n	Year	Volume of Waste Prediction (m^3)	Transported Volume Prediction (m^3)
11	2018	660.14	453.51
12	2019	663.072	467.32
13	2020	666.004	481.13
14	2021	668.936	494.94
15	2022	671.868	508.75
16	2023	674.800	522.56
17	2024	677.732	536.37
18	2025	680.664	550.18
19	2026	683.596	563.99
20	2027	686.528	577.80
21	2028	627.888	591.61
22	2029	692.392	605.42
23	2030	695.324	619.23
24	2031	698.256	633.04
25	2032	701.188	646.85
26	2033	704.120	660.66
27	2034	707.052	674.47
28	2035	709.984	688.28
29	2036	712.916	702.09
30	2037	715.900	715.90

Based on Table 4, the break event point condition is known to be the condition when the entire volume of waste can be transported occurs in 2037.

3.4 Mathematical Models Based on Arithmetic Progression

From the modeling of the volume of waste and the volume of waste transported using Arithmetic progression, a mathematical model was obtained which was developed using Arithmetic progressions. Mathematical models for predicting the volume of waste are given in Equation (7), and mathematical models for predicting the volume of transported waste are given in Equation (8). Both of these models are good to use if MAPE is between the real data and the predicted data below 20%.

The mathematical model for waste volume prediction is

$$U_{n_v} = a_v + (n_v - 1) \cdot b_v \tag{7}$$

where
$a_v = U_1$ states the real volume of waste data,
$b_v = \dfrac{U_{n_v} - U_{m_v}}{n_v - m_v}$
where $n_v > m_v$; $n_v = 2,3,4,........$; $m_v = 2,3,4,........$

The mathematical model for predicting the volume of transported waste is

$$U_{n_t} = a_t + (n_t - 1)\cdot b_t \tag{8}$$

where
$a_t = U_1$ states real data volume of waste transported,
$b_t = \dfrac{U_{n_t} - U_{m_t}}{n_t - m_t}$
where $n_t > m_t$; $n_t = 2,3,4,........$; $m_t = 2,3,4,........$

3.5 Mathematical Problems
3.5.1 Mathematical Problems 1: Determine the Time for a Break Event Point
Mathematical problem that can be derived from mathematical modeling that has been done is to determine the time when a break event point occurs, namely the condition when all waste can be transported. This condition occurs when the volume of waste is the same as the volume of waste transported. The mathematical solution gives a break event point result occurring $(n+1)$ years later from the first year, with

$$n = \frac{a_v - a_t}{b_v - b_t} - 1. \tag{9}$$

Theorem 1:
Let the volume of waste and the volume of waste transported is predicted by Arithmetic rogression. Then all the waste will be transported after $(n+1)$ years later from the time of the first year, with $n = \dfrac{a_v - a_t}{b_v - b_t} - 1.$

Proof:
It will be shown that $n = \dfrac{a_v - a_t}{b_v - b_t} - 1.$
Let U_{n_v}, U_{n_t} successively states the n-th term from Arithmetic progression for waste volumes and transported waste volumes. All waste will be transported if $U_{n_v} = U_{n_t}$

$$U_{n_v} = U_{n_t}$$
$$a_v + (n_v - 1)\cdot b_v = a_t + (n_t - 1)\cdot b_t$$
$$n_v \cdot b_v - n_t \cdot b_t = (a_v - a_t) + (b_t - b_v)$$
$$b_v(n_v + 1) - b_t(n_t + 1) = (a_v - a_t)$$

Break event points occur when $n_n = n_t = n$ where

$$n = \frac{a_v - a_t}{b_v - b_t} - 1. \qquad \blacksquare$$

Value n in theorem 1 can be positive or negative, so the general solution is $n = \left|\dfrac{a_v - a_t}{b_v - b_t} - 1\right|$. To determine the year when the break event point is as follows:

1. If $n > 0$. Let x states the year for the first data. Then the break event point will occur in the year $(x+1) + n$ or $x + (n+1)$
2. If $n < 0$. Let x states the year for the first data. Then the break event point will occur in the year tahun $(x-1) + |n|$ atau $x + (|n| - 1)$

For the case of mathematical modeling between the volume of waste and the volume of waste transported in Purwokerto, obtained $a_v = 630.82$ and $b_v = 2.93$ and $a_t = 315.41$ and $b_t = 13.81$. With Equation (9), the time when all waste can be transported is

$$n = \frac{a_v - a_t}{b_v - b_t} - 1$$

$$= \frac{630.82 - 315.41}{2.93 - 13.81} - 1 = \frac{315.41}{-10.88} - 1 = -28.99 - 1 = -29.99 \approx -30$$

In this case the first data used came from 2008. Because $n < 0$, then the prediction of the city of Purwokerto is free of waste (break event point) is in the year

$(x-1) + |n| = (2008-1) + |-30| = 2007 + 30 = 2037$ or

$x + (|n| + 1) = 2008 + (|-30| - 1) = 2008 + 29 = 2037$

3.6.2 Mathematical Problem 2: Modeling with Average of Difference

In the previous modeling, the difference is calculated only using the first data and the last data. In this way, not all data is involved in modeling. Another method that can be used is to calculate the difference as the average of sequential different values using Equation (10)

$$b = \frac{1}{k}\sum_{i=1}^{k} b_i \tag{10}$$

where k express the amount of difference.
With Equation (10) obtained

$$b_v = \frac{1}{9}(0.39 + 13.75 - 22.23 + ... + 27.01) = \frac{26.39}{9} = 2.93$$

$$b_t = \frac{1}{9}(0.19 + 6.88 - 11.12 + ... + 30.07) = \frac{124.29}{9} = 13.81$$

The results of calculations with Equation (10) and Equation (4). Mathematically, if the similarity of the results is not accidental, it must be shown that Equations (10) and (4) are the same, meaning

$$\frac{1}{k}\sum_{i=1}^{k} b_i = \frac{U_n - U_m}{n - m} \tag{11}$$

Theorem 2

Let U_m and U_n successive states of the first and last terms in Arithmetic progression. Let b_i states the difference between two terms successively in Arithmetic progression, where $i = 1,2,....,k$. The average of the different numbers in the sequence is equal to the difference between the last number and the first number divided by the number of differences between the first number and the last number.

Proof:

It will be shown that $\frac{1}{k}\sum_{i=1}^{k} b_i = \frac{U_n - U_m}{n - m}$

Suppose Arithmetic progression is

$U_1, U_2, ..., U_n = U_m, U_2, ..., U_n$

The difference between two consecutive numbers is

$b_1 = U_2 - U_1 = U_2 - U_m$

$b_2 = U_3 - U_2$

$b_3 = U_4 - U_3$

.

.

$b_k = U_n - U_{n-1}$

From the pattern above, the amount of differences are k pieces, i.e. $b_1, b_2, ..., b_k$. In this Arithmetic progression $U_1, U_2, ..., U_n = U_m, U_2, ..., U_n$, the constant k can be stated as $k = n - m$ such that $b_k = b_{n-m}$.

$b_1 = U_2 - U_1 = U_2 - U_m$
$b_2 = U_3 - U_2$
$b_3 = U_4 - U_3$
.
.
$b_k = U_n - U_{n-1} = b_{n-1} = b_{n-m}$

Therefore $k = n - m$, then the proof is enough to show that

$$\sum_{i=1}^{k} b_i = U_n - U_m$$

Next,

$$\sum_{i=1}^{k} b_i = b_1 + b_2 + ... + b_k$$
$$= (U_2 - U_m) + (U_3 - U_2) + (U_4 - U_3) + ... + (U_n - U_{n-1})$$
$$= U_n - U_m$$ ■

If U_m and U_n successively declares the first and last terms, then Theorem 2 can be generalized, meaning the numbers in between U_m and U_n does not have to have differences that are all the same. An example of the generalization of Theorem 1 is modeling the volume of waste and the volume of waste transported.

3.7.3 Mathematical Problem 3: Analysis of the Difference

To predict the volume of waste and the volume of waste transported, two different uses are used, these are b_v and b_t. Determination of the two values uses Equation (4) whose value depends on the first data, the last data and the number of differences obtained. In the case of waste in the city of Purwokerto, it is known that the first data for the waste volume and the volume of waste transported are, respectively $a_v = 630.82$ and $a_t = 315.41$, and the last consecutive data are 657.21 and 439.70. With Equation (3) obtained $b_v = 2.93$ dan $b_t = 13.81$. In this case, in 2037 a prediction of volume of waste and volume of transported waste is same.

The purpose of the prediction is to find the same volume of waste as the volume of waste transported. There are several possible relationships between b_v and b_t, these are $b_t > b_v$, $b_t < b_v$ and $b_t = b_v$.

These three conditions can produce mathematical problems, each with its solution, namely the time when all the waste can be transported, or the break point of the waste transportation event. Thus, the break event point occurs n next year from now, $U_{n_t} = U_{n_v}$.

Case **1:** $b_t > b_v$

Sub-case 1.1 $a_t > a_v$

As an illustration, for example $b_t = 3$ and $b_v = 2$ and $a_t = 10$ and $a_v = 7$

n	1	2	3	4	5	6	7	……
U_{n_t}	10	13	16	19	22	25	28	……
U_{n_v}	7	9	11	13	15	17	19	……

From these illustrations, break event points never occur.

Sub-case 1.2 $a_t = a_v$

As an illustration, for example $b_t = 3$ and $b_v = 2$ and $a_t = a_v = 10$

n	1	2	3	4	5	6	7	……
U_{n_t}	10	13	16	19	22	25	28	……
U_{n_v}	10	12	14	16	18	20	22	……

From this illustration, the break event point ever occurs in the first term. That is, all the existing waste is immediately transported away right away.

Sub-case 1.3 $a_t < a_v$

As an illustration, for example $b_t = 3$ and $b_v = 2$ and $a_t = 10$ and $a_v = 18$

n	1	2	3	4	5	6	7	8	9	10					……
U_{n_t}	10	13	16	19	22	25	28	31	34	37					……
U_{n_v}	18	20	22	24	26	28	30	32	34	36					……

From this illustration, the break event point occurs at n = 10.

Case 2: $b_t < b_v$

Sub-case 2.1 $a_t > a_v$

As an illustration, for example $b_t = 2$ and $b_v = 3$ and $a_t = 10$ and $a_v = 7$

n	1	2	3	4	5	6	7	……
U_{n_t}	10	12	14	16	18	20	22	……
U_{n_v}	7	10	13	16	15	17	19	……

From this illustration, the break event point occurs at n = 4.

Sub-case 2.2 $a_t = a_v$

As an illustration, for example $b_t = 2$ and $b_v = 3$ and $a_t = a_v = 10$

n	1	2	3	4	5	6	7	……
U_{n_t}	10	12	14	16	18	20	22	……
U_{n_v}	10	13	16	19	22	25	28	……

From this illustration, the break event point ever occurs in the first term. That is, all the existing waste is immediately transported away right away.

Sub-case 2.3 $a_t < a_v$

As an illustration, for example $b_t = 2$ and $b_v = 3$ and $a_t = 10$ and $a_v = 18$

n	1	2	3	4	5	6	7	8	9	10					……
U_{n_t}	10	12	14	16	18	20	22	24	26	28					……
U_{n_v}	18	21	24	27	30	33	36	39	42	45					……

From these illustrations, break event points never occur.

Case 3: $b_t = b_v$

Sub-case 3.1 $a_t > a_v$

As an illustration, for example $b_t = b_s = 3$ and $a_t = 10$ and $a_v = 7$

n	1	2	3	4	5	6	7	……
U_{n_t}	10	13	16	19	21	24	27	……
U_{n_v}	7	10	13	16	15	17	19	……

From these illustrations, break event points never occur.

Sub-case 3.2 $a_t = a_v$

As an illustration, for example $b_t = b_s = 3$ and $a_t = a_v = 10$

n	1	2	3	4	5	6	7	……
U_{n_t}	10	13	16	19	22	25	28	……
U_{n_v}	10	13	16	19	22	25	28	……

From this illustration, the break event point ever occurs in the first term. That is, all the existing waste is immediately transported away right away.

Sub-case 2.3 $a_t < a_v$

As an illustration, for example $b_t = b_s = 3$ and $a_t = 10$ and $a_v = 18$

n	1	2	3	4	5	6	7	8	9	10					……
U_{n_t}	10	13	16	19	22	25	28	31	34	37					……
U_{n_v}	18	21	24	27	30	33	36	39	42	45					……

From these illustrations, break event points never occur.

Kasus ini dapat dipandang dengan cara berbeda sebagai rasio antara beda volume sampah terangkut dengan beda volume sampah, atau rasio adalah $r = \frac{b_t}{b_v}$. Kasus 1 adalah $r > 1$. Kasus 2 adalah $0 < r < 1$. Kasus 3 adalah $r = 1$.

Theorem 3

Suppose $b_v > 0$ *and* $b_t > 0$ *successively states different volumes of waste and different volumes of waste transported. Suppose* a_v *and* a_t *successively declares the first term the volume of waste and the first term volume of waste transported. Let* $r = \frac{b_t}{b_v}$.

a. *If* $r > 1$ *and* $a_t \le a_v$, *then a break event point occurs;*
b. *If* $0 < r < 1$ *and* $a_t \ge a_v$, *then a break event point occurs;*
c. *If* $r = 1$ *and* $a_t = a_v$, *then break event points occur every year*

Proof:

Given $r = \frac{b_t}{b_v} > 1$

Therefore $b_t > b_v$ or $b_t = \alpha \cdot b_v$ where $\alpha > 1$

Given $a_t \le a_v$ or $a_t = \beta a_v$ where $0 < \beta < 1$

The first Arithmetic progression up to $(n+1)$ first term is

$$a_t, a_t + b_t, a_t + 2b_t, ..., a_t + n \cdot b_t \qquad (*)$$

The second Arithmetic progression up to $(n+1)$ first term is

$$a_v, a_v + b_v, a_v + 2b_v, ..., a_v + n \cdot b_v \qquad (**)$$

Because $b_t = \alpha \cdot b_v$ where $\alpha > 1$ and $a_t = \beta a_v$ where $0 < \beta < 1$, then the first line of arithmetic becomes

$$\beta a_v, \beta a_v + \alpha b_v, \beta a_v + 2\alpha b_v, ..., \beta a_v + n\alpha \cdot b_v \qquad (***)$$

Equalize (**) and (***) was obtained

$$a_v, a_v + b_v, a_v + 2b_v, ..., a_v + n \cdot b_v = \beta a_v, \beta a_v + \alpha b_v, \beta a_v + 2\alpha b_v, ..., \beta a_v + n\alpha \cdot b_v$$

$$(1-\beta)a_v + (1-\beta)a_v + (1-\alpha)b_v + (1-\beta)a_v + (1-\alpha)2b_v + ... + (1-\beta)a_v + (1-\alpha)n \cdot b_v = 0$$

$$n = -\frac{(n+1)(1-\beta)a_v + \frac{n}{2}(n-1)(1-\alpha)b_v}{(1-\alpha)b_v}$$

Proof for theorem 3 parts b and c submitted to the reader.

4. Conclusions

Some conclusions that can be obtained include:

1. Mathematical modeling for the case of the volume of waste and the volume of waste transported based on data in the City of Purwokerto from 2008 to 2017 can be modeled by arithmetic sequences. In 2037 it is predicted that all waste in Purwokerto will be transported to DFS.
2. Modeling the volume of waste and the volume of waste transported by Arithmetic progression is also solved by the difference $b = \frac{U_n - U_m}{n - m}$.
3. From the waste problem can be built mathematical knowledge in the form of predictions of the year when all the waste can be transported (break event point). Suppose $n = \frac{a_v - a_t}{b_v - b_t} - 1$ states the time until all waste can be transported. If x states the year for the first data, then the break event point will occur in the year $(x+1)+n = x+(n+1)$ for $n > 0$ or $(x-1)+|n| = x+(|n|-1)$ for $n < 0$.
4. Modeling the volume of waste and the volume of waste transported by Arithmetic progression can also be solved with the difference calculated by the equation $b = \frac{1}{k}\sum_{i=1}^{k} b_i$

As a suggestion, a mathematical model can be built for the case of predicted waste volume with Arithmatic progression and predicted transported waste volume with geometric progression, and vice versa. Another suggestion is that the volume of waste and the volume of waste transported are both predicted by geometric progression.

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