Number Sequences Likes Fibonacci

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Abstract

The initial conditions of the Fibonacci and Lucas sequences can be used to generate a new number sequence. The new number sequence has a similar form and pattern to the recursive formula of the Fibonacci sequence. It is shown from the recursive formula of the new number sequence which is similar to the recursive formula of the Fibonacci sequence. The new number sequence is called as the Fibonacci-like sequence. The variations of the Fibonacci-like sequence are, inter alia, the Fibonacci and Fibonacci-like sub-sequences. The existences of the properties of the Fibonacci-like sequence and both variations, such as the convergence to the golden ratio, the Binet formula form, and the relative prime property, will be investigated.

Keywords:
Binet formula, Golden ratio, relatively prime, sub-sequence of Fibonacci sequence, sub-sequence of sequence like Fibonacci.

1. Introduction

The Fibonacci sequence was first reported in the book Liber Abacci, published in 1202 by Leonardo da Pisa (Fernando and Prabowo, 2019). Another name for Leonardo da Pisa is Leonardo Fibonacci so that the sequence of numbers he discovered is called the Fibonacci sequence (Prabowo, 2014). Historically, Fibonacci sequence were obtained from the ideal growth of rabbit populations studied by Fibonacci until they finally produced the Fibonacci sequence (Burton, 2007; Tung, 2008). Nevertheless, Donald E. Knuth in his book *The Art Computer Programming* explained that the Fibonacci sequence was explained earlier by Indian mathematicians Gopala and Hemachandra in 1150 (Tung, 2008).

Beside Fibonacci sequence, there is Lucas sequence. The ratio of the successive Fibonacci sequence term and Lucas sequence term are known to converge towards the golden ratio. Both sequences can be presented in the form of the Binet formula (Koshy, 2001).

Many examples of events in the universe that follow the Fibonacci sequence and its golden ratio, such as biological arrangements such as branching on trees, phyllotaxis (arrangement of leaves on the stem), pineapple sprouts, flowering artichokes, ferns that are not rounded and bract arrangement of pine cones and others. Sinha (2017) provides an additional example of using Fibonacci sequences in coding theory. Fibonacci sequences in various forms are widely applied in making security codes. Pirillo (2019) discusses some results regarding Fibonacci Word and Sturmian Words, some geometries built from Fibonacci sequences, and presents some ideas that might be useful in mathematics education.

This article will be discussed a new sequence called Fibonacci-like sequence that has been obtained Harne, Bijendra, and Shubhraj (2014). From the study of Fibonacci-like sequences it can be reported several things in the form of the Binet formula for Fibonacci-like sequences and convergence of Fibonacci-like sequences to the golden ratio. The same properties will be examined in two variations of Fibonacci sequence, these are sub-sequence of Fibonacci sequence and sub-sequence of Fibonacci-like sequence.

2. Research Methodology

The methodology in this research is literature study and describes some of the results that have been obtained by researchers. The results obtained by the author and explained in this article are convergence of Fibonacci-like sequences to the golden ratio and the form of the Binet formula for Fibonacci-like sequences, sub-sequences of Fibonacci sequences and sub-sequences of Fibonacci-like sequences and relative prime validity. The steps in this research are.

Stage 1: Fibonacci and Lucas Sequence
Formulate recursive equations for Fibonacci and Lucas Sequence.

Stage 2: Golden Ratio for Fibonacci and Lucas Sequence
Shows that the ratio of two consecutive numbers in Fibonacci and Lucas sequence converge to the golden ratio.

Stage 3: Binet Formula for Fibonacci and Lucas Sequence
Construct the Fibonacci and Lucas equation with the Binet formula.

Stage 4: Three New Sequences
Construct three new sequence, these are Fibonacci-like sequences, sub-sequences of Fibonacci sequences and sub-sequences of Fibonacci-like sequences.

Stage 5: Recursive Formula for Three New Sequences
Construct a recursive formula for Fibonacci-like sequences, sub-sequences of Fibonacci sequences and sub-sequences of Fibonacci-like sequences.

Stage 6: Binet Formula for Three New Sequences
Construct Fibonacci-like sequences, sub-sequences of Fibonacci sequences and sub-sequences of Fibonacci-like sequences with Binet formula.

Stage 7: Golden Ratio for Three New Sequences
Show that the ratio of two consecutive numbers on Fibonacci-like sequences, sub-sequences of Fibonacci sequences and sub-sequences of Fibonacci-like sequences converge to the golden ratio.

Stage 8: Prime Relative Properties of Three New Sequences
Investigate the relatively prime on Fibonacci-like sequences, sub-sequences of Fibonacci sequences and sub-sequences of Fibonacci-like sequences.

3. Discussion Results

3.1 Fibonacci Sequences

Fibonacci sequence is a sequence with terms are 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, ... Burton (2007) gives fifty first Fibonacci sequence. Except for the first and second terms, the other terms are obtained from the sum of the two previous terms. That is, the third term (1) is obtained from the sum of the first term (0) with the second term (1). Likewise the fourth term (2) is obtained from the sum of the second term (1) with the third term (1).

The existence of the first and second terms causes the initial conditions in Fibonacci sequences. If F_n is a term n-th in the Fibonacci sequence, the initial conditions are written with F_1 = 0 and F_2 = 1. The process of forming the next terms is obtained by an iteration process $F_3 = F_1 + F_2$, $F_4 = F_2 + F_3$, $F_5 = F_3 + F_4$ and so on. This process produces a formula that is used to obtain the next terms of the Fibonacci sequence in the form of a recursive formula (1):

$$F_n = F_{n-1} + F_{n-2} \;;\; n \geq 2$$
$$F_0 = 0 \text{ and } F_1 = 1 \tag{1}$$

In the recursive formula (1) a slightly different formulation is performed, starting with the term 0, 1, 2 and so on. To be consistent with the explanation in the previous section, the recursive formula (1) can be presented in a recursive form in equation (2):

$$F_n = F_{n-1} + F_{n-2} \ ; \ n \geq 3$$
$$F_1 = 0 \text{ and } F_2 = 1 \tag{2}$$

Burton (2007) states that two consecutive terms in a Fibonacci sequence are relatively prime so the greatest common divisor $(F_n, F_{n+1}) = 1$ for every $n \geq 1$.

3.2 Lucas Sequence

Lucas's sequence is composed by 2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322.... (Prabowo, 2014). Except for the first and second terms, the other terms are obtained from the sum of the two previous terms. As in Fibonacci sequence, Lucas sequence can be expressed in the form of recursive formulas (3) or (4) as follows:

$$L_n = L_{n-1} + L_{n-2} \ ; \ n \geq 2$$
$$L_0 = 2 \text{ and } L_1 = 1 \tag{3}$$

or

$$L_n = L_{n-1} + L_{n-2} \ ; \ n \geq 3$$
$$L_1 = 2 \text{ and } L_2 = 1 \tag{4}$$

Burton (2007) states that the terms in the Lucas sequence are 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322.... In recursive form, it can be written with equation (5) or (6) as follows:

$$L_n = L_{n-1} + L_{n-2} \ ; \ n \geq 2$$
$$L_0 = 1 \text{ and } L_1 = 3 \tag{5}$$

or

$$L_n = L_{n-1} + L_{n-2} \ ; \ n \geq 3$$
$$L_1 = 1 \text{ and } L_1 = 3 \tag{6}$$

As is the case with Fibonacci sequence, two consecutive terms in a Lucas sequence are relatively prime so the greatest common divisor $(L_n, L_{n+1}) = 1$ for every $n \geq 1$.

3.3 Golden Ratio

The quotient (ratio) of two consecutive numbers in the Fibonacci sequence and Lucas sequence will converge to a constant called the golden ratio. This is shown in Table 1. The golden ratio constant is symbolized by the Greek capital letter Phi (Φ) and the value of Φ = 1.6180339887..... (Prabowo, 2014).

Table 1. Ratio of Two Consecutive Terms in Fibonacci Sequence and Lucas Sequence

Ratio of Fibonacci Sequence		Ratio of Lucas Sequence	
1/0	-	1/2	0,5
1/1	1	3/1	3
2/1	2	4/3	1.333333333
3/2	1.5	7/4	1.75
5/3	1.66666	11/7	1.571428571
8/5	1.6	18/11	1.636636636
13/8	1.625	29/18	1.611111111
21/13	1.61538461538	47/29	1.620689655
34/21	1.61904761905	76/47	1.617021277
55/34	1.61764705882	123/76	1.618421053
89/55	1.61818181818	199/123	1.617886179
144/89	1.6179775280	322/199	1.618090452
Phi = 1.6180339887....			

3.4 Binet Formula

In 1843, a French mathematician named Jacques-Philippe-Marie Binet (1783-1856) discovered the formula for expressing terms n-th or u_n in Equation (7):

$$u_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right] \tag{7}$$

The Binet formula for Fibonacci sequences 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, ... is (Burton, 2007):

$$F_n = \frac{\alpha^n - \beta^n}{\alpha - \beta} = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right] \quad ; \; n \geq 1 \tag{8}$$

While the Binet formula for Lucas sequences 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322.... is (Burton, 2014):

$$L_n = \alpha^n + \beta^n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n \quad ; \; n \geq 1 \tag{9}$$

Constant α and β in Equation (8) and (9) are $\alpha = \frac{1+\sqrt{5}}{2} \approx 1.618$ called *golden ratio* and $\beta = \frac{1-\sqrt{5}}{2} \approx -0.618$.

3.5 Three New Sequences

Harne, Bijendra, dan Shubhraj (2014) call this new sequence with Fibonacci-like sequence which is stated in the recursive formula (10) and similar to the recursive formula for BBF (1).

$$D_n = D_{n-1} + D_{n-2} \; ; \; n \geq 2 \tag{10}$$

The initial requirements for Fibonacci-like sequence are

$$D_0 = 2 \text{ and } D_1 = 1 + m \tag{11}$$

where m integer.

Initial condition $D_0 = 2$ and $D_1 = 1 + m$ with integer m is a special case obtained from the sum of the initial conditions in Fibonacci sequences and Lucas sequences, i.e

$$D_0 = mF_0 + L_0 \text{ and}$$
$$D_1 = mF_1 + L_1 \tag{12}$$

where: m integer, $F_0 = 0$ and $F_1 = 1$: initial condition on Fibonacci sequence on Equation (1), and $L_0 = 2$ and $L_1 = 1$: initial condition on Lucas sequence on Equation (3).

Recursive formula for Fibonacci-like sequences is (Harne, Bijendra, dan Shubhraj, 2014).

$$D_n = mF_n + L_n \; ; \; n \geq 0 \tag{13}$$

Therefore $F_0 = 0$ and $L_0 = 2$ then $D_0 = mF_0 + L_0 = L_0 =$ 2. That is, for every positive integer m, then $D_0 = 2$. As well, for every positive integer m, then $D_1 = m+1$ (Table 2). From table 2, for each positive integer m, Equation (11) is initial condition for Fibonacci-like sequence with recursive formula rekursif in Equation (10).

Table 2. First and Second Terms at Fibonacci-like Sequence

m	$D_0 = mF_0 + L_0$ $F_0 = 0$; $F_1 = 1$	$D_1 = mF_1 + L_1$ $L_0 = 2$; $L_1 = 1$
1	$D_0 = 2$	$D_1 = m+1$
2	$D_0 = 2$	$D_1 = m+1$
3	$D_0 = 2$	$D_1 = m+1$
4	$D_0 = 2$	$D_1 = m+1$
.	$D_0 = 2$	$D_1 = m+1$

By paying attention to the initial conditions and recursive formula for Fibonacci-like sequence (13), infinitely obtained Fibonacci-like sequences for every m = 1, 2, 3, ... as shown in Table 3. The process of obtaining a sequence of numbers can be continued in Table 3 for m = 10, 11, 12, ...

Table 3. Some Fibonacci-like Sequences Depend on Value m

m	Initial Condition $D_0 = 2$; $D_1 = m+1$	New Sequence (until 9 Terms) $D_n = D_{n-1} + D_{n-2}$	Explanation
1	$D_0 = 2$; $D_1 = 2$	2, 2, 4, 6, 10, 16, 26, 42, 68, ...	Fibonacci-like Sequence

2	$D_0 = 2$; $D_1 = 3$	2, 3, 5, 8, 13, 21, 34, 55, 89, ...	Sub-Sequences of Fibonacci Sequence
3	$D_0 = 2$; $D_1 = 4$	2, 4, 6, 10, 16, 26, 42, 68, 110, ...	Sub-Sequences of Fibonacci-like Sequence
4	$D_0 = 2$; $D_1 = 5$	2, 5, 7, 12, 19, 31, 50, 81, 131, ...	-
5	$D_0 = 2$; $D_1 = 6$	2, 6, 8, 14, 22, 36, 58, 94, 152, ...	-
6	$D_0 = 2$; $D_1 = 7$	2, 7, 9, 16, 25, 41, 66, 107, 173, ...	-
7	$D_0 = 2$; $D_1 = 8$	2, 8, 10, 18, 28, 46, 74, 120, 194, ...	-
8	$D_0 = 2$; $D_1 = 9$	2, 9, 11, 20, 31, 51, 82, 133, 215, ...	-
9	$D_0 = 2$; $D_1 = 10$	2, 10, 12, 22, 34, 56, 90, 146, 236, ...	-
and so on.			

The position of each term can be expressed by D_{mn} which means the term n-th for a certain value of m. That is, for $m = 1$ was obtained $D_{11} = 2$, $D_{12} = 2$, $D_{13} = 4$, $D_{14} = 6$, $D_{15} = 10$, $D_{16} = 16$ and so on. The difference of n-th tern for every m will form a Fibonacci sequence:

$$D_{21} - D_{11} = D_{31} - D_{21} = D_{41} - D_{31} = ... = 0 \qquad D_{(i+1)1} - D_{(i)1} = 0 \ ; \ i = 1,2,3,....$$
$$D_{22} - D_{12} = D_{32} - D_{22} = D_{42} - D_{32} = ... = 1 \qquad D_{(i+1)2} - D_{(i)2} = 1 \ ; \ i = 1,2,3,....$$
$$D_{23} - D_{13} = D_{33} - D_{23} = D_{43} - D_{33} = ... = 1 \qquad D_{(i+1)3} - D_{(i)3} = 1 \ ; \ i = 1,2,3,....$$
$$D_{24} - D_{14} = D_{34} - D_{24} = D_{44} - D_{34} = ... = 2 \qquad D_{(i+1)4} - D_{(i)4} = 2 \ ; \ i = 1,2,3,....$$
$$D_{25} - D_{15} = D_{35} - D_{25} = D_{45} - D_{35} = ... = 3 \qquad D_{(i+1)5} - D_{(i)5} = 3 \ ; \ i = 1,2,3,....$$
$$D_{26} - D_{16} = D_{36} - D_{26} = D_{46} - D_{36} = ... = 5 \qquad D_{(i+1)6} - D_{(i)6} = 5 \ ; \ i = 1,2,3,....$$

In general, it can be stated by equation (14) below

$$D_{(i+1)j} - D_{(i)j} = F_j \tag{14}$$

where $i = 1,2,3,....$, $j = 1,2,3,....$, and F_j : j-th term on Fibonacci sequence on Equation (2).

3.6 Recursive Formula for Three New Sequences

From Table 3, for m = 1 obtained Fibonacci-like sequence terms 2, 2, 4, 6, 10, 16, 26, 42, 68, ... Thus, BBSF can be presented with a recursive formula (15) or (16):

$$D_n = D_{n-1} + D_{n-2} \ ; \ n \geq 2 \ ; \ D_0 = 2 \text{ and } D_1 = 2 \tag{15}$$

or

$$D_n = D_{n-1} + D_{n-2} \ ; \ n \geq 2 \ ; \ D_1 = 2 \text{ and } D_2 = 2 \tag{16}$$

Next, we will analyze the formula for terms n-th on Fibonacci-like sequence 2, 2, 4, 6, 10, 16, 26, 42, 68, 110, ... in the following way:

2, 2, 4, 6, 10, ...
2, $1+m$, $3+m$, $4+2m$, $7+3m$, ...

The next terms are obtained by adding up the two previous terms, so that they are obtained $11+5m$, $18+8m$, $29+13m$, $47+21m$, $65+29m$, ... with m = 1. So, the formula for the term n-th ($n \geq 1$)on Fibonacci-like sequence 2, 2, 4, 6, 10, 16, 26, 42, 68, ...is
2, $1+m$, $3+m$, $4+2m$, $7+3m$, $11+5m$, $18+8m$, $29+13m$, ... where m = 1.

From Table 3, for m = 2 obtained by the terms contained in Fibonacci sequence 2, 3, 5, 8, 13, 21, 34, 55, 89, ... This sequence is called Sub-equences of Fibonacci sequence. In recursive form, it can be formulated with (17) or (18):

$$F_n = F_{n-1} + F_{n-2} \ ; \ n \geq 2 \ ; \ F_0 = 2 \text{ and } F_1 = 3 \tag{17}$$

or

$$F_n = F_{n-1} + F_{n-2} \ ; \ n \geq 3 \ ; \ F_1 = 2 \text{ and } F_2 = 3 \tag{18}$$

Formula for term n-th ($n \geq 1$) on Sub-Sequences of Fibonacci Sequence 2, 3, 5, 8, 13, 21, 34, 55, 89, ... is
2, $1+m$, $3+m$, $4+2m$, $7+3m$, $11+5m$, $18+8m$, $29+13m$, ... where m = 2.

From Table 3, for m = 3 obtained by the terms contained in Fibonacci-like sequence 2, 4, 6, 10, 16, 26, 42, 68, ... This sequence is Sub-sequences of Fibonacci-like sequence. In recursive form, it can be formulated with (19) or (20):

$$D_n = D_{n-1} + D_{n-2} \; ; \; n \geq 2 \; ; \; D_0 = 2 \text{ and } D_1 = 4 \tag{19}$$

or

$$D_n = D_{n-1} + D_{n-2} \; ; \; n \geq 2 \; ; \; D_1 = 2 \text{ and } D_2 = 4 \tag{20}$$

Formula for term n-th ($n \geq 1$) for Sub-sequences of Fibonacci-like sequence 2, 4, 6, 10, 16, 26, 42, 68, ... is 2, $1+m$, $3+m$, $4+2m$, $7+3m$, $11+5m$, $18+8m$, $29+13m$, ... where m = 3.

In general, the formula for terms n-th ($n \geq 1$) for m = 1, 2, 3, 4, 5, ... given in Equation (21):

$$2,\ 1+m,\ 3+m,\ 4+2m,\ 7+3m,\ 11+5m,\ 18+8m,\ 29+13m,\ \ldots.. \tag{21}$$

3.7 Binet Formula for Three New Sequences

Quadratic equation $x^2 - x - 1 = 0$ has roots $x_1 = \frac{1+\sqrt{5}}{2}$ and $x_2 = \frac{1-\sqrt{5}}{2}$. If we write x_1 by α and x_2 by β, quadratic equation $x^2 - x - 1 = 0$ can be stated with

$$\alpha^2 - \alpha - 1 = 0 \text{ or } \alpha^2 = \alpha + 1 \tag{22}$$

$$\beta^2 - \beta - 1 = 0 \text{ or } \beta^2 = \beta + 1 \tag{23}$$

Next, for (22) and (23) obtained

$$\alpha^{n+2} = \alpha^{n+1} + \alpha^n \tag{24}$$

$$\beta^{n+2} = \beta^{n+1} + \beta^n \tag{25}$$

Reduce (24) to (25) and divide by $\alpha - \beta$, obtained

$$\frac{\alpha^{n+2} - \beta^{n+2}}{\alpha - \beta} = \frac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta} + \frac{\alpha^n - \beta^n}{\alpha - \beta} \tag{26}$$

Let $H_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$, Equation (26) can be expressed by Equation (27)

$$H_{n+2} = H_{n+1} + H_n \; ; \; n \geq 1 \tag{27}$$

Sub-sequences of Fibonacci sequence is 2, 3, 5, 8, 13, 21, 34, 55, ... For $\alpha = \frac{1+\sqrt{5}}{2}$ and $\beta = \frac{1-\sqrt{5}}{2}$ then $\alpha - \beta = \sqrt{5}$, $\alpha^2 - \beta^2 = \sqrt{5}$, $\alpha^3 - \beta^3 = 2\sqrt{5}$, $\alpha^4 - \beta^4 = 3\sqrt{5}$, $\alpha^5 - \beta^5 = 5\sqrt{5}$ such that Sub-sequences of Fibonacci sequence can be stated with

$$H_3 = \frac{\alpha^3 - \beta^3}{\alpha - \beta} = 2$$

$$H_4 = \frac{\alpha^4 - \beta^4}{\alpha - \beta} = 3$$

$$H_5 = \frac{\alpha^5 - \beta^5}{\alpha - \beta} = 5 \text{ and so on.}$$

That is, if Sub-sequences of Fibonacci sequence stated by term $H_3, H_4, H_5, \ldots.$, then if Sub-sequences of Fibonacci sequence 2, 3, 5, 8, 13, 21, 34, 55, 89, ... can be written in the form of the Binet formula:

$$F_n = \frac{\alpha^n - \beta^n}{\alpha - \beta} \; ; \; n \geq 3 \tag{28}$$

Harne, Bijendra, and Shubhraj (2014) has given the Binet formula for Fibonacci-like sequence 2, 2, 4, 6, 10, 16, 26, 42, 68, ... is

$$D_n = \frac{m\left(\alpha^n - \beta^n\right)}{\sqrt{5}} + \left(\alpha^n + \beta^n\right) \; ; \; n \geq 0 \tag{29}$$

Next, Binet's formula for Sub-sequences of Fibonacci-like sequence 2, 4, 6, 10, 16, 26, 42, 68, 110, ... obtained from (19) with the first term starting at $n = 1$, namely

$$D_n = \frac{m\left(\alpha^n - \beta^n\right)}{\sqrt{5}} + \left(\alpha^n + \beta^n\right) \; ; \; n \geq 1 \tag{30}$$

3.8 Golden Ratio for Three New Sequences

Convergence of the golden ratio of Fibonacci-like sequence, Sub-sequences of Fibonacci sequence, and Sub-sequences of Fibonacci-like sequence can be tracked through Table 4. The three sequence converge toward the golden ratio.

Table 4. Ratio of two successive terms of Fibonacci-like sequence, Sub-sequences of Fibonacci sequence, and Sub-sequences of Fibonacci-like sequence

Fibonacci-like sequence		Sub-sequences of Fibonacci sequence		Sub-sequences of Fibonacci-like sequence	
2/2	1	3/2	1,5	4/2	2
4/2	2	5/3	1.66666	6/4	1.5
6/4	1.5	8/5	1.6	10/6	1.666666667
10/6	1.666666667	13/8	1.625	16/10	1.6
16/10	1.6	21/13	1.61538461538	26/16	1.625
26/16	1.625	34/21	1.61904761905	42/26	1.615384615
42/26	1.615384615	55/34	1.61764705882	68/42	1.619047619
68/42	1.619047619	89/55	1.61818181818	110/68	1.617647059
110/68	1.617647059	144/89	1.6179775280	178/110	1.618618618
Phi = 1.6180339887....					

3.7 Sifat Relatif Prima pada BBSF, Sub-BBF dan Sub-BBSF

From Table 3, more complete terms can be presented in terms Fibonacci-like sequence, Sub-sequences of Fibonacci sequence, and Sub-sequences of Fibonacci-like sequence as follows

Fibonacci-like sequence : 2, 2, 4, 6, 10, 16, 26, 42, 68, 110.....
Sub-sequences of Fibonacci sequence : 2, 3, 5, 8, 13, 21, 34, 55, 89, 144.....
Sub-sequences of Fibonacci-like sequence : 2, 4, 6, 10, 16, 26, 42, 68, 110, 178....

The relatively prime nature is only owned by Sub-sequences of Fibonacci sequence. Therefore Fibonacci sequence has relatively prime properties and Sub-sequences of Fibonacci sequence obtained by eliminating the first three terms in Fibonacci sequence. The removal of these terms does not change the relatively prime nature of the remaining terms (defended terms).

Terms on Fibonacci-like sequence and Sub-sequences of Fibonacci-like sequence are even genap. The greatest common divisr of two even numbers is not equal to 1.Thus, Fibonacci-like sequence and Sub-sequences of Fibonacci-like sequence does not have relatively prime properties.

4. Conclusion

From the initial conditions Fibonacci sequence and Lucas sequence obtained recursive formula $D_n = D_{n-1} + D_{n-2}$ for $n \geq 2$, with initial condition $D_0 = 2$, $D_1 = 1 + m$ for $m = 1,2,3,....$ From the recursive formula obtained Fibonacci-like sequence ($m = 1$), Sub-sequences of Fibonacci sequence ($m = 2$) and Sub-sequences of Fibonacci-like sequence ($m = 3$).

In general, the terms n-th for m = 1, 2, 3, 4, 5, ... is 2, $1+m$, $3+m$, $4+2m$, $7+3m$, $11+5m$, $18+8m$, $29+13m$, $47+21m$, $65+29m$, ...

The terms n-th for $m = 1,2,3,....$ stated with D_{mn}. Difference between two successive terms forming Fibonacci, stated with $D_{(i+1)j} - D_{(i)j} = F_j$ for $i = 1,2,3,....$, $j = 1,2,3,....$, and F_j is j-th term on Fibonacci sequence.

Binet formula for Fibonacci-like sequence and Sub-sequences of Fibonacci-like sequence is equal, namely $D_n = \frac{m(\alpha^n - \beta^n)}{\sqrt{5}} + (\alpha^n + \beta^n)$, where $n \geq 0$ for Fibonacci-like sequence and $n \geq 1$ for Sub-sequences of Fibonacci-like sequence, with $m = 1,2,3,....$ and $\alpha = \frac{1+\sqrt{5}}{2}, \beta = \frac{1-\sqrt{5}}{2}$.

Binet formula for Sub-sequences of Fibonacci sequence is equal with Binet formula for untuk Fibonacci sequence, namely $F_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$, $n \geq 3$.

The first and second terms are the prerequisites of Fibonacci-like sequence and Sub-sequences of Fibonacci-like sequence are even number. As a result, all tribes are on Fibonacci-like sequence and Sub-sequences of Fibonacci sequence are even, because the sum of two even numbers is even.

Next, two successive terms on Fibonacci-like sequence not relatively prime so the greatest common divisor $(D_n, D_{n+1}) \neq 1$ for $n \geq 0$. The same thing happened to Sub-sequence of Fibonacci-like sequence. Relatively prime is only owned by Sub-sequences of Fibonacci sequence.

The ratio of two consecutive terms in Fibonacci-like sequence, Sub-sequences of Fibonacci sequence, and Sub-sequences of Fibonacci-like sequence has been shown to be converging to the golden ratio.

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