Estimating Expected Time for Recruitment of Human Resource Companies Health Insurance based on Lomax Distribution

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Abstract
In a company, especially insurance companies, human resources play an important role to maintain the company's business continuity. The increasing number of insurance customers, of course, is very positive effect for insurance companies, but it must also be balanced by the company's human resources, so that recruitment of workers needs to be done, but the recruitment will be related to time and cost. In this paper, statistical model developed to obtain the expected time using Lomax Distribution models for reaching threshold level. In the time of recruitment, the assumptions were assumed that independent and identically distributed (i.i.d) random variables for time between decision epochs.

Keywords :
Human resources, Recruitment, Expected time, Lomax Distribution

1. Introduction
Human resources are important assets for the company because humans are dynamic resources and must be owned by the company in achieving its goals. Likewise in a private-owned insurance company, human resources play an important role to maintain the continuity of the company's business. The increasing number of health insurance customers, of course, has a very positive effect on health insurance companies, but this must also be balanced by the quantity and quality of human resources owned by the company, especially in the health insurance operational section in particular, so they feel the need to recruit employees. however in terms of recruitment it will require time and cost.
Seeing this phenomenon so it takes thought to determine the average recruitment time. Where it is assumed that the threshold level having the Lomax distribution . Lomax (1954) proposed Pareto Type – II Distribution, also known as Lomax Distribution, and used it for the analysis of the business failure life time data. Ghitany and Al-Awadhi

(2007) The Lomax Distribution is widely applicable in reliability and life testing problems in engineering as well as in Survival Analysis as an alternative distribution. Nayak (1987) proposed Lomax Distribution and discussed its various properties and usefulness in reliability theory.

There are many organizations which are engaged in productions, marketing and other business activities. All these organizations are very much dependent on the availability of enough manpower. Decisions regarding the targets, work scheduled, pay revisions, promotions and codes of conduct are taken by the management at random epoch. Hence the time interval between successive decisions which is the so called inter arrival times is of random character. After every decisions epoch, due to the decisions taken by the management, it may result in leaving of personnel due to unsatisfactory packages. Hence the depletion of manpower has to be compensated by suitable recruitments in order to keep the manpower availability at stable level. Recruiting employees is an important task for management team in all sectors. The expected time needs to be predicted to make decisions on recruitment, it has been a usual practice for the last few years to modify the existing probability distributions so as to improve the flexibility of the existing models.

In this paper estimasi the expected time to recruitment is derived using the concept of shock model and cumulative damage processes due to Esary et.al., (1973). It is assumed that the threshold level is a random variable which follows the so called lomax distribution. Such models have been discussed by the authors Sathiyamoorthi (1980), Pandian et.al., (2010), Kannadasan et.al., (2013), Rao and Rao (2013).

2. Assumptions of the Model

- Exit of person from an organization takes place whenever the policy decisions regarding targets, incentives and promotions are made.
- The exit of every person from the organization results in a random amount of depletion of manpower (in man hours).
- The process of depletion is linear and cumulative.
- The inter arrival times between successive occasions of wastage are i.i.d. random variables.
- If the total depletion exceeds a threshold level which is itself a random variable, the breakdown of the organization occurs. In other words recruitment becomes inevitable.
- The process, which generates the exits, the sequence of depletions and the threshold are mutually independent

2.1 Notations

X_i : a continuous random variable denoting the amount of damage/depletion caused to the system due to the exit of persons on the i[th] occasion of policy announcement, $i = 1,2,3,\dots,k$ X'_i s are i.i.d and $X_i = X$ for all i

Y : a continuous random variable denoting the threshold level having the Lomax distribution

$g(.)$: The probability density functions (p.d.f) of X_i

$g_k(.)$: The k- fold convolution of $g(.)$ i.e., p.d.f. of $\sum_{i=1}^{k} X_i$

$g*(.)$: Laplace transform of $g(.)$; $g_k^*(.)$: Laplace transform of $g_k(.)$

h (.): The probability density functions of random threshold level 'Y' which has the Lomax is the corresponding probability distribution function and $H(.)$)is the corresponding probability generating function.

U : a continuous random variable denoting the inter-arrival times between decision epochs. p.d.f. of random variable Page Layout

$\boldsymbol{f}(.)$: p.d.f. of random variable $\boldsymbol{U}$ with corresponding c.d.f $\boldsymbol{F}(.)$

$V_k(t)$: $F_k(t) - F_{k+1}(t)$

$F_k(t)$: Probability that there are exactly 'k' policies decisions in (0, t]

$S(.)$: The survivor function i.e. $P\,[T > t]$

$L(t) = 1 - S(t)$

3. Model Description

Ghitany and Al-Awadhi presented The Lomax distribution has the following cumulative distribution function (CDF) is

$$F(x,\theta) = 1 - (1+x)^{-\theta}$$

and the corresponding probability density function (PDF) is

$$f(x;\alpha) = \theta\,(1+x)^{-(\theta+1)};\quad 0 < x < \infty,\ \theta > 0$$

The corresponding Survival Function is (SF)

$$\bar{H}(x) = 1 - F(x)$$
$$= (1+x)^{-\theta}$$

The shock survival probability are given by

$$P(X_i < Y) = \int_0^\infty g_k(x)\bar{H}(x)dx$$
$$= \int_0^\infty g_k(x)(1+x)^{-\alpha}dx$$

On simplification,

$$P(X_i < Y) = (g^*(\alpha))^k$$

The survival function which gives the probability that the cumulative threshold will fail only after time *t*.

$S(t) = P(T > t) =$ Probability that the total damage survives beyond *t*

$$= \sum_{k=0}^{n} P(There\ are\ exactly\ k\ contacts\ in(0,t])$$
$$= P(the\ total\ cumulative\ threshold\ (0,t]$$

It is also known from renewal process that,

$$P(exactly\ k\ policy\ decesions\ in\ (0,t]) = F_k(t) - F_{k-1}(t) \text{ with } F_{(0)}(t) = 1$$

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t)P(X_i < Y)$$

Now, the life time is given by

$P(T < t) = L(t) =$ the distribution function of life time (T)

$$= 1 - S(t)$$
$$= 1 - \sum_{k=0}^{\infty}[F_k(t) - F_{k-1}(t)]P(X_k < Y)$$
$$= 1 - \sum_{k=0}^{\infty}[F_k(t) - F_{k-1}(t)]\ (g^*(\alpha))^k$$

Let the random variable U denoting the inter arrival time which follows exponential distribution with parameter ' .

Now, $f^*(s) = \frac{c}{c+s}$, substituting in the above equation we get,

$$L^*(s) = \frac{[1 - g^*(\alpha)]\frac{c}{c+s}}{[1 - g^*(\alpha)\frac{c}{c+s}]}$$

$$= \frac{[1 - g^*(\alpha)]c}{[c + s - g^*(\alpha)c]}$$

$E(T) = -\frac{d}{ds}L^*(s)$ given s = 0 ,

$$E(T) = \frac{1}{c[1 - g^*(\alpha)]}$$

$E(T^2) = \frac{d^2}{ds^2}L^*(s)$ given s = 0,

$$E(T^2) = \frac{d^2}{ds^2}L^*(s)$$
$$E(T^2) = \frac{1}{[1 - g^*(\alpha)]^2c^2}$$

From which the variance can be obtained,

$$V(T) = E(T^2) - (E(T))^2$$
$$= \frac{1}{c[1 - g^*(\alpha)]} - \frac{1}{[1 - g^*(\alpha)]^2c^2}$$

On simplification one can get

$$V(T) = \frac{c[1 - g^*(\alpha) - 1}{c^2[1 - g^*(\alpha)]^2}$$

4. Conclusions

The mathematical models have been discussed by various authors taking into consideration, many hypothetical assumptions. Such models provide the possible clues relating to the consequences of infections, the time taken for recruitment etc. Model obtained states that by setting c in the exponential distribution the average time will increase according to the increasing convolution function that depends on α. The same case is observed in the threshold of recruitment of Variance V(T)

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