# Solution of Fractional Ordinary Differential Equations Using the Elzaki-Adomian Decomposition Method

**Ira Sumiati**
Master Program of Mathematics, Faculty of Mathematics and Natural Sciences,
Universitas Padjadjaran, Indonesia
Email: irasumiati@gmail.com

**Sukono**
Department of Mathematics, Faculty of Mathematics and Natural Sciences,
Universitas Padjadjaran, Indonesia
Email: sukono@unpad.ac.id

**Kalfin**
Doctor Program of Mathematics, Faculty of Mathematics and Natural Sciences,
Universitas Padjadjaran, Indonesia
kalfin17001@mail.unpad.ac.id

**Abdul Talib Bon**
Department of Production and Operations,
University Tun Hussein Onn Malaysia, Malaysia
talibon@gmail.com

## Abstract
Differential equations have an important role to solve problems in various fields, for example engineering, physics, biology, and economics. Fractional calculus is a generalization of classical calculus, which includes derivatives, integrals, and differential equations of order not integers (fractional). This paper aims to solve fractional ordinary differential equations using a combination of Adomian decomposition method and Elzaki integral transform. Based on the two numerical examples presented in this paper, the results show that the Elzaki-Adomian decomposition method is very effective, useful, and easy to use to solve linear and nonlinear fractional ordinary differential equations.

**Keywords:**
Differential equation, fractional calculus, Adomian decomposition method, Elzaki transform.

## 1. Introduction

Differential equations are mathematical equations that contain a function with one or more variables and connect the function with its derivatives. Differential equations have an important role to solve problems in various fields, such as engineering, physics, biology, and economics. Fractional calculus deals with derivatives and integrals with the order not integers (fractional). Fractional calculus is a generalization of classical calculus. Today, the development and research on fractional calculus are extended to the field of differential equations, so that ordinary and partial differential equations can be ordered by fractional numbers (Podlubny, 1999; Kilbas et al., 2006; Lakshmikantham & Vatsala, 2008; Rahimy, 2010; Dahmani & Tabharit, 2014; Mathai & Haubold, 2017).

Various methods continue to be developed to solve fractional differential equations to obtain both the approach and the exact solution. For example, homotopy perturbation method (HPM), homotopy analysis method (HAM), variational iteration method (VIM), differential transform method (DTM), and Adomian decomposition method (ADM) (Ray & Bera, 2005; Abdulaziz et al., 2008; Hu et al., 2008; Hashim et al., 2009; Nazari & Shahmorad, 2010; Duan et al., 2012; Wen et al., 2012; Mohammed & Khlaif, 2014; Singh & Kumar, 2017; Ahrabi & Momenzadeh, 2018; Javeed et al., 2019; Saratha et al., 2020). These iterative methods can also be combined with integral transforms, such as the Laplace, Sumudu, and Elzaki transform. In this paper, we aim to combine the Adomian decomposition method with the Elzaki transform to solve fractional ordinary differential equations. Other research on the application of the Elzaki-Adomian decomposition method can be seen in Elzaki & Alkhateeb (2015), Mahgoub & Sedeeg (2016), Ige et al. (2019a), Ige et al. (2019b), Mohamed & Elzaki (2020), Varsoliwala & Singh (2020).

## 2. Literature Review

This section presents the basic theories and concepts related to fractional calculus and Elzaki transform.

**Definition 1.** (Elzaki, 2011) Given a set of functions

$$A = \left\{ f(t) \colon \exists M, k_1, k_2 > 0, |f(t)| < M e^{\frac{|t|}{k_j}}, t \in (-1)^j \times [0, \infty) \right\},$$

Elzaki transform is defined as

$$E[f(t)] = T(v) = v \int_0^{\infty} f(t) e^{-\frac{t}{v}} dt \, , t \geq 0, k_1 \leq v \leq k_2.$$

Next, the inverse of Elzaki transform is denoted by $E^{-1}[T(v)] = f(t)$, $t \geq 0$. For $\alpha$ is a fractional number, valid

$$E[t^{\alpha}] = \Gamma(\alpha + 1) v^{\alpha+2}. \tag{1}$$

**Definition 2.** (Podlubny, 1999) The Caputo fractional derivative of the function $f$ with $t$ in the order $\alpha$, where $\alpha > 0$, is defined as

$${}_{a}^{C}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t (t-u)^{n-\alpha-1} f^{(n)}(u) du \, , n-1 < \alpha \leq n.$$

**Definition 3.** (Elzaki, 2011) The Elzaki transform of Caputo fractional derivative for $a = 0$ is defined as

$$E[{}^{C}D_t^{\alpha} f(t)] = \frac{T(v)}{v^{\alpha}} - \sum_{k=0}^{n-1} v^{k-\alpha+2} f^{(k)}(0) \, , n-1 < \alpha \leq n.$$

## 3. Elzaki-Adomian Decomposition Method

This section presents the solution of fractional ordinary differential equations using the Elzaki-Adomian decomposition method.

Given the fractional ordinary differential equation as follows

$$D_t^{\alpha} y(t) = g(t) + Ny(t) + Ry(t), \tag{2}$$

and initial condition $y(0) = c$, where $D_t^{\alpha} \equiv {}^{C}D_t^{\alpha}$ is a Caputo fractional derivative operator with $0 < \alpha \leq 1$, $N$ is a nonlinear operator, $R$ is a linear operator, $g$ is a function that shows the homogeneity of the differential equation, and $y$ is a function of $t$ to be determined. Using the Elzaki transform in equation (2), thus based on Definition 3, is obtained

$$y(v) = v^2 y(0) + v^{\alpha} E[g(t)] + v^{\alpha} E[Ny(t)] + v^{\alpha} E[Ry(t)]. \tag{3}$$

Next, using the inverse of Elzaki transform in equation (3), is obtained

$$y(t) = y(0) + E^{-1}\left[v^{\alpha} E[g(t)]\right] + E^{-1}\left[v^{\alpha} E[Ny(t)]\right] + E^{-1}\left[v^{\alpha} E[Ry(t)]\right]. \tag{4}$$

The Adomian decomposition method assumes that the $y$ function can be broken down or decomposed into an infinite series (Adomian, 1988; Al Awadah, 2016)

$$y(t) = \sum_{n=0}^{\infty} y_n(t) = y_0 + y_1 + y_2 + \cdots, \tag{5}$$

where $y_n$ can be specified recursively. This method also assumes the nonlinear operator $Ny$ can be decomposed into an infinite polynomial series

$$Ny = \sum_{n=0}^{\infty} A_n, \tag{6}$$

where $A_n = A_n(y_0, y_1, y_2, \dots, y_n)$ is a defined Adomian polynomial,

$$A_n(y_0, y_1, y_2, \dots, y_n) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{k=0}^{n} \lambda^k y_k\right)\right]_{\lambda=0}, n = 0,1,2, \dots$$

where $\lambda$ is a parameter. The Adomian polynomial $A_n$ can be described as follows

$$\begin{aligned}
A_0 &= \frac{1}{0!}\frac{d^0}{d\lambda^0}\left[N\left(\sum_{k=0}^{0} \lambda^k y_k\right)\right]_{\lambda=0} = N(y_0),\\
A_1 &= \frac{1}{1!}\frac{d^1}{d\lambda^1}\left[N\left(\sum_{k=0}^{1} \lambda^k y_k\right)\right]_{\lambda=0} = y_1 N'(y_0),\\
A_2 &= \frac{1}{2!}\frac{d^2}{d\lambda^2}\left[N\left(\sum_{k=0}^{2} \lambda^k y_k\right)\right]_{\lambda=0} = y_2 N'(y_0) + \frac{y_1^2}{2!} N''(y_0),
\end{aligned}$$

Substitute equations (5) and (6) to equation (4), obtained

$$\sum_{n=0}^{\infty} y_n(t) = y(0) + E^{-1}\left[v^{\alpha} E[g(t)]\right] + E^{-1}\left[v^{\alpha} E\left[\sum_{n=0}^{\infty} A_n\right]\right] + E^{-1}\left[v^{\alpha} E\left[R\sum_{n=0}^{\infty} y_n(t)\right]\right]. \tag{7}$$

If both sides of equation (7) are described, then successively is obtained

$$\begin{aligned}
y_0 &= y(0) + E^{-1}\left[v^{\alpha} E[g(t)]\right],\\
y_1 &= E^{-1}\left[v^{\alpha} E[A_0]\right] + E^{-1}\left[v^{\alpha} E[Ry_0]\right],\\
y_2 &= E^{-1}\left[v^{\alpha} E[A_1]\right] + E^{-1}\left[v^{\alpha} E[Ry_1]\right],\\
y_3 &= E^{-1}\left[v^{\alpha} E[A_2]\right] + E^{-1}\left[v^{\alpha} E[Ry_2]\right],\\
&\vdots
\end{aligned}$$

thus generally obtained the recursive relation of the fractional ordinary differential equation solution (2) using the Elzaki-Adomian decomposition method as follows

$$\begin{aligned}
y_0 &= y(0) + E^{-1}\left[v^{\alpha} E[g(t)]\right],\\
y_{n+1} &= E^{-1}\left[v^{\alpha} E[A_n]\right] + E^{-1}\left[v^{\alpha} E[Ry_n]\right], n = 0,1,2, \dots
\end{aligned} \tag{8}$$

## 4. Numerical Examples

This section presents two numerical examples of applying the Elzaki-Adomian decomposition method to solve fractional ordinary differential equations.

**Example 1.** Given a linear fractional ordinary differential equation as follows

$$D_t^{\alpha} y(t) = -\frac{1}{2} y(t) + 1, t > 0, 0 < \alpha \le 1, \tag{9}$$

with initial condition $y(0) = 10$.

Based on the recursive formula in equation (8), approach solutions of linear fractional ordinary differential equation (9) using the Elzaki Adomian decomposition method is

$$\begin{aligned} y_0 &= 10 + E^{-1}\left[v^{\alpha} E[1]\right], \\ y_{n+1} &= -\frac{1}{2} E^{-1}\left[v^{\alpha} E[y_n]\right], n = 0,1,2, \dots \end{aligned} \tag{10}$$

If the recursive solution in equation (10) is described, then based on equation (1) and the inverse of Elzaki transform is obtained

$$\begin{aligned}
y_0 &= 10 + E^{-1}\left[v^{\alpha} E[1]\right] = 10 + E^{-1}[v^{\alpha+2}] = 10 + \frac{t^{\alpha}}{\Gamma(\alpha+1)}, \\
y_1 &= -\frac{1}{2} E^{-1}\left[v^{\alpha} E[y_0]\right] = -\frac{1}{2} E^{-1}\left[v^{\alpha} E\left[10 + \frac{t^{\alpha}}{\Gamma(\alpha+1)}\right]\right] = -\frac{1}{2} E^{-1}[10 v^{\alpha+2} + v^{2\alpha+2}] \\
&= -\frac{5t^{\alpha}}{\Gamma(\alpha+1)} - \frac{t^{2\alpha}}{2\Gamma(2\alpha+1)}, \\
y_2 &= -\frac{1}{2} E^{-1}\left[v^{\alpha} E[y_1]\right] = -\frac{1}{2} E^{-1}\left[v^{\alpha} E\left[-\frac{5t^{\alpha}}{\Gamma(\alpha+1)} - \frac{t^{2\alpha}}{2\Gamma(2\alpha+1)}\right]\right] = -\frac{1}{2} E^{-1}\left[-5 v^{2\alpha+2} - \frac{1}{2} v^{3\alpha+2}\right] \\
&= \frac{5t^{2\alpha}}{2\Gamma(2\alpha+1)} + \frac{t^{3\alpha}}{4\Gamma(3\alpha+1)}, \\
&\vdots
\end{aligned}$$

so we get the following approach solutions

$$y(t) = y_0 + y_1 + y_2 + \cdots = 10 - \frac{4t^{\alpha}}{\Gamma(\alpha+1)} + \frac{2t^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{t^{3\alpha}}{4\Gamma(3\alpha+1)} + \cdots. \tag{11}$$

Figure 1 shows a graph of the approach solution of a linear fractional ordinary differential equation using the Elzaki-Adomian decomposition method (11) with different $\alpha$ values.

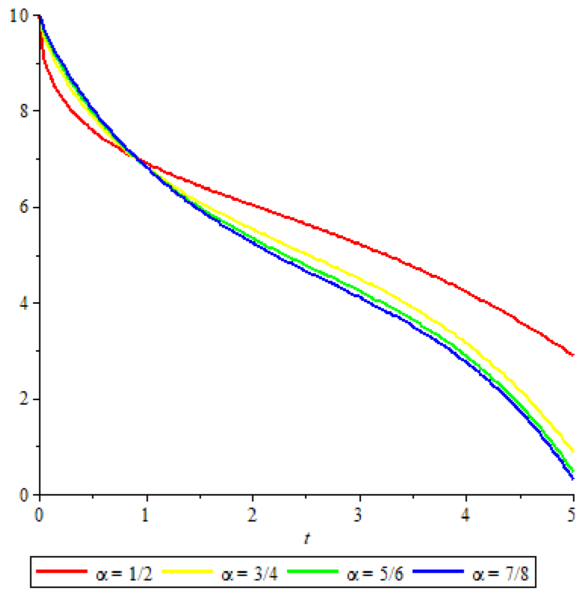


**Figure 1.** Graph of the solution of linear fractional ordinary differential equation using the Elzaki-Adomian decomposition method.

**Example 2.** Given a nonlinear fractional ordinary differential equation as follows

$$D_t^{\alpha} y(t) = 2y(t) - y^2(t) + 1, t > 0, 0 < \alpha \leq 1, \tag{12}$$

with initial condition $y(0) = 0$.

Based on the recursive formula in equation (8), approach solutions of nonlinear fractional ordinary differential equation (12) using the Elzaki Adomian decomposition method is

$$\begin{aligned} y_0 &= E^{-1}\left[v^{\alpha} E[1]\right], \\ y_{n+1} &= 2E^{-1}\left[v^{\alpha} E[y_n]\right] - E^{-1}\left[v^{\alpha} E[A_n]\right], n = 0,1,2, \ldots \end{aligned} \tag{13}$$

If the recursive solution in equation (13) is described, then based on equation (1) and the inverse of Elzaki transform is obtained

$$\begin{aligned} y_0 &= E^{-1}\left[v^{\alpha} E[1]\right] = E^{-1}\left[v^{\alpha+2}\right] = \frac{t^{\alpha}}{\Gamma(\alpha+1)}, \\ y_1 &= 2E^{-1}\left[v^{\alpha} E[y_0]\right] - E^{-1}\left[v^{\alpha} E[A_0]\right] = 2E^{-1}\left[v^{\alpha} E[y_0]\right] - E^{-1}\left[v^{\alpha} E[y_0^2]\right] \\ &= 2E^{-1}\left[v^{\alpha} E\left[\frac{t^{\alpha}}{\Gamma(\alpha+1)}\right]\right] - E^{-1}\left[v^{\alpha} E\left[\frac{t^{2\alpha}}{\Gamma^2(\alpha+1)}\right]\right] = \frac{2t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}, \\ y_2 &= 2E^{-1}\left[v^{\alpha} E[y_1]\right] - E^{-1}\left[v^{\alpha} E[A_1]\right] = 2E^{-1}\left[v^{\alpha} E[y_1]\right] - E^{-1}\left[v^{\alpha} E[2y_0 y_1]\right] \\ &= 2E^{-1}\left[v^{\alpha} E\left[\frac{2t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}\right]\right] \\ &\quad - E^{-1}\left[v^{\alpha} E\left[2\left(\frac{t^{\alpha}}{\Gamma(\alpha+1)}\right)\left(\frac{2t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}\right)\right]\right] \\ &= \frac{4t^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{2\Gamma(2\alpha+1)t^{4\alpha}}{\Gamma^2(\alpha+1)\Gamma(4\alpha+1)} - \frac{4\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma(\alpha+1)\Gamma(2\alpha+1)\Gamma(4\alpha+1)} \\ &\quad + \frac{2\Gamma(2\alpha+1)\Gamma(4\alpha+1)t^{5\alpha}}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)}, \\ &\vdots \end{aligned}$$

so we get the following approach solutions

$$
\begin{aligned}
yy(t) &= y_0 + y_1 + y_2 + \cdots \\
&= \frac{t^{\alpha}}{\Gamma(\alpha+1)} + \frac{2t^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{\Gamma(2\alpha+1)t^{3\alpha}}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} + \frac{4t^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{2\Gamma(2\alpha+1)t^{4\alpha}}{\Gamma^2(\alpha+1)\Gamma(4\alpha+1)} \\
&\quad - \frac{4\Gamma(3\alpha+1)t^{4\alpha}}{\Gamma(\alpha+1)\Gamma(2\alpha+1)\Gamma(4\alpha+1)} + \frac{2\Gamma(2\alpha+1)\Gamma(4\alpha+1)t^{5\alpha}}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)} + \cdots.
\end{aligned}
\tag{14}
$$

Figure 2 shows a graph of the approach solution of a nonlinear fractional ordinary differential equation using the Elzaki-Adomian decomposition method (14) with different $\alpha$ values.

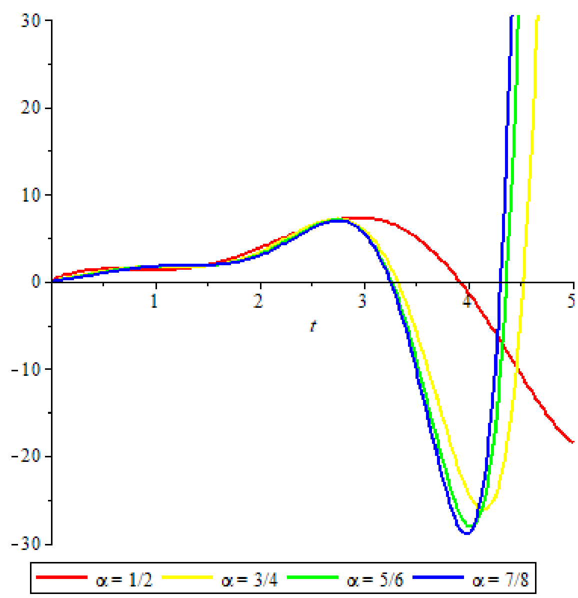


**Figure 2.** Graph of the solution of nonlinear fractional ordinary differential equation using the Elzaki-Adomian decomposition method.

## 4. Conclusion

The Elzaki-Adomian decomposition method is a combination of the Adomian decomposition method and the Elzaki integral transform. This method is very reliable and can solve ordinary or partial differential equations, linear or nonlinear, and can be an integer or fractional order. The two numerical examples presented in this paper show that the Elzaki-Adomian decomposition method is very effective, useful, and easy to use to solve linear and nonlinear fractional ordinary differential equations.

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## Biographies

**Ira Sumiati** is a magister student in the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, the field of applied mathematics, with a field of financial fractional differential.

**Sukono** is a lecturer in the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran. Currently serves as Head of Master's Program in Mathematics, the field of applied mathematics, with a field of concentration of financial mathematics and actuarial sciences.

**Kalfin** is currently a Student at the Doctoral Program at Universitas Padjadjaran, Indonesia since 2019. He received his M.Mat in Mathematics from Universitas Padjadjaran (UNPAD), Indonesia in 2019. His current research focuses on financial mathematics and actuarial sciences.

**Abdul Talib Bon** is a professor of Production and Operations Management in the Faculty of Technology Management and Business at the Universiti Tun Hussein Onn Malaysia since 1999. He has a PhD in Computer Science, which he obtained from the Universite de La Rochelle, France in the year 2008. His doctoral thesis was on topic Process Quality Improvement on Beltline Moulding Manufacturing. He studied Business Administration in the Universiti Kebangsaan Malaysia for which he was awarded the MBA in the year 1998. He's bachelor degree and diploma in Mechanical Engineering which his obtained from the Universiti Teknologi Malaysia. He received his postgraduate certificate in Mechatronics and Robotics from Carlisle, United Kingdom in 1997. He had published more 150 International Proceedings and International Journals and 8 books. He is a member of MSORSM, IIF, IEOM, IIE, INFORMS, TAM and MIM.