

# **A Deterministic, Multi-period, Multi-item Inventory Model with Supplier Selection and Emission Control**

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## **Abstract**

As we move to a more carbon-constrained world, business will ultimately have to meet customer needs in a way that reduces the carbon footprint of products across the supply chain to minimize carbon emissions and mitigate climate change. The various activities contributing to carbon emissions in a supply chain are transportation, ordering and holding of inventory. This research work develops a mixed-integer nonlinear programming (MINLP) model that considers the scenario of supply chain with multiple periods, multiple products, and multiple suppliers. The model assumes that the demand is deterministic, the buyer has a limited storage space in each period, the buyer is responsible for the transportation cost, a supplier-dependent ordering cost applies for each period in which an order is placed on a supplier and inventory shortage is permissible. The model provides an optimal decision regarding what products to order, in what quantities, with which suppliers, and in which periods to minimize the overall supply chain cost as well as associated cost of carbon emissions. For evaluating the carbon emissions, four different carbon regulating policies i.e., carbon cap-and-trade, strict cap, carbon offset and carbon tax on emissions, have been considered. The proposed MINLP has been validated using a randomly generated data set.

## **Keywords**

Supply Chain, Mixed Integer Non-Linear Program, Carbon cap-and-trade, Carbon tax, and Carbon offset.

## **1. Introduction**

In the competitive environment of the global market, there has been a steady increase in the outsourcing of raw materials, components and services to suppliers. The availability of same products with multi-suppliers along with dynamic pricing of inventory holding cost, ordering cost and transportation cost, has led customers to choose suppliers which minimize overall cost while satisfying all the requirements. Integration of lot-sizing models and supplier selection has recently achieved great attention among researchers. Several research activities in this field (Alfares & Turnadi, 2018; Cárdenas-Barrón et al., 2015; Lamba et al., 2019; Vital Soto et al., 2017; Zouadi et al., 2016) study the situation where buyers need to determine the best suppliers from which to acquire the products, the lot sizes, and the time to place the orders during a finite planning horizon while satisfying given constraints.

Along with the optimal lot-size selections from multi-suppliers, managing carbon emissions to minimize the global impact of climate change is also crucial to consider in today's supply chain management (Kaur & Singh, 2018; Lamba & Singh, 2019). In order to reduce carbon emissions, governments try to set environmental legislations to force companies to reduce their emissions while still minimizing production and transportation costs. The most important policies considered are as follows (Benjaafar et al., 2013):

- Strict carbon cap policy - where firms are subject to mandatory caps on the amount of carbon they emit.
- Carbon cap-and-trade policy - firms are subject to carbon caps but are rewarded (penalized) for emitting less (more) than their caps
- Carbon tax policy - firms are taxed on the amount of emissions they emit
- Carbon offset policy - firms can invest in carbon offsets to mitigate carbon caps to minimize the global carbon footprint.

These laws and various regulatory mechanisms are forcing the organizations to replace energy inefficient equipment and facilities, redesign products and packaging, find alternative sources of energy, or institute energy savings programs

to keep their emissions under the emission cap. The result is an economy where all companies are competing to reduce carbon emissions to meet an overall reduction target.

In this paper a multi-period, multi-product lot-sizing and supplier selection problem under storage and emission constraint is considered from a buyer's perspective. The problem is formulated as a mixed-integer nonlinear programming (MINLP) model that minimizes the overall supply chain cost of purchasing, ordering, transportation, holding and backordering as well as associated cost of carbon emissions. The results determine what products to order in what quantities with which suppliers in which periods to satisfy the overall demand.

This paper is organized as follows: Section 2 provides a glimpse of the recent researches on inventory lot-sizing problems, Section 3 presents the mathematical model, Section 4 illustrates a numerical example of the model. Finally, computational results and conclusions are presented in Section 5 and 6 respectively.

## **2. Literature Review:**

Global climate change is an important contemporary issue that is being investigated from numerous perspectives. Many prominent world leaders and scientists have raised concerns in recent years regarding increased levels of greenhouse gas (GHG) emissions and the impacts these emissions have on climate change. More than three quarters of the greenhouse gas (GHG) emissions associated with many industry sectors come from their supply chains (Huang et al., 2009). The environmental problems due to carbon emission from the industries stem from various supply chain activities such as procurement, transportation, ordering and holding of inventory (Konur & Schaefer, 2014). In addition, there are effects of these activities on the operational decisions such as supplier selection and lot-sizes, which can play an essential role for the companies to stay ahead in the competitive market.

Recently, various researchers have studied how to consider carbon emissions in production and distribution planning problems. One of the seminal work in the domain can be found in Benjaafar, Li, and Daskin (Benjaafar et al., 2013), where the authors propose a mathematical model which includes a global carbon emission constraint on the planning horizon. They perform a numerical study to derive some managerial insights. Based on the same type of constraints, Helmrich, Jans, van den Heuvel, and Wagelmans (Retel Helmrich et al., 2015) show that the problem is NP-hard and propose various solution methods to solve this problem.

Absi et al. (Absi et al., 2013) propose four types of carbon emission constraints for a multi-mode single-item uncapacitated lot-sizing problem: (1) Periodic carbon emission constraint (i.e., the amount of carbon emission that is not used in a given period is lost), (2) Cumulative carbon emission constraint (i.e., the amount of unused carbon emission of a given period can be used in future periods without exceeding the cumulative capacities), (3) Global carbon emission constraint (i.e., limit on carbon emission over the whole horizon) (4) Rolling carbon emission constraint (at each period  $t$ , that the horizon from 1 to  $t$  can be used to compensate carbon emissions between periods.). Later Absi et al. (Absi et al., 2016) analyze how fixed carbon emissions impact the problem with periodic carbon emission constraints and propose a polynomial dynamic programming algorithm when the carbon emissions parameters are stationary and the number of production modes is fixed.

Akbalik & Rapine (Akbalik & Rapine, 2014) consider an uncapacitated lot-sizing problem and choose carbon cap-and-trade as a regulatory policy. They incorporate a budget limitation on the carbon amount to purchase or to sell-and-purchase and propose a mixed integer linear programming formulation for the problem. Palak et al. (Palak et al., 2014) propose models that capture the impact of carbon regulatory mechanisms such as carbon cap, carbon tax, carbon cap-and-trade, and carbon offset, on supplier selection and transportation mode selection decisions. Konur & Schaefer (Konur & Schaefer, 2014) analyze an integrated inventory control and transportation planning problem under carbon cap, cap and trade, cap and offset, and taxing policies. They find the retailer's optimal order quantity under each regulation. Kaur & Singh (Kaur & Singh, 2018) study a procurement problem in carbon emission trading environment for a manufacturing firm pertaining to uncertainties of demand, supplier capacity and carrier capacity while meeting expected demand and simultaneously minimizing cost and carbon emissions. Phouratsamay & Cheng (Phouratsamay & Cheng, 2019) develop a dynamic programming model to determine the optimal lot-size for a single-item lot-sizing problem with inventory bounds under a carbon emissions constraint with two options for producing items: regular or green.

Lamba and Singh (Lamba & Singh, 2019) proposes a mixed-integer nonlinear programming (MINLP) model for a multi-periods, multi-products and multi-suppliers lot-sizing problem in order to provide an optimal supplier selection

and lot-sizing along with the carbon emissions using a carbon cap-and-trade policy. Lamba et al. (Lamba et al., 2019) consider the same problem and extend it by incorporating the three different carbon regulating policies viz., carbon cap-and-trade, strict cap on carbon emission and carbon tax. They study the relationships between total cost and carbon emission with carbon caps for cap-and-trade and strict cap on emission policies whereas the relationship between total cost and carbon emission with carbon tax are studied for carbon tax policy.

### **3. Mathematical Model:**

Let us assume a buyer who controls the inventory and inbound transportation for different items. The model presented in this paper is a multi-product, multi-supplier, and multi-period inventory lot-sizing problem. The buyer has a limited storage space in each period and has an initial budget which will limit their carbon transactions. This paper considers four different carbon emissions regulatory policies, namely strict cap on emission, carbon cap-and-trade, carbon offset and carbon tax. The basic assumptions are listed below.

Assumptions:

- Demands are known for each item and each period.
- Each item can be ordered at most once per period.
- Each item is sold by several suppliers offering different purchasing prices.
- Lead-time is known and constant, and each order is fully received at the start of the given period.
- Either inventory or shortage are possible in any period.
- Each vehicle can transport several items, within its given volume capacity.
- The planning horizon is known and finite.
- Several orders from several suppliers can be made, but only one supplier is included in any given order.
- The initial value for both inventory and shortage is zero.
- All shortages must be fulfilled at the end of the planning horizon.

Indices

$i$  Product Index  $i = 1 \dots n$   
 $j$  Supplier Index  $j = 1 \dots m$   
 $t$  period index  $t = 1 \dots T$

Parameters:

$D_{it}$  Demand of product  $i$  in period  $t$   
 $P_{ijt}$  Per unit purchasing cost of product  $i$  from supplier  $j$  in period  $t$   
 $O_j$  Ordering cost for supplier  $j$   
 $h_i$  Per unit holding cost of product  $i$   
 $t_j$  Transportation of cost for supplier  $j$   
 $b_i$  Per unit backordering cost of product  $i$

$w_i$  Occupied space by product  $i$  in warehouse or freight truck  
 $r_j$  Freight load capacity of the truck (tons) from supplier  $j$   
 $W$  Storage capacity of the buyer

$E_t^{order}$  Emission due to ordering in period  $t$   
 $E_t^{hold}$  Emission due to inventory holding in period  $t$   
 $E_t^{trans}$  Emission due to transportation in period  $t$   
 $C_{cap}$  Emission cap for entire planning horizon  
 $f$  Carbon tax paid on each unit emitted  
 $G$  Available budget for buying carbon credit or paying the carbon tax or investing on emission reduction activities

Decision Variable:

$X_{ijt}$  Quantity of product  $i$  ordered from supplier  $j$  in period  $t$

$Y_{jt}$  = 1 if the order is placed with supplier  $j$  in period  $t$ ; 0, otherwise  
 $Z_{jt}$  Number of freight truck required by supplier  $j$  in period  $t$   
 $I_{it}$  Inventory of product  $i$  at the end of period  $t$   
 $B_{it}$  Quantity to be backordered for product  $i$  at the end of period  $t$   
 $E_{Total}$  Total carbon emission due to ordering, transporting and inventory holding for the entire planning horizon

This paper extended the models developed by Benjaafar et al. (2013) by incorporating multiple product, multiple supplier into the model and by adding a budget constraint for controlling the emission and the limited storage space constraint.

### 3.1 MINLP formulation under strict cap on emission policy

$$\text{Minimize } \sum_{t=1}^T \sum_{j=1}^m \sum_{i=1}^n P_{ijt} X_{ijt} + \sum_{t=1}^T \sum_{j=1}^m (O_j Y_{jt} + Z_{jt} t_j Y_{jt}) + \sum_{t=1}^T \sum_{i=1}^n (h_i I_{it} + b_i B_{it}) \quad (1)$$

Subject to,

$$I_{i(t-1)} + \sum_{j=1}^m X_{ijt} - B_{i(t-1)} = D_{it} + I_{it} - B_{it} \quad \forall i \in \{1..n\}, t \in \{1..T\} \quad (2)$$

$$(\sum_{k=t}^T D_{ik}) Y_{jt} - X_{ijt} \geq 0 \quad \forall i \in \{1..n\}, j \in \{1..m\}, t \in \{1..T\} \quad (3)$$

$$\sum_{i=1}^n w_i X_{ijt} \leq r_j Z_{jt} \quad \forall j \in \{1..m\}, t \in \{1..T\} \quad (4)$$

$$\sum_{i=1}^n w_i (\sum_{k=1}^t \sum_{j=1}^m X_{ijk} - \sum_{k=1}^t D_{ik}) \leq W \quad \forall t \in \{1..T\} \quad (5)$$

$$E_{Total} = \sum_{t=1}^T \sum_{j=1}^m (E_t^{order} Y_{jt} + E_t^{trans} Z_{jt}) + \sum_{t=1}^T \sum_{i=1}^n E_t^{hold} I_{it} \quad (6)$$

$$E_{Total} \leq C_{cap} \quad (7)$$

$$X_{ijt}, I_{it}, B_{it}, Z_{jt} \geq 0 \quad \forall i \in \{1..n\}, j \in \{1..m\}, t \in \{1..T\} \quad (8)$$

$$Y_{jt} \in \{0,1\} \quad \forall j \in \{1..m\}, t \in \{1..T\} \quad (9)$$

The complete MINLP model is presented as the minimization of the objective function (1), subject to constraints (2) – (9). The first term in Eq (1) refers to the total purchasing cost, the second term refers to the ordering and transportation costs, the third term refers to the inventory holding and backlogging cost over the planning horizon of periods. Constraints (2) are net inventory balance equations. Constraints (3) ensure that  $Y_{jt} = 1$  whenever  $X_{ijt} > 0$ . Constraints (4) calculate the number of freight trucks required by the supplier for transportation. Since buyer has a limited storage space available, Eq. (5) ensures that the storage space constraints are met. Equation (6) computes the total carbon emission due to production, inventory and setup for each period. Constraint (7) controls carbon emissions under strict carbon cap regulatory mechanism applied over the planning horizon, which states that the total emissions should not exceed the available emission cap ( $C_{cap}$ ) applied over the horizon. Constraints (8) declare the nonnegativity of the variables and constraint and constraints (9) are the integrality ones.

### 3.2 MINLP formulation under cap-and-trade policy

Another alternative policy to controlling pollution is emissions trading, which is also known as carbon cap-and-trade policy. Under the carbon cap-and-trade policy, a central authority issues a limited number of annual permits that allow companies to emit a certain amount of carbon dioxide. The total amount permitted thus becomes the "cap" on emissions which is lowered each year. Companies are penalized if they produce a higher level of emissions than their permits allow. Companies that reduce their emissions can sell, or "trade," unused permits to other companies. In a realistic scenario, the companies will probably have an initial budget which will limit their carbon transactions. Let  $p$  be the market price of one unit of carbon. Note that  $p$  is assumed to be stationary in this study to avoid speculation. The additional decision variables are used as follows:

$e_t^p$  Carbon credit purchased in period  $t$   
 $e_t^s$  Carbon credit sold in period  $t$

We can reformulate the problem in (1)–(9) as follows.

$$\text{Min } \sum_{t=1}^T \sum_{j=1}^m \sum_{i=1}^n P_{ijt} X_{ijt} + \sum_{t=1}^T \sum_{j=1}^m (O_j Y_{jt} + Z_{jt} t_j Y_{jt}) + \sum_{t=1}^T \sum_{i=1}^n (h_i I_{it} + b_i B_{it}) + p \sum_{t=1}^T (e_t^p - e_t^s) \quad (10)$$

Subject to,

(2)-(6), (8), (9)

$$E_{Total} \leq C_{cap} + \sum_{t=1}^T (e_t^p - e_t^s) \quad (11)$$

$$p \sum_{t=1}^T e_t^p \leq G + p \sum_{t=1}^T e_t^s \quad (12)$$

$$e_t^p, e_t^s \geq 0 \quad \forall t \in \{1..T\} \quad (13)$$

The objective function Eq. (10) is to minimize the buyer's total cost over the entire planning horizon under carbon cap-and-trade regulation. Constraint (11) controls carbon emissions under carbon cap-and-trade regulatory mechanism applied over the planning horizon. Constraint (11) balances the total carbon emitted due to ordering, holding inventory, and transportation over the entire planning horizon against the maximum permissible carbon cap and the excess carbon bought/sold. Buying or selling carbon credits help in relaxing the carbon cap imposed over the horizon. Constraint (12) ensures that the total carbon units the firm can buy over the entire horizon must not exceed the available budget, which can be increased from incentives obtained by selling the unused carbon units. Constraints (13) correspond to non-negativity constraints.

### 3.3 MINLP formulation for carbon tax

An alternative to strict caps on emissions is not to restrict emissions but instead to penalize emissions using a *carbon tax*. A carbon tax can take on a variety of forms. In its simplest, the tax is a financial penalty linear in the number of carbon units emitted. To illustrate how a carbon tax would modify the problem formulation in (1)–(9), the problem can be restated as follows.

$$\text{Min } \sum_{t=1}^T \sum_{j=1}^m \sum_{i=1}^n P_{ijt} X_{ijt} + \sum_{t=1}^T \sum_{j=1}^m (O_j Y_{jt} + Z_{jt} t_j Y_{jt}) + \sum_{t=1}^T \sum_{i=1}^n (h_i I_{it} + b_i B_{it}) + f E_{Total} \quad (14)$$

Subject to

(2)-(6), (8), (9)

$$f E_{Total} \leq G \quad \forall t \in \{1..T\} \quad (15)$$

The objective function given by Eq. (14) is to minimize the buyer's total cost that includes the tax levied on each unit of carbon emission over the entire planning horizon. Constraint (15) ensures that the total tax the buyer can pay over the entire horizon must not exceed the available budget.

### 3.4 MINLP formulation for carbon offset

Under the cap and offset model, similar to the cap and trade model, the buyer is subject to carbon emissions cap  $C_{cap}$  per unit time; however, a carbon trading system is not available. Carbon offsets are investments a firm would make in carbon-reducing projects, typically offered by a third party, to offset emissions that exceed its specified cap. If we assume that  $g$  be the price per unit of carbon offset and  $e_t^{offset}$  be the amount of carbon emissions per unit time that the buyer decides to compensate by investing in carbon offset projects, then the problem in (1)–(9) can be reformulated as follows:

$$\text{Min } \sum_{t=1}^T \sum_{j=1}^m \sum_{i=1}^n P_{ijt} X_{ijt} + \sum_{t=1}^T \sum_{j=1}^m (O_j Y_{jt} + Z_{jt} t_j Y_{jt}) + \sum_{t=1}^T \sum_{i=1}^n (h_i I_{it} + b_i B_{it}) + g \sum_{t=1}^T e_t^{offset} \quad (16)$$

Subject to,

(2)-(6), (8), (9)

$$E_{Total} \leq C_{cap} + \sum_{t=1}^T e_t^{offset} \quad (17)$$

$$g \sum_{t=1}^T e_t^{offset} \leq G \quad (18)$$

$$e_t^{offset} \geq 0 \quad (19)$$

This formulation is similar to the one for cap-and-trade, except that the buyer does not benefit from emitting less than its specified cap. The objective function Eq. (16) is to minimize the buyer's total cost over the entire planning horizon under the carbon offset regulatory mechanism. Constraint (17) ensures that the total carbon emitted due to ordering, holding inventory, and transportation does not exceed the carbon cap plus the carbon allowances achieved through investing in carbon offset projects. Constraint (18) ensures that the total money the firm can spend for emission reduction activities over the entire horizon must not exceed the available budget. Constraints (19) correspond to non-negativity constraints.

#### 4. Computational Study:

The MINLP presented in this paper has been illustrated through a randomly generated dataset. The datasets correspond to 3 products, 3 suppliers and 4 periods (3P-3S-4T). All the models have been solved FICO’s Mosel (Xpress-IVE Version 8.92 64 bit) algebraic modeling language and solved using “mrxnlp” solver. All the test instances are run on a PC with an Intel Core i5 2.81 GHz processor, 16 GB of RAM and an L2 cache of 1 MB.

The mean value of demands ( $D_{it}$ ) is randomly generated from a uniform distribution  $U(50,100)$  in all instances. The storage capacity of the warehouse is equal to 300. Other parameters such as purchasing costs are generated from a uniform distribution  $DU(20,50)$ , ordering costs from  $U(150,300)$ , holding costs from  $DU(0.3,0.9)$ , transportation costs from  $DU(500,3000)$  and storage space required per item from  $DU(1,5)$ . A ratio of backloging costs to inventory costs such that  $b_{it} = 10h_{it} \forall i = 1,2,..n, t = 1,2,..T$ . The emission due to ordering ( $E_t^{order}$ ), transportation ( $E_t^{transport}$ ), and holding ( $\hat{h}_j$ ) activities are all generated from discrete uniform distributions  $DU(100,250)$ ,  $DU(0, 0.9)$ , and  $DU(0,0.9)$  respectively. To study the effect of increasing carbon cap on emissions, carbon cap has been varied from 300 to 900 with increments of 50.

Figure 1 depicts the supplier and lot-size distribution for all the time periods under strict emission cap policy. Table 1 and Figure 2 shows the impact of the variation of carbon emission cap on the total cost and total emission for the instances considered. It is clear from Table 1 and Figure 2 that carbon emissions increase with the increase in carbon cap while the total cost decreases. However, it is noticeable that initially the total cost decreases sharply with the increase of carbon cap. Furthermore, the total cost curve becomes almost flat after a certain value of carbon cap under the strict emission cap policy. It is evident that reducing the carbon cap by 30.7% (from 900 to 700) has resulted in carbon emission reduction by 7.8% (from 660.6 to 716.7) while increasing the costs by 7.1% (from 14165.5 to 13225.8).

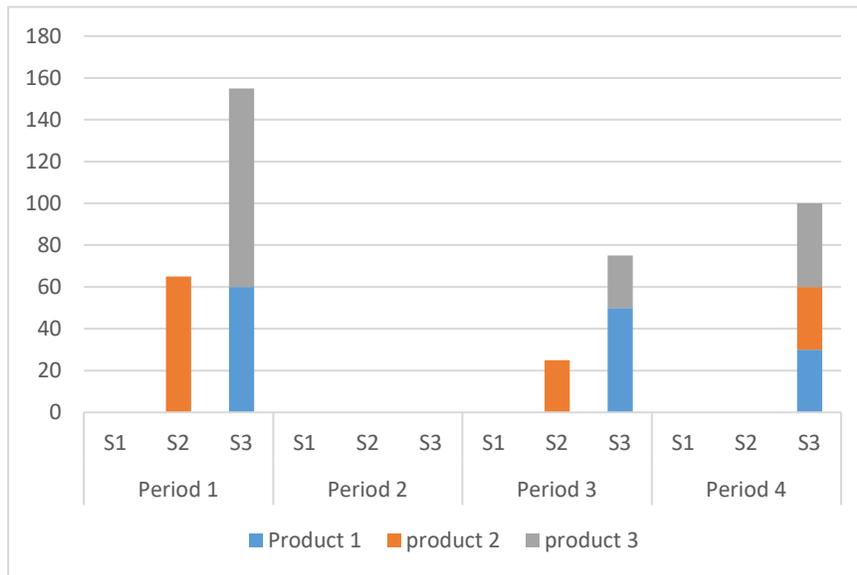


Figure 1: Order Quantity of three products over a planning horizon of 4 periods

Table 2 shows the experimental results of varying the carbon cap on total cost and the carbon emissions at different carbon price under carbon cap and trade policy. It is to be noted that the carbon cap has no effect on the emissions, and it remains constant at a given carbon price. The carbon emission value is constant at 469.8 for  $p = 20$  and 30 and 481.6 for  $p = 10$ . Figure 3 shows the variation of cost and carbon emission with carbon cap carbon cap and trade policy. It is observed from Figure 3 that if the emission cap is below the total emission, the total cost increases as the carbon price increases. However, if the emission cap is higher than the total emission, a higher carbon price can lead to lower total cost. When carbon price is relatively low, the firm is mostly engaged in the buying of carbon. Therefore, higher carbon prices increase its carbon purchasing cost. When the carbon price is sufficiently high, the firm becomes

engaged in the selling of carbon, as the firm finds it advantageous to adjust its operations and emit less carbon. Figure 4 shows the variation of total emission with budget for a given carbon price under cap and trade policy. It is noticeable from Figure 4 that total emission decrease with tighter budget.

Table 1: Total cost and carbon emission with the variation of emission cap at different carbon price under strict carbon cap policy.

Emission Cap	Total Cost	Total Emission	Emission Cap	Total Cost	Total Emission
300	23677.2	299.6	650	14845.8	640.6
350	21817.5	349.8	700	14165.5	660.6
400	19965.2	399.5	750	13876.6	680
450	18234.5	449.3	800	13657	704.5
500	17487.5	499.7	850	13456	710.5
550	16868.6	549.6	900	13225.8	716.7
600	15342.5	599.6	950	12989.8	722.8

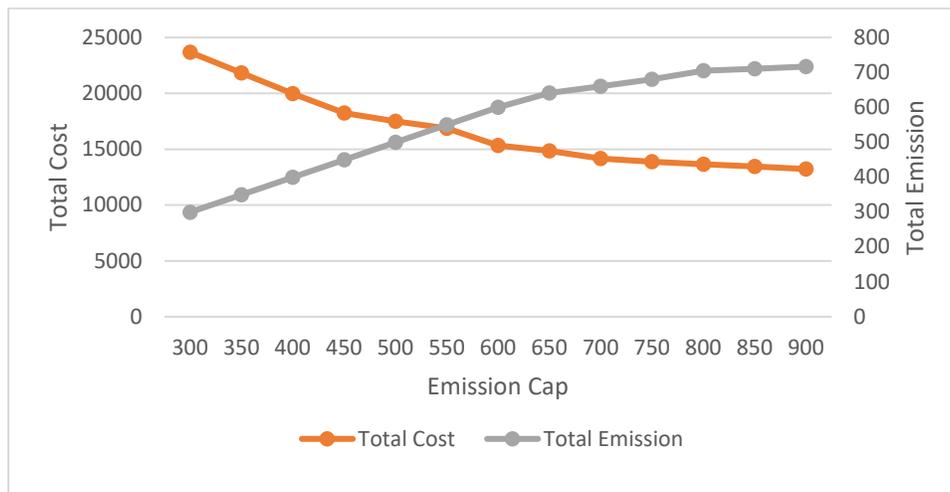


Figure 2: Variation of cost and carbon emission with carbon cap under strict emission cap policy

Table 2: Total cost and carbon emission with the variation of emission cap at different carbon price under carbon cap and trade policy.

Emission Cap	p=10		p=20		p=30	
	Total Cost	Total Emission	Total Cost	Total Emission	Total Cost	Total Emission
400	11430.6	481.6	11781.6	469.8	12821	469.2
450	10930.3	481.6	11031.6	469.8	11321.4	469.2
500	10430.3	481.6	10281.6	469.8	9821	469.2
550	9930.3	481.6	9531.6	469.8	8321	469.2
600	9430.3	481.6	8781.6	469.8	6821	469.2
650	8930	481.6	8031.6	469.8	5321	469.2
700	8430	481.6	7281.6	469.8	3821.4	469.2
750	7930.3	481.6	6531.6	469.8	2321.4	469.2
800	7430.3	481.6	5781.6	469.8	821.4	469.2

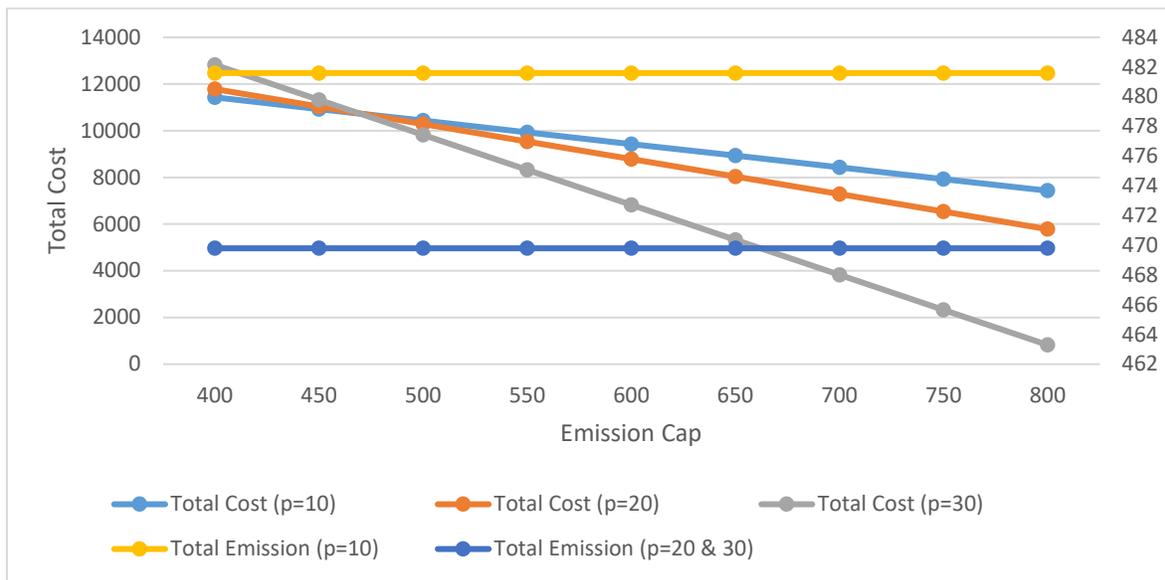


Figure 3: Variation of cost and carbon emission with carbon cap carbon cap and trade policy

Table 2: Total cost and carbon emission with the variation of emission cap at different carbon price under carbon cap and trade policy.

Emission Cap	p=10		p=20		p=30	
	Total Cost	Total Emission	Total Cost	Total Emission	Total Cost	Total Emission
400	11430.6	481.6	11781.6	469.8	12821	469.2
450	10930.3	481.6	11031.6	469.8	11321.4	469.2
500	10430.3	481.6	10281.6	469.8	9821	469.2
550	9930.3	481.6	9531.6	469.8	8321	469.2
600	9430.3	481.6	8781.6	469.8	6821	469.2
650	8930	481.6	8031.6	469.8	5321	469.2
700	8430	481.6	7281.6	469.8	3821.4	469.2
750	7930.3	481.6	6531.6	469.8	2321.4	469.2
800	7430.3	481.6	5781.6	469.8	821.4	469.2

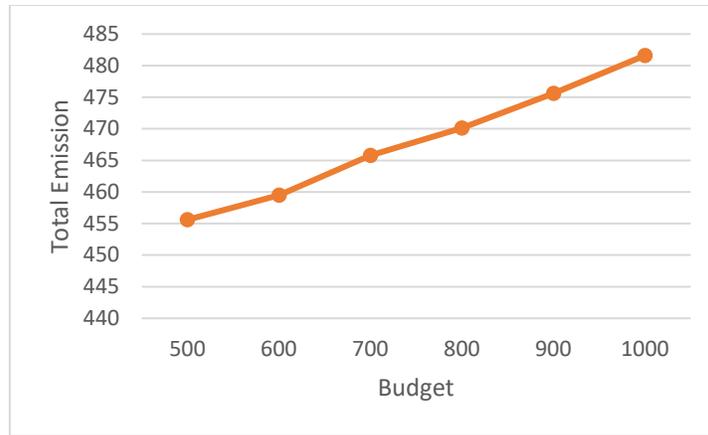


Figure 4: Variation of total emission with budget under cap and trade policy

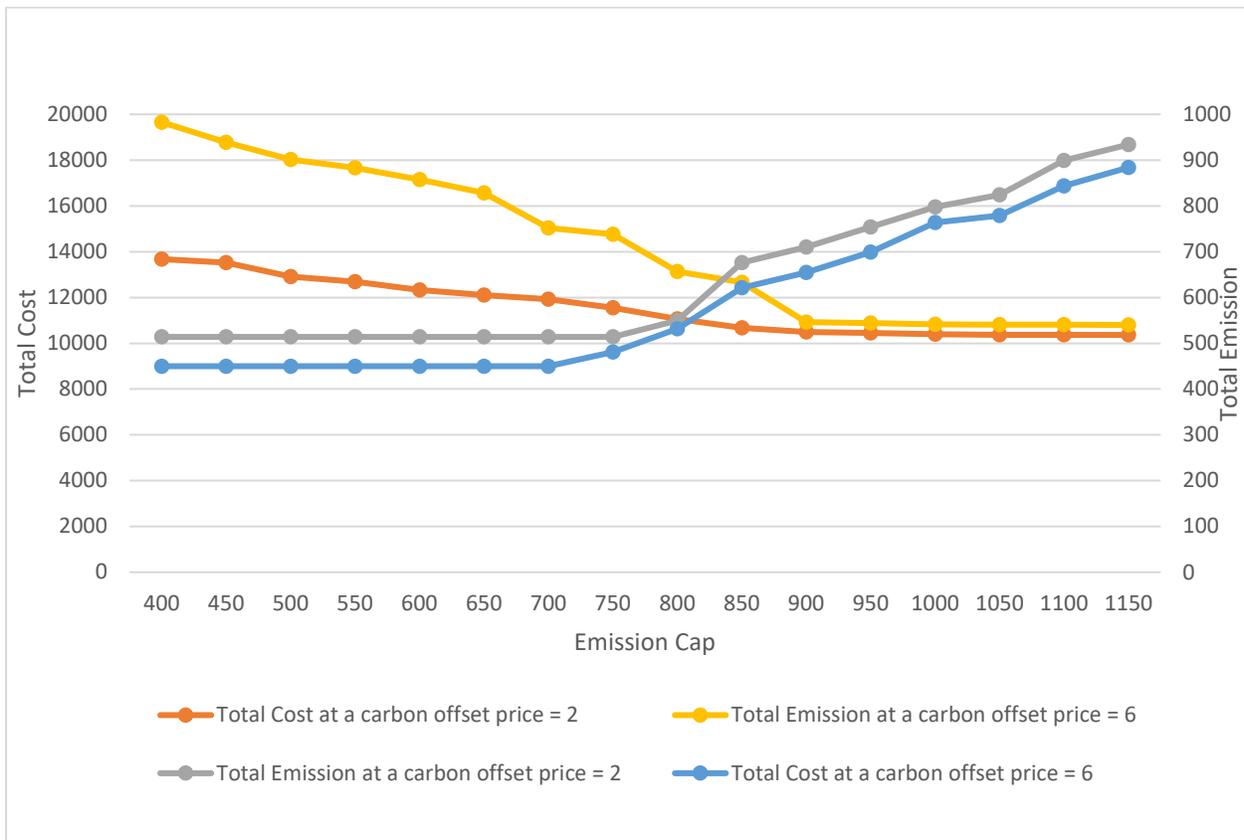


Figure 5: Effect of varying the emission cap and unit carbon offset price when a firm has the option of purchasing carbon offsets

Figure 5 illustrates the impact of varying the emission cap and unit carbon offset price when a firm has the option of purchasing carbon offsets. It is obvious from Figure 5 that total cost is higher at lower emission cap. As the cap increases the total cost decreases and flattens at a value of emission cap 800. The total carbon emission is almost stable at the beginning and start to increase at a value of emission cap 700. Figure 5 also depicts that the total emission is higher at lower carbon offset price while the total cost is lower at lower carbon offset price.

Table 4: experimental results for tax on carbon emissions model for different carbon tax values

Carbon Tax	Total Cost	Total Emission	Carbon Tax	Total Cost	Total Emission
0	6167.2	481.6	1.4	8343.4	469.8
0.2	6239	481.6	1.6	8545.4	469.8
0.4	6465.3	481.6	1.8	8,887.30	469.8
0.6	6820.9	481.6	2	9277.4	469.8
0.8	7134.8	481.6	2.5	9848.3	444.5
1	7508.2	469.8	3	10,367.30	444.5
1.2	7976.3	469.8	5	11168.5	444.5
1.4	8343.4	469.8	10	12546.5	444.5

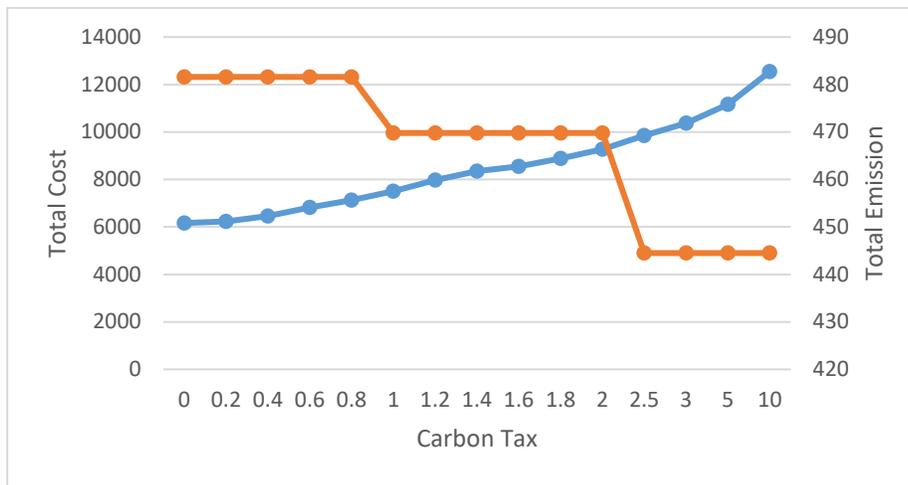


Figure 6: variation of cost and carbon emission with carbon tax

Table 5: Ordering Schedule with the variation of carbon Tax

	Supplier	Carbon tax = 0.4				Carbon tax = 1				Carbon tax = 2			
		t=1	t=2	t=3	t=4	t=1	t=2	t=3	t=4	t=1	t=2	t=3	t=4
Product 1	1	24	0	0	0	30	0	0	0	60	0	0	0
	2	0	86	0	30	0	90	0	25	0	80	0	25
	3	0	0	0	0	0	0	0	0	0	0	0	0
Product 2	1	89	0	0	0	78	0	0	0	41	0	0	0
	2	0	0	0	0	0	0	0	0	0	0	0	0
	3	0	0	12	30	0	0	30	26	0	0	49	33
Product 3	1	50	0	0	0	65	0	0	0	90	0	0	0
	2	0	0	0	0	0	0	0	0	0	0	0	0
	3	0	0	78	32	0	0	75	30	0	0	44	35

Under carbon tax policy of carbon emissions regulation, there is no cap on emissions. However, each unit of the carbon emitted is taxed at a certain price. The carbon tax rates have been varied for 0 to 10 and the experimental results of varying the carbon tax rate on total cost and carbon emission is shown in Table 4. Figure 6 shows the variation of total

costs and carbon emissions with varying rates of carbon tax. The carbon emission curve shows a piece-wise behavior with overall reduction while the cost continues to increase. Thus, a carbon tax provides a simpler mechanism for reducing carbon emissions quickly and reliably. Table 5 shows the ordering schedule with varying carbon tax rate. It is clear from Table 5 that carbon tax does not have any impact on the supplier selection decision.

## **5. Conclusion:**

This paper develops a mixed-integer nonlinear programming (MINLP) model that considers the scenario of supply chain with multiple periods, multiple products, and multiple suppliers. The model assumes that the demand is deterministic, the buyer has a limited storage space in each period, the buyer is responsible for the transportation cost, a supplier-dependent ordering cost applies for each period in which an order is placed on a supplier and inventory shortage is permissible. The model provides an optimal decision regarding what products to order, in what quantities, with which suppliers, and in which periods to minimize the overall supply chain cost as well as associated cost of carbon emissions. This paper evaluates the impact of carbon regulatory mechanisms such as carbon cap, carbon tax, carbon cap-and-trade, and carbon offset, on inventory replenishment decision. While analyzing the strict cap on emission policy, carbon emissions increase with the increase in carbon cap while the total cost decreases. For carbon cap and trade emission policy, it has been observed that the carbon cap has no effect on the emissions, and it remains constant at a given carbon price. Furthermore, it has been noticed that if the emission cap is below the total emission, the total cost increases as the carbon price increases. However, if the emission cap is higher than the total emission, a higher carbon price can lead to lower total cost. The analysis of carbon offset policy indicates that total cost is higher at lower emission cap. As the cap increases the total cost decreases and flattens at a specific value of emission cap. The total carbon emission is almost stable at the beginning and start to increase at a specific value of emission cap. Moreover, it has been observed that the total emission is higher at lower carbon offset price while the total cost is lower at lower carbon offset price. Under carbon tax policy of carbon emissions regulation, the carbon emission curve shows a piece-wise behavior with overall reduction while the cost continues to increase. Thus, a carbon tax provides a simpler mechanism for reducing carbon emissions quickly and reliably. The analysis of the ordering schedule with varying carbon tax rate shows that carbon tax does not have any impact on the supplier selection decision.

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