Single-Asset Portfolio Allocation Using Markov Decision Process – A Case from the Saudi Stock Market

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Abstract

This paper presents a Markov Decision Process (MDP) model for single portfolio allocation in Saudi Exchange Market. The model consists of six states small increase, medium increase, large increase, small decrease, medium decrease and large decrease with three decisions, buy, sell and keep. The model is developed for three major Saudi companies namely SABIC, Rajhi Bank and SEC along with TASI index. 5-years data were analyzed to developed Markovien transition probabilities. Analysis for both the Markov Chain, and the MDP, provides the investors with the necessary insights to make informed and optimum decisions.

Keywords
Single-Asset Portfolio, Markov Decision Process, Stock Market

1. Introduction

Stochastic investment models attempt to relate the variations of prices and returns on assets and asset classes, such as bonds and stocks, over time. As for investments, stochastic models can be classified differently, having different models for single assets and multiple assets. Such modeling is, much of the time, used for financial planning and actuarial work that allows investors and traders to optimize asset allocation as well as asset-liability management. Stochastic modelling is crucial in the field of asset allocation as new variables may come into play at any time, and because the number of variables that may have an effect could be high, stochastic models sometimes run hundreds or even thousands of times, offering potential outcomes for nearly every situation a business, industry, portfolio or agency may face.

The paper addresses the following research questions:

- How Saudi leading companies' stocks (i.e. Saudi Arabian Basic Industries Corporation (SABIC), Rajhi Bank and Saudi Electricity Company (SEC) are correlated with Tadawul All Share Index (TASI)'s Index?
- What is the steady states for Saudi leading companies' stocks (SABIC, Rajhi Bank and SEC) and TASI?
- What are the best course of actions for a single asset portfolio? (Buy/Sell Decision Problem)

The methodology is develop a financial stochastic model for Saudi leading companies' stocks (SABIC, Rajhi Bank and SEC) and TASI index to predict the stock behaviour and its steady state. Consecutively, the second part, is to develop a policy for trading to maximize the profit using Markov Decision Process (MDP).
The remainder of the paper is structured as follows: The next section summarizes the related literature, and Section 3 presents the proposed financial model formulation. Section 4 illustrates the proposed MDP model via numerical example. Finally, Section 5 presents future research opportunities and concludes the paper.

2. Literature Review

Stock exchanges markets have been known for their uncertainty. However, it does not discourage the creation predictive models for various stock markets (Agwuegbo et al., 2010). Investors and investment bankers use a wide range of tools to predict the market trend and consequently build their portfolio allocation strategies (Atsalakis & Valavanis, 2009). In this section, all Markov chain related work on stock market prediction and portfolio allocation is surveyed. The papers are classified in single Markov chain models and integrated techniques with it.

(Ghezzi et al.,2003) was the seminal paper of using Markov chain in the stock markets. a Markov chain model to establish a stock value estimation model based on dividends and stock price, which l enables the reader to grasp the rationale underlying a stock market without any involvement in misleading analytical subtleties. The main insight of the paper is that bankruptcy cannot be escaped in the long run and that the mean time to bankruptcy can be readily computed.(Hassan & Nath, 2005) used hidden Markov models (HMM) approach for forecasting stock price for interrelated markets. The prediction result is tested against the comment method used in the literature artificial neural networks (ANN), the mean absolute percentage errors (MAPE) values of the two methods are quite similar. Whilst, the primary weakness with ANNs is the inability to properly explain the models. The proposed method using HMM to forecast stock price is explainable and has solid statistical foundation. Several papers followed on various stock markets., (Agwuegbo et al., 2010) developed a random walk model for stock market prices prediction, they analyzed the behaviour of daily return of Nigeria stock market prices. The sample included daily market prices of all securities listed in the Nigeria Stock Exchange. The result from the study provided evidence that the Nigeria stock exchange is not efficient even in weak form and that NSE follow the random walk model. The idealized stock price in the Nigeria stock exchange is a martingale. They concluded saying that martingale defines the fairness or unfairness of the investment and no investor can alter the stock price as defined by expectation. (Vasanthi et al.,2011) conducted an empirical study on stock index trend prediction using markov chain analysis. In this study, the First Order Markov Chain Model is applied to indices of various stock exchanges round the globe. Indices from markets like the American stock markets (DJIA, S&P 500), European Markets (FTSE, FTSH), Australian markets (AUSTA^ORD), China (SSE^), South East Asian markets (Hang Seng), Pakistan (KSE), India (BSE, NSE) etc. are chosen for the study. All major stock market indices representing popular investment destinations are included in the study. The results of the trend prediction using Markov Chain analysis is compared with the results obtained through traditional trend prediction tools. The prediction of the trend using Markov Chain Model is done using short term (one-year data), medium term (3 year data) and long term (5 year data) and the results are compared. The results of the study show that majority of the time, Markov model outperforms the traditional trend prediction methods. Followed by a model by (Doubleday and Julius ,2011) to determine the correlation between a diverse portfolio of stocks and the market for the US market. (Škrinjarić, V Kojić, 2014) modelled stock returns on Zagreb stock exchange by using Markov chains. (Svoboda,2016) developed a Stochastic model of short-term prediction of stock prices and its profitability in the Czech Stock Market. (Farshchian and Jahan, 2015) used Hidden Markov Model (HMM) to predict the changes in Tehran Stock Exchanges. In this technique, the normal factors that affect stock prices are used along with abnormal conditions such as political effect and other factors. All the factors in the data are trained with the help of Baum-Welch Algorithm after that the progressive prediction was achieved by HMM method. The overall accuracy, specificity and sensitivity will be increased up to 2% compared to other previous systems. The periodic accuracy is found to be non-linear compared to original stock value data's (Saadat & Rahmani, 2016).

The Markov decision process is also used combined with other theories on stock investment with good effect. For example, (Hassan,2009) combined hidden Markov chains and fuzzy theory to devise a model to predict stock market volatility and find the best fuzzy rules. The paper presented experimental results clearly show an improved forecasting accuracy compared to other forecasting models such as, Auto Regressive Integrated Moving Average (ARIMA), artificial neural network (ANN) and another HMM-based forecasting model.(Hsu et al.,2009) integrated the Markov chain, gray theory and Fourier series to forecast turning points of the stock market weighted index. The forecast accuracy of the combination HMM–fuzzy model is found to be always better when compared to the ARIMA and ANN. (Wang et al.,2010) applied Markov chain and fuzzy theory to create a predictive model for the stock market index. Following data validation, the results not only demonstrate the ability to improve return on investment, but also to stop losses. (Chang & Lee, 2017) proposed a novel application for incorporating Markov decision process on genetic algorithms to develop stock trading strategies and formulate correct timing (portfolio adjustment) and trading strategies (buy or sell). The experimental results demonstrated model proposed in this study, obtains better rate of return than the buy and hold strategy, the Taiwan market index and TWN50.

The work closest related to the paper at hand is the one of (Doubleday and Julius , 2011), who developed a model to determine the relationship between a diverse portfolio of stocks and the market as a whole for the US market , however, the paper did not develop a model to choose the best decision. This paper therefore contributes to the literature by proposing a decision making model for the buy, sell and keep problem in the stock market, after proving the relationship between the major Saudi companies with TASI index.

3. Financial Model Formulation

The main objective of the financial model is to predict how financial market behaves using Marko Chains.

3.1 2x2 States

The model states are {0,1} = {Down, up}, when the stock goes down it will be in state 0, while when it goes up it will be in state 1, regardless of the amount.

3.2 2x2 Transition Probabilities

The transition probabilities were developed by dividing number of visits to states over the total number of moves. Five years Saudi market data was collected and analyzed per the following transition probability matrices categories.
Transition Probabilities:

Table 1: TASI Transition Probabilities

TASI		0	1
Transition Probability	0	51%	49%
	1	39%	61%

Table 2: SABIC Transition Probabilities

SABIC		0	1
Transition Probability	0	49%	51%
	1	53%	47%

Table 3: SEC Transition Probabilities

SEC		0	1
Transition Probability	0	54%	46%
	1	49%	51%

Table 4: Rajhi Bank Transition Probabilities

Rajhi Bank		0	1
Transition Probability	0	53%	47%
	1	45%	55%

3.2 Transition Probability & Steady States Discussion

All probabilities are around 50%, which is expected, as the stock market is random by nature. When the Saudi stock market (TASI) goes up, there is 61% probability it will further increase next day.

It is important to note when SABIC stock goes down, it is hard to decide the stock behavior the next day, while when SEC stock goes down, there is 54% chance it will further decrease next day. When Rajhi bank stock goes up, there is 55% probability it will further increase next day.

Saudi stock market (TASI) steady state probability is 55.7% chance it will go up regardless its current stock prices, while the other major stocks have steady state probability around 50% to go up or down. Moreover, SABIC & SEC stocks have almost the same steady states probability.

In order to have more insights and to get more detailed picture of the Saudi financial stock market behavior, 6x6 matrices were developed as following:

3.3 6x6 States

In order to have a deeper insights and analysis, the states space is further expanded as shown in Table 5. The States are {0s, 0m, 0h,1s,1m,1h}, the Interpretations are detailed below:

Table 5: 6x6 States Interpretations

State	Interpretations
0s	0-3.33% Decrease
0m	3.34-6.6% Decrease
0h	6.7-10% Decrease
1s	0-3.33% Increase
1m	3.34-6.6% Increase
1h	6.7-10% Increase

3.4 6x6 Transition Probabilities

The transition probability matrices were developed as following:

Table 6: TASI 6x6 Transition Probabilities

TASI		0s	0m	0h	1s	1m	1h
Transition Probability	0s	56.3%	1.1%	0.2%	43.5%	0.1%	0.0%
	0m	19.8%	19.8%	0.0%	52.9%	0.0%	5.3%
	0h	0.0%	56.0%	0.0%	0.0%	45.0%	0.0%
	1s	32.9%	0.5%	0.1%	68.3%	0.3%	0.0%
	1m	21.2%	21.2%	0.0%	28.3%	0.0%	28.3%
	1h	42.5%	0.0%	0.0%	56.5%	0.0%	0.0%

Table 7: SABIC 6x6 Transition Probabilities

SABIC		0s	0m	0h	1s	1m	1h
Transition Probability	0s	57.8%	2.6%	0.4%	41.6%	1.0%	0.3%
	0m	46.4%	15.5%	3.9%	31.7%	7.9%	0.0%
	0h	0.0%	0.0%	20.6%	42.2%	14.1%	14.1%
	1s	59.2%	2.9%	0.2%	38.4%	1.4%	0.3%
	1m	53.4%	0.0%	4.9%	25.1%	14.4%	3.6%
	1h	19.4%	19.4%	0.0%	28.7%	14.4%	14.4%

Table 8: SEC 6x6 Transition Probabilities

SEC		0s	0m	0h	1s	1m	1h
Transition Probability	0s	45.9%	3.6%	0.2%	50.0%	0.9%	0.2%
	0m	33.1%	10.2%	0.0%	37.3%	18.7%	3.1%
	0h	0.0%	68.7%	0.0%	0.0%	28.0%	0.0%
	1s	41.6%	1.0%	0.2%	55.7%	2.8%	0.9%
	1m	28.9%	0.0%	0.0%	56.6%	17.4%	4.4%
	1h	43.3%	21.7%	10.8%	14.7%	0.0%	0.0%

Table 9: Rajhi Bank 6x6 Transition Probabilities

Rajhi Bank		0s	0m	0h	1s	1m	1h
Transition Probability	0s	41.9%	1.0%	0.0%	62.8%	1.0%	0.2%
	0m	30.3%	5.0%	0.0%	42.7%	25.6%	8.5%
	0h	0.0%	80.7%	0.0%	0.0%	0.0%	0.0%
	1s	37.8%	0.5%	0.1%	65.3%	2.4%	0.5%
	1m	44.0%	4.0%	0.0%	32.5%	8.1%	8.1%
	1h	24.0%	24.0%	0.0%	48.7%	0.0%	0.0%

4. Markov Decision Process (MDP) Model

In this section, we will model the decision of optimizing the portfolio decision for holding, buying or selling the asset as a Markov Decision Process. The application of MDP will be based on the 2x2 matrix, since it is less complex for implementation. The below subsections will highlight the states, actions space, reward criteria and optimal policy. The last subsection will discuss the obtained results.

4.1 States

The state space S of the MDP consists of the combinations of stock price state and {Down, Up} and weights of the specific stock or index in the portfolio. For simplicity, we allow a discrete set of three possible weights {0, 0.5, 1}. Therefore, the MDP states will be as follows:

S = {(D,0), (D, 0.5), (D, 1), (U,0), (U,0.5), (U,1)}

4.2 Action Space

The action space A consists of all possible actions that can be taken to change the weight of the stock portfolio form the previous a state.

A= {Do Nothing, sell 50% of asset, sell all asset, buy 50% of asset, Buy all asset)

However, when we apply the actions, they are not applicable for all of the MDP states. For example, let's say the system is currently in state (D,0), which means that the stock is down, and we invested nothing in the stock. We can apply actions 1, 4 & 5, which are either Do Nothing or Buy a certain weight in the stock. We can't sell what we don't have.

It was decided to apply the MDP firstly on TASI and observe the market behaviour overall.
Building on the above, the following are the transition matrices for each action for TASI:

Table 10: Action 1: Do Nothing Transition Matrix

	D, 0	D, 0.5	D, 1	U, 0	U, 0.5	U, 1
D, 0	0.50			0.50		
D, 0.5		0.50			0.50	
D, 1			0.50			0.50
U, 0	0.40			0.60		
U, 0.5		0.40			0.60	
U, 1			0.40			0.60

Table 11: Action 2: Invest 50% Transition Matrix

	D, 0	D, 0.5	D, 1	U, 0	U, 0.5	U, 1
D, 0		0.50			0.50	
D, 0.5			0.50			0.50
D, 1						
U, 0		0.40			0.60	
U, 0.5			0.40			0.60
U, 1						

Table 12: Action 3: Invest All Transition Matrix

	D, 0	D, 0.5	D, 1	U, 0	U, 0.5	U, 1
D, 0			0.50			0.50
D, 0.5						

D, 1						
U, 0			0.40			0.60
U, 0.5						
U, 1						

Table 13: Action 4: Sell 50% Transition Matrix

	D, 0	D, 0.5	D, 1	U, 0	U, 0.5	U, 1
D, 0						
D, 0.5	0.50			0.50		
D, 1		0.50			0.50	
U, 0						
U, 0.5	0.40			0.60		
U, 1		0.40			0.60	

Table 14: Action 5: Sell All Transition Matrix

	D, 0	D, 0.5	D, 1	U, 0	U, 0.5	U, 1
D, 0						
D, 0.5						
D, 1	0.50			0.50		
U, 0						
U, 0.5						
U, 1	0.40			0.60		

4.2 Reward

Let W_i be the assigned weight in MDP state i, and L_k be the mean change in the stock/index price where k ∈ {Down, Up}. The reward criteria R for each action is the expected change of the stock price given the weight change applied. For example, let's say the system was in state (U,1), which means the stock price is up, and we invested all our portfolio in it already. If we apply action "Sell All", and the system transitions into state (D,0), it means that we sold, and the stock price was down. So, the reward will be penalized by the average loss that was determined from the data.

$$\boldsymbol{r_{ij} = |W_i - W_j| * L_j}$$

4.3 Optimal Policy:

"*A policy is a function that maps an action to every state. We say a policy is optimal if it generates at least as much total reward as all other possible policies*" (Bookstaber, 2005).

We applied the value iteration model in the proposed paper, and like Taha book model and obtained an interesting result. No matter how many stages we analyzed, the actions for each state remained the same and didn't change. If we choose TASI for example (Figure 1) to apply this model for 4 months of 2018, the total gain was 21% over a default policy (Figure 2) of just purchasing the stocks across and leaving it with no action.

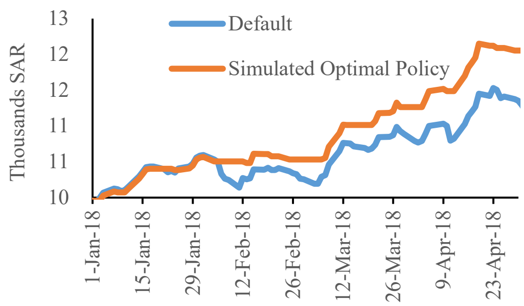

Figure 2: Value of Optimum Policy vs. Default

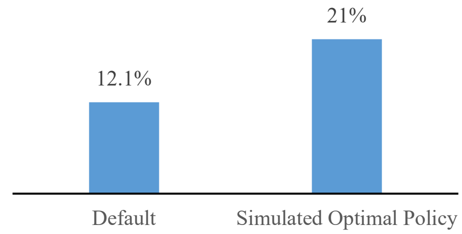

Figure 3: ROI for Optimum Policy vs. Default

5. Conclusion and Future research

We opted to analyze the Saudi Stock Market as a Markov Chain, and then modelled the buy/sell decision as a Markov Decision Process. For the Markov Chain, we modelled the increase and decreases first as only two states (Up, Down). All probabilities were around 50%, especially in the long run. Only TASI overall showed that there is a 61% probability it will further increase next day, given it had increased yesterday.

We've further split the decrease & increase states into 3 more states, totalling to 6 states overall, where each one represents a small, medium or large increase/decrease. We've obtained similar results, but this gave more granular level of details which is of extreme paramount to investors. For example, if Rajhi Bank experiences a large hit, there is a 100% chance it will face a medium hit in the next day.

Lastly, we applied the value iteration model in the second paper. The analysis revealed that no matter how many stages we analyzed, the actions for each state remained the same and unchanged. However, the simulated policy that was tested on TASI only, showed a total gain of 21% over a default policy of just purchasing the stocks across and leaving them with no action. Such analysis for both the Markov Chain, and the MDP, provides the investors with the necessary insights to make informed and optimum decisions. Future research can be applied to the model such as adding more states (enlarged state space and probably higher order Markov chain) or more actions joint consideration of more than one stock.

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Mohammad M. AlDurgam received the B.Sc. and M.Sc. degrees in industrial engineering from The University of Jordan, Amman, Jordan, in 2002 and 2005 respectively; and obtained the Ph.D. degree in systems engineering from King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia, in 2009. Since 2010, he joined the Systems Engineering Department, King Fahd University of Petroleum and Minerals, Dhahran, KSA as an assistant Professor. His research interests include maintenance planning and control, supply chain management and project management. He served as principal Investigator in different research and consulting projects. Dr. Mohammad AlDurgam is member of the Jordanian Engineers Association.