A Multi-Period Inventory Replenishment Policy for ATM Network

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Abstract

Automated Teller Machines (ATMs) are one of the most important cash distribution channels for the banks. Banks are expanding their ATM network to satisfy customer demand and are facing operating costs. This work focuses on the inventory management for optimizing the replenishment policy of the Automated Teller Machine (ATM) network. We introduce a multi-period strategy that helps to minimize the total cost of ATMs replenishment. In addition, the paper discusses how to obtain the shortest tour to replenish ATMs. We formulate the problem as a Mixed Integer Nonlinear Programming (MINLP) model and solve it using GAMS solver BARON, to minimize the total cost associated with replenishing ATMs. Then, we linearize the MINLP and compare the results of both models.

Keywords

Cash management, ATM replenishment, Multi-period, Mixed integer nonlinear programming, Mixed integer linear programming.

1. Introduction

The use of Automated Teller Machines (ATMs) has increased over the years and ATMs have become one of the important banking service channels. In the inventory management of the ATMs, several actions; such as forecasting, replenishments and the denomination mix used, influence the total cost of the inventory system. With a good demand forecast, the challenge is to determine replenishment and denomination mix strategies. To the best of our knowledge, the first work presented the cash management problem as an inventory problem was proposed in a deterministic environment by Baumol (1952). One of the earliest studies of the cash inventory problems was presented by Girgis (1968). The author considered a continuous net cash flows with both fixed and linear transaction costs. That study was followed by a set of studies discussing different inventory policies under deterministic and stochastic conditions (Bell and Hamidi-Noori, 1984; Hamidi-Noori and Bell, 1982; Neave, 1970). da Costa Moraes et al. (2015) and Hasheminejad and Reisjafari (2017) presented two of the most recent literature reviews of the cash management problems. Most recently, the cash management and ATM replenishment problem have attracted the attention of many researchers and many features of the problem have been investigated such as investigating the effect of parameters uncertainty on the cash management problem (Castro, 2009; Ferstl and Weissensteiner, 2010; García Cabello and Lobillo, 2017; López Lázaro et al., 2018; Righetto et al., 2016), the incorporation of multi-objective approach (Salas-Molina et al., 2017, 2018a, 2018b), and the utilization of recent artificial intelligence approaches in cash management problem (López Lázaro et al., 2018; Venkatesh et al., 2014). One approach to model the cash management problem is to formulate the problem as a mathematical program (Barbosa and Pimentel, 2001; Cardona and Moreno, 2012; Larrain et al., 2017; López Lázaro et al., 2018; Righetto et al., 2016). This work focuses on the inventory management for optimizing the replenishment policy of the Automated Teller Machine (ATM) network. It also discusses how to determine the shortest route for replenishing the ATMs that minimizes the total cost. The decisions of this problem are, quantity of each denomination, the replenishment frequency and the route to replenish the ATM. Varying these decisions will affect the holding cost, fixed cost and stock-out costs. The holding cost depicts the loss from the missed interest rate that couldn't be made due to the money tied up in the ATM. Fixed costs are those incurred whenever a replenishment is needed, where driver salary, fuel, and insurance constitute the majority of the fixed costs. Thus, fixed cost is influenced by the length of the path taken when to replenish the ATM. Finally, we consider stock-out as a cost associated with the fail to satisfy the requested cash amount. In the literature, there are several studies considered the problem of joining inventory management and routing problems (Ekinci et al., 2015).

In this study, we consider the forecasted demand for the requested cash at ATM as the number of bills of a particular class being withdrawn form an ATM at a certain period (Ekinci et al., 2015; Venkatesh et al., 2014). More elaboration of the lost-demand will be discussed in following sections. Most banks use predetermined denomination mix which is based on the least note strategy, the smallest number of bills to satisfy a cash withdrawal (Larrain et al., 2017; van Anholt et al., 2016). The difficulty of determining the optimal denomination mix is acknowledged in the literature. The suggested ATM replenishment strategies in the literature are still inefficient (Larrain et al., 2017; van Anholt et al., 2016).

In this paper, we present a multi-period, varying denomination mixes to determine an optimal strategy to satisfy the demand. The presented model considers the above mentioned aspects and analyzes the effect of the denomination mixes on the various types of costs and inventory. This leads to a reduction in the frequency of visiting the ATMs, and hence, decreasing the total cost.

The remainder of this paper is organized as follows: problem statement is discussed in Section 2. Mathematical models formulations are presented in Section 3. The numerical results and are reported in Section 4. Finally, Section 5 concludes the paper and presents limitations and future work.

2. Problem Statement

The objective of this work is to introduce a policy that enables analyzing the cost of replenishing ATMs over a planning horizon. The goal is to formulate a mathematical model to minimize the costs associated with replenishing ATMs distributed in a network. The total cost consists of fixed cost, inventory cost, and stock-out cost. This model considers a forecast demand representing the number of bills withdrawn from each ATM at different periods. We first begin our analysis by defining the underlying assumptions of the problem.

Consider a network contains a set I of ATMs distributed in a city, and let $i \in I = \{1, 2, \ldots a\}$, where each ATM has a set of J boxes, and let $j \in J = \{1, 2, ..., b\}$. For simplicity, we will assume three boxes for each ATM, each box contains bills of class n, where $n \in N = \{500, 100, 50, ...\}$. We assume that box 1 has bills of class 500, box 2 has bills of class 100, and box 3 has bills of class 50 for each ATM. Furthermore, filling of ATMs will be conducted at several periods t, where $t \in T = \{1, 2, \ldots, p\}$. Periods are assumed to be quarterly, therefore in this study we have twelve quarters. Now, the demand at each period is fixed and known and is defined as $\boldsymbol{b_{ijt}}$ which represents the number of bills requested from box j of ATM i at period t. The maximum capacity of box j of ATM i is represented by $\boldsymbol{Q_{ij}}$. The remaining bills of each box j is defined as $\boldsymbol{S_{ijt}}$. Obtaining the remaining inventory for each period will give us the inventory cost. $\boldsymbol{h}$ represents the holding cost as interest rate for the money remaining in each ATM and not used. The monetary value $\boldsymbol{l_j}$ represents the class of monetary value of bills. Also, when we need to replenish ATM i, we define this state by a binary variable $\boldsymbol{y_{it}}$ where its value is 1 when ATM i is visited at period t, and zero otherwise. A fixed cost $\boldsymbol{c}$ is incurred whenever an ATM is visited, and the box to be replenished is defined by a binary variable $\boldsymbol{w_{ijt}}$. The binary variable $\boldsymbol{x_{ijt}}$ represents the state of stock-out in box j of ATM i at period t. The stock-out occurs for example when demand for three bills of class 500, and the ATM has only two bills. When there is a shortage to satisfy demand, $\boldsymbol{x_{ijt}}$ equals to 1, and 0 otherwise. A penalty $\boldsymbol{M}$ is applied in case of stock-out due to the loss of goodwill. The bank's policy to replenish ATMs depends on the number of bills to transport to each ATM, and using positive integer variable $\boldsymbol{q_{ijt}}$ to represents the number of bills to be transported to ATM i, but it is not necessary to replenish ATMs every period, depending on the demand.

3. Mathematical Models

In this section, we present the mathematical models to minimize the cost associated with the replenishment of an ATM network.

3.1. Mixed Integer Nonlinear Program

The proposed MINLP model for the ATM network replenishment policy is presented as follows:

$$\min \sum_{i=1}^{a} \sum_{t=1}^{p} \left[c.y_{it} + \sum_{j=1}^{b} \left(h.l_j.S_{ijt} + M.x_{ijt} \right) \right]$$

Subject to:

$$\sum_{j=1}^{b} w_{ijt} \leq y_{it}, \forall\, i\ \in I \wedge t \in T \tag{1}$$

$$S_{ijt} \leq Q_{ij}, \forall\, i\ \in I \wedge j \in J \wedge t \in T \tag{2}$$

$$S_{ij(t+1)} = (S_{ijt} - b_{ijt}).(1 - x_{ijt}) + q_{ijt}, \forall\, i\ \in I \wedge j \in J \wedge t \in T \tag{3}$$

$$S_{ijt} - b_{ijt} \leq M.(1 - x_{ijt}), \forall\, i\ \in I \wedge j \in J \wedge t \in T \tag{4}$$

$$b_{ijt} - S_{ijt} \leq M.x_{ijt}, \forall\, i\ \in I \wedge j \in J \wedge t \in T \tag{5}$$

$$q_{ijt} \leq w_{ijt}.Q_{ij}, \forall\, i\ \in I \wedge j \in J \wedge t \in T \tag{6}$$

$$y_{it}, x_{ijt}, w_{ijt} \in \{0, 1\}, \forall\, i\ \in I \wedge j \in J \wedge t \in T \tag{7}$$

$$S_{ijt} \in R, \forall\, i\ \in I \wedge j \in J \wedge t \in T \tag{8}$$

$$q_{ijt} \in Z^{+}, \forall\, i\ \in I \wedge j \in J \wedge t \in T \tag{9}$$

The objective function is to minimize the total cost composed of replenishment, holding cost and stock-out costs. $\boldsymbol{M}$ should be large enough to prevent stock-out unless there is no other option. Constraint (1) ensure that boxes can only be replenished if their ATMs are visited. Constraint (2) and (3) impose the inventory in each ATM. constraint (2) ensure that the inventory in each box is less than or equal its maximum capacity. Constraint (3) ensure that the inventory in the next period is equal to the difference between the inventory and the remaining bills plus the quantity to be delivered if there is no stock-out. If there is a stock-out, the quantity to be delivered in the next period $(t + 1)$ will constitute the inventory at that period, which can vary from zero to the maximum capacity. Constraints (4), and (5) define the states of stock-out, that is when the inventory is less than the demand, those constraints will force $\boldsymbol{x_{ijt}}$ value to be 1, therefore, lost-demand is detected. Constraint (6) ensure that the quantity to be delivered in each box does not exceed its maximum capacity, given that a visit is required. Constraint (7), (8), and (9) define the domain of the variables**.**

3.2. Linearization of the Proposed MINLP

In Constrain (3), the term $\boldsymbol{S_{ijt}x_{ijt}}$ causes the previous model to be nonlinear. The MINLP model presented in the previous section is linearized by introducing a new positive variable $\boldsymbol{SS_{ijt}}$ and replacing Constraint (3) with Constraint (10). Constraint (11) enforce the introduced variable $\boldsymbol{SS_{ijt}}$ to take any value as long as no stock-out occurs. Constraint (12) insures that $\boldsymbol{SS_{ijt}}$ is less than or equal to the inventory as long as the inventory value is positive. Constraint (13) triggers whenever a stock-out occurs which enables $\boldsymbol{SS_{ijt}}$ to be greater than the inventory and preserve the condition on the domain of $\boldsymbol{SS_{ijt}}$ in constraint (14).

$$\min \sum_{i=1}^{a} \sum_{t=1}^{p} \left[c.y_{it} + \sum_{j=1}^{b} (h.l_j.S_{ijt} + M.x_{ijt}) \right]$$

Subject to: Constraints (1), (2), (4), (5), (6), (7), (8), (9)

$$S_{ij(t+1)} = S_{ijt} - b_{ijt} - SS_{ijt} + b_{ijt}.x_{ijt} + q_{ijt}, \forall\, i\ \in I \wedge j \in J \wedge t \in T \tag{10}$$

$$SS_{ijt} \leq M.x_{ijt}, \forall\, i\ \in I \wedge j \in J \wedge t \in T \tag{11}$$

$$SS_{ijt} \leq S_{ijt}, \forall\, i\ \in I \wedge j \in J \wedge t \in T \tag{12}$$

$$SS_{ijt} \leq S_{ijt} - M.(1 - x_{ijt}), \forall\, i\ \in I \wedge j \in J \wedge t \in T \tag{13}$$

$$SS_{ijt} \geq 0, \forall\, i\ \in I \wedge j \in J \wedge t \in T \tag{14}$$

4. Solution and Results

This section presents the computational experiments. We run 50 different randomly generated trials consisting of 3, 4, …, 12 periods, and 10, 20, …, 50 ATMs. Next, we present the results obtained from solving the proposed models. The results show the performance of the MINLP model, linearized model, and compare their results based on the total cost.

4.1 Analysis of Mixed Integer Nonlinear Program

This section discusses the analysis of the results of implementing the MINLP model. We first solve the MINLP model using Baron solver in Gams software.

Figure 1 shows the cost of each set of planning periods for ten, twenty, thirty, forty and fifty ATMs. One can notice that as long as the number of ATMs increases, the cost increases dramatically. On the other hand, we can notice that the cost for a set of three periods with a set of ten ATMs is 773,793 however, the cost for a set of twelve periods is 3,072,100. This shows that the increase of the number of planning periods will significantly affect the increase in the total cost by 297%. The cost is 3,962,155 for a set of fifty ATMs with three planning periods with an increase of 412% compared to the cost for a set of ten ATMs with the same number of planning periods. In addition, the cost is 16,108,890 for a set of fifty ATMs with twelve planning periods which results in 306% increase compared to the cost if the number of planning periods is three. Finally, we conclude that the cost increase associated to the increase in the number of ATMs is much higher than the increase resulting from increasing the number of planning periods.

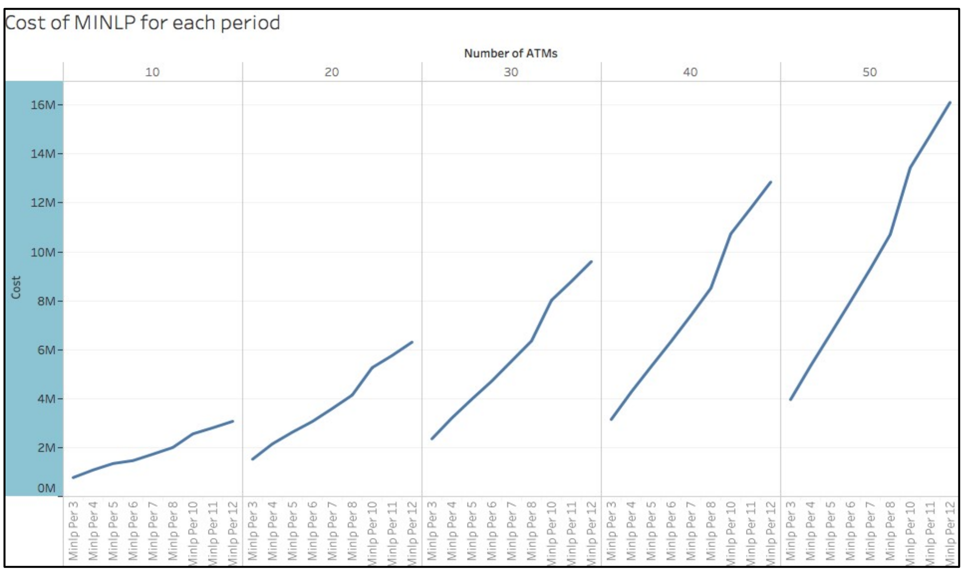

Figure 1. Total cost of the MINLP

4.2 Analysis of Mixed Integer Program

In this section we analyze the results of implementing the linearized model using CPLEX solver in GAMS software. Figure 2 shows the total cost of the MIP model for each set of planning periods with different numbers of ATMs. Based on the results in Figure 2, we can notice that the cost increases if we increase the number of planning periods. We also can realize that the cost increases dramatically if we increase the number of ATMs. If the number of planning periods is three for a set of ten ATMs the cost is 799,945 while it is 3,137,667 if the number of planning periods is twelve, which results in 292% increasing in the cost value. However, for a set of three planning periods with a set of fifty ATMs the cost is 3,997,585 this is 400% increase in the cost compared to the result with a set of ten ATMs. In addition, the cost increase for a set of twenty ATMs with three planning periods and twelve planning periods is 309% whereas it is 304% for a set of thirty ATMs and it is 305% for a set of forty ATMs. If we want to compare the cost for each set of periods with a set of ten ATMs and a set of fifty ATMs we realize the following; there is 394% cost increase if the number of planning periods is four, 386% cost increase when the number of planning periods is five and 423% cost increase when the number of planning periods is six.

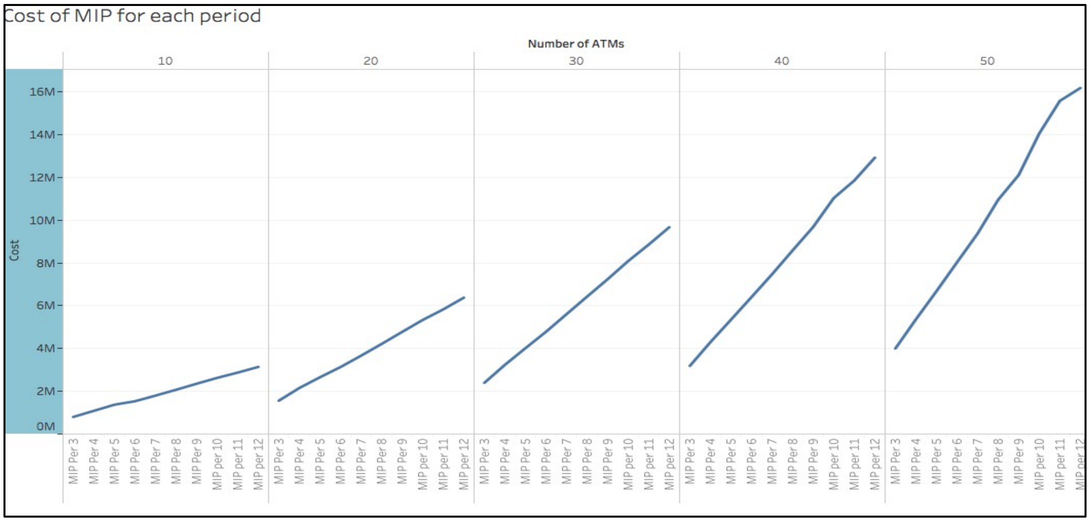

Figure 2. Total cost of the MIP

4.3. Comparison of the Results of MINLP and MILP

In this section, we compare the obtained results of implementing both MINLP and MILP. Figure 3 illustrates the comparison between the cost results of implementing MINLP and MIP model for each set of ATMs with different sets of planning periods. We realize that the costs determined by applying the MIP are almost equivalent to the costs of the MINLP model. For example, the cost determined by the MIP model for a set of ten ATMs for a set of three periods is 799,945 whereas it is 773,792 for the MINLP model which is a 3.38% difference. Also, for a set of six periods, the cost determined by the MIP model is 1,533,252 whereas the cost determined by the MINLP model is 1,467,685 for a set of ten ATMs which indicates a 4.40% difference.

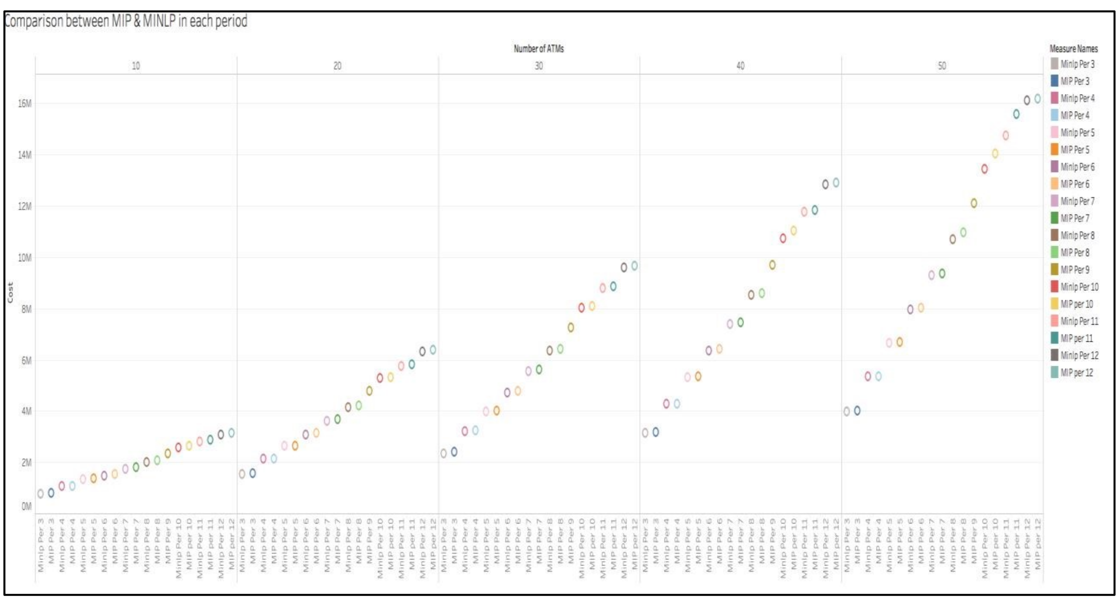

Figure 3. Comparison between the solutions of MINLP and MIP

5. Conclusion and Future Work

To conclude our work, we introduce the ATM replenishment problem. We formulate the problem as a Mixed Integer Nonlinear Program (MINLP) model. Then, we linearize the MINLP and obtain the results from both models and compare them. We can conclude that using the MINLP for replenishment problem delivered better results in term of the total cost, but the computational time consumed to solve the model is somehow larger than the time taken by the MIP. A significant increase in the total cost is observed going from set of numbers of ATMs into a larger number. Finally, the data used in this project is not obtained from any real bank, it was randomly generated for purposes of testing the model. It would be of great interest if we can get a real problem and test our proposed models with real data. Future work directions include considering stochastic demand and the associated safety stock, and the effect of stock-out of ATMs at special times such as nights or weekends, where bank branches are not working to reload ATMs.

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