Find The Maximum Clique by Graph Coloring Using Heuristic Greedy

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Abstract

Graph coloring problem is to find the minimal number of colors to color vertex of a graph in such a way that every two vertex linked by an edge have different colors. A vertex coloring algorithm has been presented. As a result of applying vertex coloring algorithm no two vertex are to be allocated in same color if they are adjacent in graph. In this paper, Graph coloring used for to find maximum clique in a graph with heuristic greedy.

Keyword: Graph Coloring, Vertex Coloring, Clique Maksimum, Heuristic Greedy.

1. Introduction

Graph coloring problem is one of the most useful models in graph theory. This problem has been used to solve problems in school Timetabling (De Werra, 1998), computers enrolling the allocation (Chow and Hennessy, 1990), electronic broadband allocation (Gamst, 1986), and many others. This application shows that an effective algorithm to solve the problem of color staining will be very important. Although this link exists, relatively few methods are available to solve the graph precisely, but are limited to solving the graph with a slight vertex. In contrast, heuristic techniques have been designed that break the graphs with hundreds or thousands of vertices (M. R. Garey and D. S. Johnson,1976) on the fees of suboptimal completion on a regular basis.
Given an undirected graph $G(V, E)$ with n vertices and vertex set denoted by $V(G)$, a clique is any complete subgraph of G. A clique of order r is denoted by K_r. A maximal clique in a *graph* $G(V, E)$ is a *clique* that can not be entirely contained within another *clique* (Wasserman and Faust1994), while a maximum clique is the largest complete subgraph of G. The order of the maximum clique of G is called the clique number of G, and is denoted by $\omega(G)$.

2. Greedy Coloring

As defined in the introduction, the coloring problem consists of finding a minimum partition of the vertices of a graph into independent sets. Since partitions can be represented as functions, there is an equivalent formulation of the problem in terms of functions. A coloring c of a graph $G = (V, E)$ is a function from V to the set of positive integers. A coloring is proper if adjacent vertices of G are assigned different numbers (also referred to as colors). The coloring problem consists of finding a coloring with the minimum numbers of colors[Jose M. Bonnin Cadogan, 2011].

Remark If C is a clique of G, then $|C| \leq \chi(G)$.

The following greedy algorithm finds a proper coloring of any graph. Color the vertices one by one so that the color of each vertex is the smallest positive integer not used by its colored neighbors.

Remark If we greedily color a vertex v, $c(v)$ uses a color in $[1, \delta(v) + 1]$.

Consider a vertex v whose neighborhood is a clique. $N[v]$ is a clique of size $\delta(v) + 1$, so $\delta(v) + 1 \leq \chi(G)$. It follows that if we greedily color v, $c(v) \leq \chi(G)$. We can use this fact to reduce the problem of coloring G to the problem of coloring $G - v$.

Remark If G is a subgraph of H, then $\chi(G) \leq \chi(H)$.

Remark Let c be a proper coloring of a graph G. If for every vertex v of G, $c(v) \leq \chi(G)$, then c is an optimum coloring.

Definition A simplicial vertex is a one whose neighborhood is a clique.

Let v be a simplicial vertex. We will show that the problem of coloring G can be reduced to the problem of coloring $G - v$. Let c be an optimum coloring of $G - v$. We color each vertex w different from v with color $c(w)$. We know that $c(w) \leq \chi(G - v) \leq \chi(G)$. Next, we greedily color v. Since $N[v]$ is a clique, $c(v) \leq \chi(G)$. The resulting coloring of G is optimum.

If we have a graph G in which the coloring problem can be successively reduced to the coloring problem in smaller graphs up to a trivial graph, then we can optimally color G by just greedily coloring simplicial vertices of induced subgraphs of G.

Definition Let G be a graph with n vertices. A perfect elimination ordering is a sequence of vertices $v_1, v_2, \ldots, v_n$ such that for every $i \in [1, n]$, v_i is a simplicial vertex in the subgraph of G induced by $v_i, v_{i+1}, \ldots, v_n$.

3. Proseedur Greedy algorithm

If a graph has a perfect elimination ordering, we can greedily color the vertices in the ordering from right to left and obtain an optimum coloring. Graphs with a perfect elimination ordering are called chordal graphs. The name comes from the fact that a graph has a perfect elimination ordering if and only if all its cycles of length four or greater have a chord [Fulkerson, Gross 1965]. A chord is an edge between non-consecutive vertices of a cycle.

The running time of the greedy coloring algorithm is $O(m+n)$, where m is the number of edges and n is the number of vertices of the graph being colored. Given a chordal graph, a perfect elimination ordering can be found in $O(m+n)$ time [D. Rose, George Lueker, Robert E. Tarjan 1976], so the coloring problem on chordal graphs can be solved in $O(m + n)$ time.

One way to color a G graph with not too many colors is to follow a greedy algorithm:
starting from determining the sequence $v_1, \ldots, v_n$ of vertices G, consider vertices in the rotation and color of each v_i with the first available color (for example with the smallest integer that has not been used to color each neighbor of v_i between $v_1, \ldots, v_{i-1}$) .

Proseedur Greedy algorithm:

1. Select a vertex with the maximum degree (at most an adjacent with another vertex). The color of the vertex with color 1.
2. Select a v vertex that has not been colored. Color it with the lowest number of colors that have not been used at each of the previous vertices that have been colored, move to v. (If all colors have been used in the previous vertices that are adjacent to v, this means that a new color and a new number must be entered.)
3. Repeat the previous step until all vertices are colored.

Definition 4. (Chromatic Number). The chromatic number of a graph is the minimum number of colors in the proper coloring of the graph denoted by $\chi(G)$.

Definition 7. The maximum subset of $V(G)$ of adjacent vertices called clique in graph G. The number of clique $\omega(G)$ of graph G is the number of vertex-vertices in the largest clique in G.
The maximum subset of $V(G)$ of adjacent vertices is called clique in graph G. The number of clique $\omega(G)$ of graph G is the sum of vertex-vertices in the largest clique in G. This gives a trivial lower bound for the chromatic number , that is:

$$\omega(G) \leq \chi(G)$$

Because in a clique all vertices must get a different color. There are several graphs expressly according to the above inequality. Every odd Cycle C_{2k+1} is having $\omega(C_{2k+1}) = 2$ and $\chi(C_{2k+1}) = 3$ according to the above inequality.

4. Results

A graph $G = (V, E)$ is a set of vertices and edges. (Along this paper, the vertex will be represented by letters and colors for the vertices will be represented by integers). A clique in G is part of the vertices $C \subseteq V$ where each pair of vertices are joined with an edge, for example, see Figure 4.1. In other words, all the vertices in the clique must adjasen one another. The size of the maximum clique in G indicated by $\omega(G)$. An illustration of a nice, simple, this problem is with a dinner party. Suppose the vertices of the graph represent the dinner guests. Suppose edge connects guests if they already know each other. Then the maximum clique will be the biggest set of guests at a dinner where everyone knew each other [R. Beigel 1999].
A vertices coloring is to assign a color to the vertices of a graph G such that no two adjacent vertices receive the same color. A minimum coloring is a coloring that uses the least amount possible for the entire graph coloring, as an example, see Figure 2. The size of the minimum coloring in G indicated by $\chi(G)$. Minimum Coloring can also be illustrated using the example of a dinner party described above. Suppose that the host wants her friends to meet each other. Then it will just sit people at tables where they are not familiar with the other guests. If we let the different colors represent the different tables at the party, then the only guests at no edge connecting them (ie, guests who do not know each other) will be allowed to color / same table. Coloring minimum in accordance with the minimum number of tables required to ensure no guests sit with people they've known ([Babel and Tinhofer 1990], [Beigel 1999]).
Consider a triangle, which is a clique of size 3. Each of the three vertices must have different colors, if not, then there will be no edge has endpoints with the same color and coloring will be valid. In general, in any coloring a graph G, each pair of adjacent vertices must have different colors. Therefore, $\omega(G) \leq \chi(G)$.
The *neighbourhood* of a subset V', denoted by $N(V')$, of a vertex set V of graph $G(V, E)$ is the set of all vertices i not in V' such that for some vertex j in V the edge (i, j) is in E.
$N(V') = \{ i \in V \setminus V' \mid (i, j) \in E,\ j \in V' \}$
In Figure 1, if $V' = \{$ C, E, F $\}$ then $N(V') = \{$ A, B, D, G $\}$

More importantly, every clique in G is a lower bound on $\omega(G)$, and every shade in G is an upper bound on $\omega(G)$.

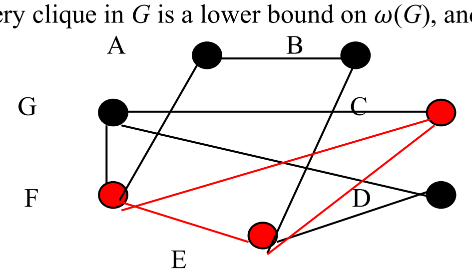

Figure 1. Vertices C, E, and F form clique size $\omega(G) = 3$.

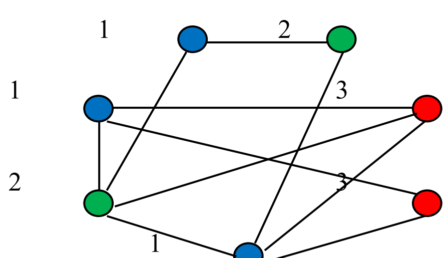

Figure 2. Shows the Coloring of Size $\chi(G)$= 3

Therefore the results show that $\omega(G) = 3 \leq \chi(G) = 3$.

5. Conclusions

The main motive of this paper is to present the new Graph Coloring Algorithm with its space and time complexity, this algorithm can be applied to so many applications based on Graph coloring.
In general, in any coloring a graph G, each pair of adjacent vertices must have different colors. Therefore, $\omega(G) \leq \chi(G)$.

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