Estimation of Aggregate Claim Risk Model on Insurance for Damage to Buildings Due to Flooding of the Citarum River in Bandung Indonesia

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Abstract

Claim risk for flood insurance is a payment made by an insurance company to policyholders. Actuaries in insurance companies must be able to measure and control the risk of claims, so there will be no loss to insurance companies. In this paper, an estimation of the risk model of aggregate claim risk on damage loss insurance due to flooding of the Citarum river in Bandung Indonesia. Estimates are made on the claim frequency model, and the model for the amount of the claim. Using the claim frequency model estimator, and the claim amount model estimator, then used to estimate the risk of aggregate claims. The results of the analysis show that the claim frequency follows the Poisson distribution model, and the amount of the claim follows the normal distribution model. So that, based on the claim frequency estimator and the claim amount estimator can be used to measure the risk of claims on insurance losses caused by flooding of the Citarum river.

Keywords:
Aggregate claims risk, flood insurance, damage to buildings, Poisson distribution, normal distribution.

1. Introduction

Don't ask about losses every time a flood occurs. The furniture is gone and the house is damaged. The automatic motor has been damaged by two units. Certainly, not earning money during the floods, murmured a victim. The Bandung Regency Regional Disaster Management Agency (BPBD) noted that the total number of houses and all kinds of flooded facilities this time reached 5,800 units in four sub-districts: Baleendah, Dayeuhkolot, Bojongsoang, and Banjaran (Bachrudin et al., 2018; Sidi (a) et al., 2017; Sidi (b) et al., 2017). This condition triggered hundreds of residents to evacuate. Total damage to buildings due to flooding can reach IDR 3 billion. The problem is, not many people understand that insurance protection can guarantee the risk of flooding of customer assets. In fact, not only protects customers' property and motorized vehicles, insurance that guarantees flood risk is an extension of the main guarantees that can guarantee the risk of customer property contents (Sidi et al., 2018; Sukono et al., 2018).

Flood insurance can be seen as a tool whereby individuals can transfer risk to other parties, where the insurer accumulates funds from individuals to meet financial needs related to losses incurred (Dickson, 2005; Djuric, 2013). As a takeover institution and risk recipient, insurance companies must of course be able to take into account risks if there are many claims. Because, if not, it will cause losses that can make the insurance company go bankrupt (Nino and Paolo, 2010; Sukono et al., 2016). In risk management, insurance companies must know the character of the risk. The point is to predict the losses that will occur in the future. The risk character can be studied in a claim distribution model (Polanski et al., 2013; Saputra et al., 2018; Sukono et al., 2017). There are two standard approaches to form a claim distribution model over the insurance period, namely the aggregate risk model or collective risk. Mahmoudvand and Edalati (2009), study the distribution of discounted collective risk models, where the counting process is Poisson. Using this approach, aims to get the mean function, variance, and moment generating function of the model. In this study two approaches were used. The first approach is done using the classical method to obtain mean and variance values. The second approach is done using the martingale model to obtain the moment generating function of the total loss. In addition, the Fast Fourier Transform approach is used for numerical calculation of the discounted collective risk distribution model.

Referring to the description above, this paper discusses the "estimation of aggregate risk model claims on insurance for damage to buildings due to the flooding of the Citarum river in Bandung Indonesia". The aim is to do an estimation of aggregate damage to buildings due to flooding. The object of conducting this research is the loss arising from the flood of the Citarum river in Bandung Indonesia. This object was chosen because the Citarum river basin is flooded every year, and causes damage to buildings.

2. Methodology

This section intends to discuss the model of flood insurance, and aggregate risk or collective risk of flood insurance.

2.1 Claim distribution model

This section discusses the Poisson distribution used in the research here. Poisson distribution is chosen here because it has several interesting characteristics that can make it easy to use in various actuarial application analyzes. Referring to Dickson (2005) and Bon et al. (2018), for example Poisson distribution is used to analyze the frequency of flood insurance claims. When a random variable of frequency has a Poisson distribution with parameters, the probability function is given by:

$$P(N=n)=\frac{e^{-\lambda}\lambda^{n}}{n!},\ n=0,1,2,...$$

The moment generating function of Poisson distribution is

$$M_N(t)=\sum_{n=0}^{\infty}e^{tn}\frac{e^{-\lambda}\lambda^{n}}{n!}=e^{-\lambda}\sum_{n=0}^{\infty}\frac{(\lambda e^{t})^{n}}{n!}=\exp\{\lambda(e^{t}-1)\},\qquad(1)$$

and the probability generating function is

$$P_N(r)=\sum_{n=0}^{\infty}r^{n}e^{-\lambda}\frac{\lambda^{n}}{n!}=\exp\{\lambda(r-1)\}.$$

The moments of N can be found from the moment generating function. For example:

$$M'_N(t)=\lambda e^{t}M_N(t)\text{, and}$$

$$M''_N(t)=\lambda e^{t}M_N(t)+(\lambda e^{t})^{2}M_N(t),$$

From which it follows that $E[N]=\lambda$ and $E[N^2]=\lambda+\lambda^2$ so that $V[N]=\lambda$. Furthermore, we use the notation $P(\lambda)$ to denote a Poisson distribution with parameter λ.

According to Dickson (2005) and Bortoluzzo et al. 2017), in insurance the amount of a flood insurance claim is assumed to be a continuous random variable. For example X is the amount of flood insurance claim which is assumed to be lognormal with parameters μ_X and σ_X, where $-\infty < \mu_X < \infty$ and $\sigma_X > 0$, its density function is given by:

$$f(x) = \frac{1}{x\sigma_X\sqrt{2\pi}} \exp\left\{-\frac{(\log x - \mu_X)^2}{2\sigma_X^2}\right\},\ x > 0$$

The distribution function can be obtained by integrating the density function as follows:

$$F(x) = \int_0^x \frac{1}{y\sigma_X\sqrt{2\pi}} \exp\left\{-\frac{(\log x - \mu_X)^2}{2\sigma_X^2}\right\} dy,$$

and the substitution $z = \log y$ yields

$$F(x) = \int_{-\infty}^{\log x} \frac{1}{\sigma_X\sqrt{2\pi}} \exp\left\{-\frac{(z - \mu_X)^2}{2\sigma_X^2}\right\} dz.$$

As the integrand is the $N(\mu_X, \sigma_X^2)$ density function,

$$F(x) = \Phi\left(\frac{\log x - \mu}{\sigma}\right).$$

Thus, probabilities under a lognormal distribution can be calculated from the standard normal distribution function.

This relationship between normal and lognormal (*LN*) distributions is extremely useful, particularly in deriving moments. If $X \sim LN(\mu_X, \sigma_X)$ and $Y = \log X_i$, then

$$E[X_i^k] = E[e^{kY}] = M_Y(k) = \exp\{k\mu_X + \tfrac{1}{2}k^2\sigma_X^2\}, \quad (2)$$

where the final equality follows by moment of normal distribution.

2.2 Collective risk model

Distribution of S

For example S random variable magnitude of flood insurance aggregate claims in one year, N random variable amount of flood insurance claims within one year, and X_i random variable amount of flood insurance claims to -i. The amount of flood insurance aggregate claims can be stated as:

$$S = \sum_{i=1}^{N} X_i, \quad (3)$$

where $S = 0$ when $N = 0$. Assumed that $\{X_i\}_{i=1}^{\infty}$ is a sequence of independent random variables and identical (iid) lognormal distributions, and N Poisson distribution. Random variable N and X_i is mutually independent.

For example $G(x) = P(S \le x)$ is a distribution function of flood insurance aggregate claims, $F(x) = P(X_i \le x)$ is a large distribution function of individual insurance claims for floods, and for example $p_n = P(N = n)$ such that $\{p_n\}_{n=0}^{\infty}$ is a probability function of many flood insurance claims. Event $\{S \le x\}$ this is a combination of mutually exclusive events $\{S \le x \text{ and } N = n\}$, so that:

$$\{S \le x\} = \bigcup_{n=0}^{\infty} \{S \le x \text{ and } N = n\}$$

Therefore,

$$G(x) = P(S \le x) = \sum_{n=0}^{\infty} P(S \le x \text{ and } N = n).$$

Need to know that,

$$P(S \le x \text{ and } N = n) = P(S \le x \mid N = n)P(N = n), \text{ and}$$

$$P(S \le x \mid N = n) = P\left(\sum_{i=1}^{n} X_i \le x\right) = F^{n*}(x) .$$

So, for $x \ge 0$,

$$G(x) = \sum_{n=0}^{\infty} p_n F^{n*}(x) , \tag{4}$$

with $F^{0*}(x)$ defined equal to 1 for $x \ge 0$, and 0 for x others.

Moments of S

The moments and moment generating function of S can be determined using the properties of conditional expectations. In principle, if you know two random variables Y and Z, then the properties that are in accordance with the discussion in this paper are that:

$$E[Y] = E[E(Y \mid Z)] \text{, and} \tag{5}$$

$$V[Y] = E[V(y \mid Z] + V[E(Y \mid Z)]. \tag{6}$$

Using equation (5), expectations of S is:

$$E[S] = E[E(S \mid N)] .$$

When it is said $m_k = E[X_i^k]$ for $k = 1,2,3,4$, then:

$$E[S \mid N = n] = E\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} E[X_i] = nm_1 ,$$

and for $n = 0,1,2...,$ $E[S \mid N] = Nm_1$, so that:

$$E[S] = E[Nm_1] = E[N]m_1 . \tag{7}$$

So, expectations of the amount of aggregate claims S is the result of the expectations of many claims and expectations of each of the large claims.

The same way, using the assumption that $\{X_i\}_{i=1}^{\infty}$ is a mutually independent random variable, obtained:

$$V[S \mid N = n] = V\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} V[X_i] = n(m_2 - m_1^2) ,$$

so that $V[S \mid N] = N(m_2 - m_1^2)$. Furthermore, using equation (6) is obtained:

$$\begin{aligned} V[S] &= E[V(S \mid N)] + V[E(S \mid N)] \\ &= E[N(m_2 - m_1^2)] + V[Nm_1] \\ &= E[N](m_2 - m_1^2) + V[N\}m_1^2 . \end{aligned} \tag{8}$$

Equation (8) shows that the variance of, consists of the average rates and the variance of the distribution of many claims and the large distribution of individual claims for flood insurance. Equation (8) is referred to as the collective risk or aggregate riks, when the risk of flood insurance claims is measured using variance.

If the general generating function technique is used, the moment generating function is obtained as follows:

$$M_S(t) = E[e^{tS}] = E[E(e^{tS} \mid N)] \text{, and}$$

$$E[e^{tS} \mid N = n] = E\left[\exp\left\{t \sum_{i=1}^{n} X_i\right\}\right] = \prod_{i=1}^{n} E[\exp\{tX_i\}] ,$$

where obtained by using the nature of independence from $\{X_i\}_{i=1}^{\infty}$. Next, because $\{X_i\}_{i=1}^{\infty}$ is identically distributed,

$$E[e^{tS} \mid N = n] = M_X(t)^n ,$$

where $M_X(t) = E[\exp\{tX_i\}]$. Based on these:

$$\begin{aligned} M_S(t) &= E[M_X(t)^N] \\ &= E[\exp\{\log M_X(t)^N\}] \\ &= E[\exp\{N \log M_X(t)\}] \\ &= M_N(t)[\log M_X(t)]. \end{aligned} \tag{9}$$

So, M_S expressed in terms of terms $M_N(t)$ and $M_X(t)$, where $M_N(t)$ given in the equation (1) and $M_X(t)$ obtained from equation (2). The above models are then used to analyze flood insurance claims data in the Citarum watershed in Bandung Indonesia.

3. Result and Analysis

The data used in this study is secondary data obtained from the Bandung Branch Office insurance company on February 15-26 2018 as many as 9 flood insurance claims. In this preliminary study obtained data on flood insurance claims, which only provide an initial description. Flood insurance claims that occur have an average frequency of 7 incidents, and the average value of claims is IDR 8,7313 million with a variance in the amount of claims of 0.4616. Because the data obtained is seen as very small, then for the sake of analysis based on the data obtained, the generation of data on frequency and size of claims is generated, as discussed in the following sections.

3.2 Numerical Illustration

The numerical illustration here is intended to give an example of how to calculate Collective risk. In this numerical illustration the number of claim data are generated through a simulation of random variables that follow 900 times the Poisson P(7) distribution. The discrete distribution histogram resulting from the many claim data simulations is given in Figure 1.

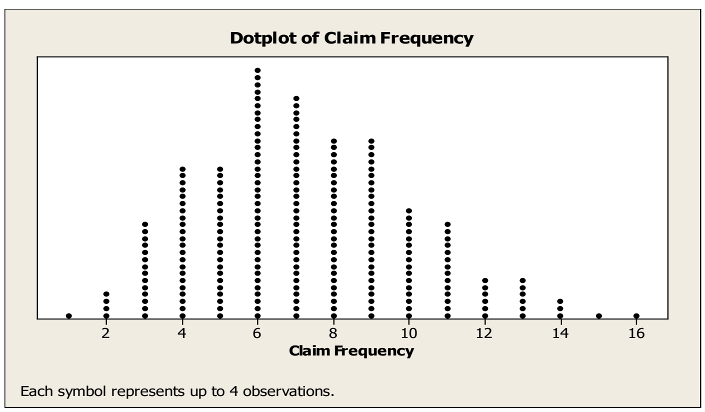

Figure 1. Histogram of Poisson claim frequency distribution

Claim frequency data that follows the Poisson distribution P(7) has descriptive statistics that are given as follows:

Descriptive Statistics: Claim Frequency

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median
Claim Frequency	900	0	7.0033	0.0919	2.7567	1.0000	5.0000	7.0000

Variable	Q3	Maximum
Claim Frequency	9.0000	16.0000

Poisson distribution results from the generation of simulations have a probability function as:

$$P(N=n) = \frac{e^7 7^n}{n!}, \ n = 0,1,2,... \tag{18}$$

Based on the equation (18) it is obtained that $E[N] = V[N] = \lambda = 7$.

While the large data claim is generated through a simulation of random variables that follow the Lognormal distribution as LN(8.7313, 0.213075) as well as 900 times. The continuous distribution histogram resulting from the simulation of large data claims is given in Figure 2.

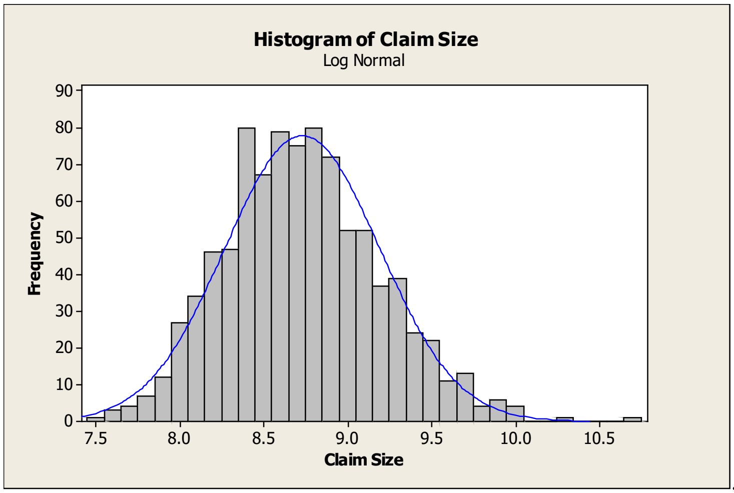

Figure 2. Histogram distribution of the amount of the claim Lognormal

Claim amount data that follows the LN(8.7313, 0.213075) has descriptive statistics that are given as follows:

Descriptive Statistics: Claim Size

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
Claim Size	900	0	8.7313	0.0154	0.4616	7.5132	8.4127	8.6988	9.0258

Variable	Maximum
Claim Size	10.6969

Lognormal distributions of the simulation results have a probability density function as:

$$f(x) = \frac{1}{(0.4616)x\sqrt{2\pi}} \exp\left\{ -\frac{(\log x - 8.7313)^2}{2(0.213075)} \right\}, \ x > 0 \qquad (19)$$

Based on equation (19) and using equation (2) values are obtained m_1 = 8.7313 and m_2 = 76.44867469.

Use the values of $E[N]$ and $V[N]$, and values m_1 and m_2. The value of collective risk can be calculated using equation (8) as follows:

$$V[S] = (7)\{ 76.44867469-(8.7313)^2\}+(7)(8.7313)^2$$
$$= 535.1407228$$

Also use values λ, m_1 and m_2, parameter values can be obtained for the average aggregate claim μ_S = 61.1191 and aggregate variance σ_S^2 = 535.1407228.

4. Conclusions

In this paper a discussion has been conducted on estimation of aggregate risk claims model on insurance for damage to buildings due to flooding of the Citarum river. Based on the results of the analysis it can be shown that the frequency of Citarum river flood insurance claims follows the Poisson distribution with λ = 7. Whereas the amount of Citarum river flood insurance claims follows the lognormal distribution with parameter values μ_S = IDR

61.1191 million and variance parameters σ_S^2 = IDR 535.1407228. This value can certainly be taken into consideration for actuaries in analyzing aggregate risk in flood insurance. In addition, it can also be used to determine the pure magnitude of the Citarum river flood insurance.

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