Dynamical Analysis of Carbon Concentration Model Due to the Interaction with Biomass Production Based on Predator-Prey

Ruly Budiono
Department of Biology, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran
Jl. Raya Jatinangor Km 21, Jatinangor 45363, Sumedang, Jawa Barat, Indonesia
ruly@unpad.ac.id

Hafizan Juahir
East Coast environmental Research Institute (ESERI), Universiti Sultan Zainal Abidin,
21300 Kuala Terengganu, Terengganu, Malaysia
hafizanjuahir@unisza.edu.my

Mustafa Mamat
Faculty of Informatics and Computing, Universiti Sultan Zainal Abidin
21300 Kuala Terengganu, Terengganu, Malaysia
musmat567@gmail.com

Sukono
Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran
Jl. Raya Jatinangor Km 21, Jatinangor 45363, Sumedang, Jawa Barat, Indonesia
sukono@unpad.ac.id

Adam Sukma Putra
Department of Physics, Faculty of Mathematics and Natural Sciences, Universitas Gajah Mada
Sekip Utara, Bulaksumur, Yogyakarta 55281, Indonesia
adam_sukmaputra@yahoo.com

Abdul Talib Bon
Department of Production and Operations, University Tun Hussein Onn Malaysia, Malaysia
talibon@gmail.com

Abstract

The high energy demands and the development of technology produce transcendent concentration of atmospheric Carbon dioxide gas emission. Many studies have been conducted to develop a model of the carbon dioxide cycle to understand its characters. Here, we proposed the model of the interaction between Carbon consumption due to the biomass production. The model is currently developed to characterize the dynamical behaviour of the climate vector of the gas to predict its stability. We simulated the Rungge-Kutta method to analyze the bifurcation and define the Jacobian matrix to see the eigen value. The result gave the idea that the gas concentration can be controlled by manipulating its uncertain parameters and could be a provisional prediction for several years ahead.

Keywords:
Carbon dioxide, biomass production, dynamical behavior, Rungge-Kutta method

1. Introduction

The carbon emission has been widely a serious problem during the last decades. Many efforts has been conducted by the people all over the world. Many scientist has modeled the carbon emission in order to predict the atmospheric concentration. Moreover to reduce the carbon emission, One of the safest and sustainable way is to use the biomass as the 'reductor', so that carbon dioxide (CO2) wastes can be converted into useful compounds for the living life (Knizley et al., 2014; Puliafito et al., 2006).

Global warming (global warming) is basically a phenomenon of global temperature increase from year to year due to the greenhouse effect caused by increasing greenhouse gas emissions. One way to avoid and overcome the increase of carbon dioxide emissions is the existence of an effort capable of fixation of carbon dioxide emission gas in the atmosphere using natural and safest biomass agent, phytoplankton (Trevisan, 2008; Klinthong et al., 2015).

Associated with the dynamic analysis of carbon dioxide concentration models, several studies have been conducted. Sundar (2015), examined the mathematical model of the effect of high concentrations of carbon dioxide on plant growth. Mathematical models were analyzed using the stability theory of ordinary differential equations and numerical simulations. The results show that plant biomass density increases as carbon dioxide concentration increases. Simelane et al. (2014), explores the dynamics of respiratory gas interactions accompanied by water loss through insect spirals equilibrium. Sundar et al. (2014), modeling the dynamics of carbon dioxide depletion in the atmosphere. The goal is to determine the limits for gas volume, and to discuss complexity and stability. Here a nonlinear mathematical model is used to study the reduction of carbon dioxide by using an appropriate absorbent near the emission source. The results show that the concentration of carbon dioxide is reduced by a large amount if adequate amounts of absorbent are used near emission sources. Budiono et al. (2018), modeling the interaction of CO2 and algae biomass concentrations due to anthropogenic carbon reductions based on predator-prey models. This study, the observations made to increase the production of cell biomass Nannochloropsis oculata, by providing some concentration of carbon dioxide, and setting different light intensity can be judged most effective in the growth of algae biomass.

Here, in this resaerch, we continue the study conducted by Budiono et al. (2018) proposing a mathematical model of the interaction between the carbon concentration and the phytoplankton. He assumed that the carbon emission can be consumed as the *fuel* for the biomass production. The model is derived from the predator-prey model. Here, we will study more about the dynamical behaviour of the model. This work focus to analyse the equation proposed by Budiono et al. (2018) to see the dynamical behaviour of the model. Further study, is to optimize the model to fit with the experimental data.

2. Materials and Methods

In this section, first discussed about the material that is the model and data used in this study. The second discussed the methods used in the analysis of carbon concentration model due to the interaction with biomass production.

2.1. Materials

The experimental setting to collect the biomass abundance using specific growth rate the and the carbon concentration (Ibrahima *et al*, 2014).

$$\mu = \frac{(LnN_t - LnN_0)}{T_t - T_0} \tag{1}$$

Where the N_0 and N_t is the biomass density in $t=0$ and $t=T$. While the carbon concentration follow Boyd (1982) Formulation.

$$[CO_2] = \frac{(m \times N \times p \times l)}{V} \tag{2}$$

Where the concentartion is in mg/dm^3, m is the titrant volume (NaOH), N is constant (0.0227), p is the molarity of CO_2 , l is the ratio per liter of water, and V is the solvent volume. By setting the parameters for the experiment during 25 days, we extracted the optimum data for 5 samples of 5 controlled treatments. The initial density of the phytoplankton is 10^6 cell/ml in 2 liters of culture volume and the CO_2 concentration is set at 60 gr/days for 25 days (Singh, 2014; Hernandez-Mireles et al., 2014).

2.2. Methods

This study is based on the experimental research conducted by Chiu *et al.* (2008) which used the *Nanochloropsis occulata* as a biomass phytoplankton for the carbon absoreber in order to reduce the carbon emission in the atmosphere. The sample data which is used for the model is limited to the 1 experimental result for early 25 days of data collection. This data is then to be used for built the mathematical model based on modification of Lotka-

Volterra predator-prey model. This research is to draws a provision of the interaction between the biomass plant and the anthropogenic carbon in the atmosphere.

2.2.1. Mathematical Model

Here, we take into account to build a simple mathematical model to illustrate the dynamic relation between the carbon concentration *C* and the growth rate of the biomass *N*. We proposed our model (Becker, 1994) (ARS Model) as following equation:

$$\frac{dC}{dt} = -vN + aC \tag{3}$$

$$\frac{dN}{dt} = -eC + fNA(L,T) \tag{4}$$

In our model, we adapted the predator-prey model for the CO_2 interaction problem. We consider the carbon concentration as prey, while the biomass density is predator. The carbon concentration (*mg/l*) is denoted by *C* which depends on adduction rate *a* (mg/days). The vegetal biomass distribution *v* is proportional to the density of produced phytoplankton *N* (ppm). *A(t)* is the parameter related to the *in vitro* treatment such as the light density *L* and the temperature *T*. (T_0 is room temperature):

$$A(L,T) = \frac{bLT}{126 + \frac{T_0}{T}} \tag{5}$$

The T_0 is the room temperature, while *b and* 126 is measured constant during the simulation.

2.2.2. Dynamical Analysis

For further study, we also analyze the dynamical characteristics of the model which is to find the type of bifurcation of the system by evaluating the jacobian matrix and the eigen value. First we need to define the stability point which occurs when dC/dt and dH/dt is equal to zero, hence:

$$0 = -vN + aC$$

$$C = \frac{vN}{a} \tag{6}$$

$$0 = -eC + fNA(L,T)$$

$$N = \frac{eC}{fA(L,T)} \tag{7}$$

The equations (3) and (4) becomes equations (6) and (7). By substituting the number of all parameters (confidential). We obtain the C_{null} = -4.529 and the N = 0.183. (we set the temperature at 25^0C and the light density at 6000 Lux.

Next we build a jacobian matrix for the system which can be formulated as follow:

$$J = \begin{pmatrix} \frac{\partial f(C,N)}{\partial C} & \frac{\partial f(C,N)}{\partial N} \\ \frac{\partial g(C,N)}{\partial C} & \frac{\partial g(C,N)}{\partial N} \end{pmatrix}$$

$$J = \begin{pmatrix} a & -v \\ -e & fA \end{pmatrix} \tag{8}$$

Where function f and g represents the equation (1) and (2) and by defining all of the parameters, we obtain the eigen value for each C and N, are -8.371 and 0.113

3. Results and Discussion

This section may be divided by subheadings. It should provide a concise and precise description of the experimental results, their interpretation as well as the experimental conclusions that can be drawn.

3.1. Fitted Model and Experimental Data

In this section, we used the sampled data of the experiment conducted by Ruly *et al* (2017) with the condition of 15% CO 2 concentration with CO2 gas flow rate (0.0199 g / l / m) at 180 buble / hour and 35% carbon dioxide injection with CO2 gas flow rate (0.0599 g / l / l), speed 480 buble / hour, it is applied nutrition formula walne every 2 days once as much as 1 ml, during Growth of nannochloropsis oculata performed for 25 days. (controlled light density of 9000 Lux at room temperature (25^0).

The experimental data and modeled data are fitted based on the equation (3) and (4) as it is shown in Figure 1.

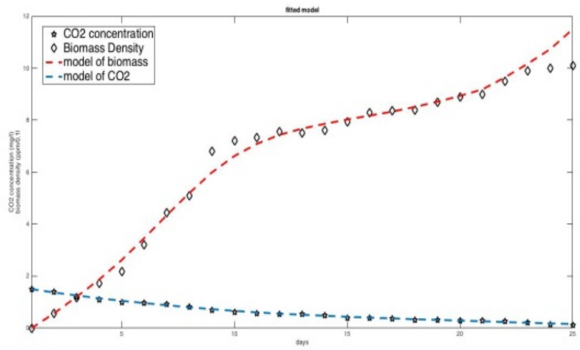

Figure 1. The experimental data fitted with the model data.

Based on the results obtained from the simulation, the proposed model may represent the results of a carbon conversion experiment into biomass with plankton media. Can be seen in experimental data of carbon concentration, the fit simulation model is near perfect. On the other hand, biomass density simulation faces little problem in emulating experimental data. This occurs because the biomass experimental data is slightly anomalous in the modeling. There may be several in vitro factors during the chemical treatment, in the experimental sample. However, in this real case it is within reasonable limits.

3.2. Dynamical Analysis

Based on the results obtained from the calculation, the eigen value for *C* is negative, while *N* has positive sign. Since the eigen value has a negative sign, it means that *C* has the stable node point. While *N* is unstable with the positive sign of eigen value. According to this result, we can conclude that the system has the *saddle-Node* stability point.

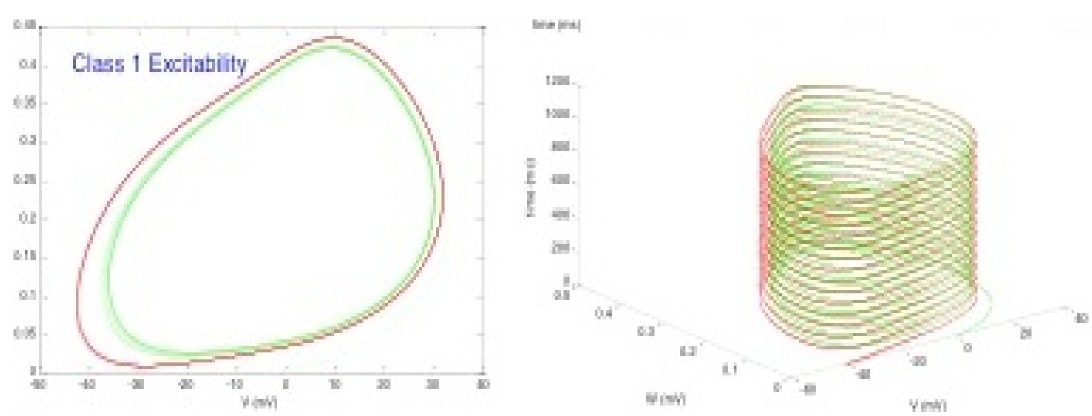

Figure 2. The saddle-Node Stability point of the system of *Class 1 Excitability*

The *Node* stability point is stable. In contrary the *saddle* point is not stable. However, this combination of the satbility points can define the type of the bifurcation. For this case, the system has the *saddle-node* which is not stable.

In order to see another behaviour of the model, we change the parameters (confidential) somehow to be able to produce the *focus* stability denoted by the class 2 excitability system that can be compared to the class 1

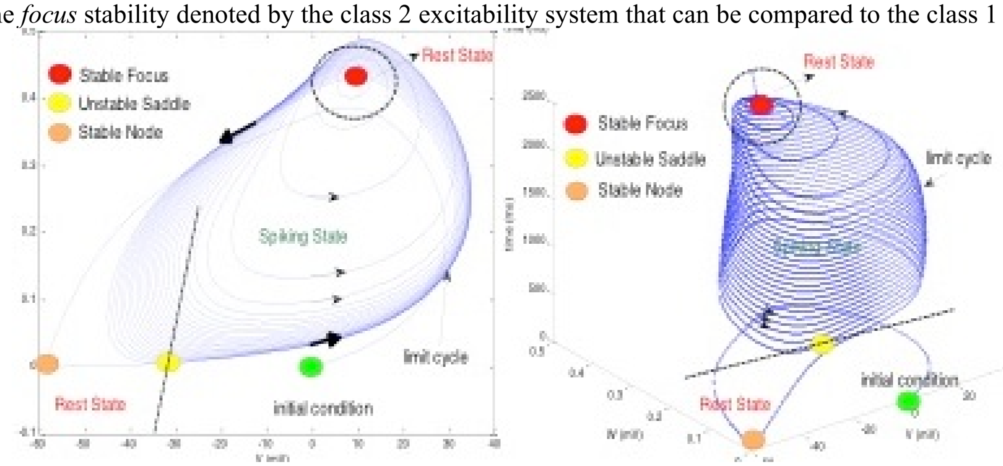

Figure 2. The saddle-Node Biffurcation with stable node point and unstable saddle point.

To see further about the characteristic of the system, we also modified the value for each variable to see the behavior of the sytem. One of the type of the bifurcation which is to characterize a chaotic state of the system is the focus bifurcation or called *adronov-hopf* bifurcation.

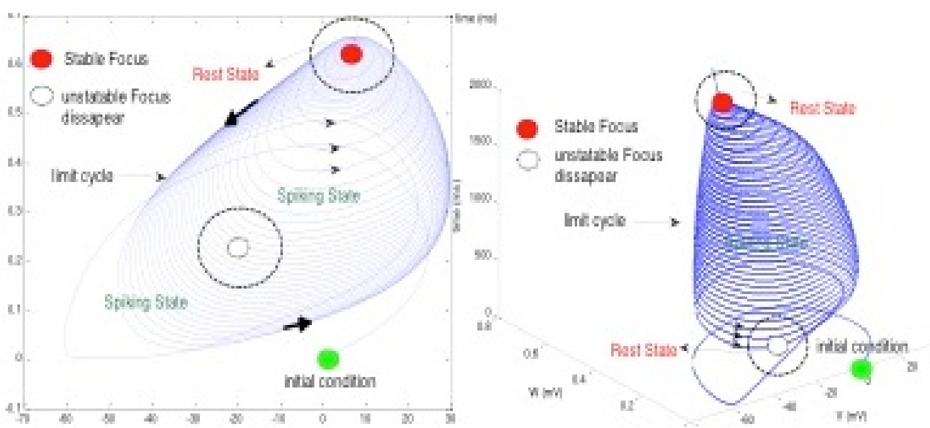

Figure 2. Focus Biffurcation with unstable focus point.

The eigen value of the system for the focus point is -0.0525+0.04871i and -0.0525-0.04871i. Since the real parts have the negative sign, hence the system is stable.

3.3 Discussion

Since the model is a preliminary formulation, it is important to know its dynamycal charateristics. According to the result, the system is stable for the chaotic state which is relevant for the experimental study which has many uncertainty and errors while collecting the data. However, this model needs more validation of data which is still continuing for 1 years instead of 25 days.

4. Conclusions

In this paper we have conducted a dynamical analysis of carbon concentration model due to the interaction with biomass production based on predator-prey. Based on the result of research, it can be concluded that: We proposed the mathematical model based on modification of predator-prey model to the experimental data of CO_2 consumption by the phytoplankton due to reduce the carbon emission. The result give us the idea that the model is well-matched to the data. Nevertheless, this represents only for a small data. The model may need a larger data set in order to validate its robustness so that it can be a provision for the CO_2 gas emission in the atmosphere. We have done the dynamical analysis of the model proposed by Ruly (2017) which draws the information about the interaction between the Carbon concentration and the biomass production. The model is stable and may be used for the forecasting of the experimental studies.

Acknowledgments: Acknowledgments are conveyed to the Rector, Director of DRPMI, and the Dean of FMIPA, Universitas Padjadjaran, who has granted an Internal Grant program of Universitas Padjadjaran, which is a means of enhancing research and publication activities for researchers at Universitas Padjadjaran.

References

Becker, E.W. 1994. *Microalgae Biotechnology and Microbiology*, University Press. Cambridge.

Boyd. 1988. *Water Quality In Warmwater Fish Pounds*. Fourth Printing. Alabama Iniversity Agricultural Experiment Station, Alabama, USA.

Budiono, R.; Juahir, H.; Mamat, M.; Sukono, Nurzaman, M. 2018. Modelling Interaction of CO_2 Concentration and the Biomass Algae Due to Reduction of Anthropogenic Carbon Based on Predator-Prey Model, *International Journal of Applied Environmental Sciences*, ISSN 0973-6077 Volume 13, Number 1 (2018), pp. 27-38, © Research India Publications, http://www.ripublication.com

Chiu, S.Y.; Chien, Y.K.; Ming, T.T.; Seow, C.O.; Chiun, H.C.; and Chih, S.L. 2008. Lipid Accumulation and CO_2 Utilization of *Nannochloropsis oculata* in Response to CO_2 Aeration, *Bioresource Tech*, 100: 833-838, https://pdfs.semanticscholar.org/95fc/415ba6a17fe0c3890694d00914a8703a8978.pdf.

Hernandez-Mireles, I., van der Stel, R., and Goetheer, E., 2014, New Methodologies for Integrating Algae with CO2 Capture, *Energy Procedia,*63 (2014), 7954-7958. https://www.sintef.no/contentassets/ 6fa12dc1224e404b9ea23668a1168eea/hipercap_hernandez_mireles.pdf

Ibrahima, M., Skaugenb, G., and Ertesvåg, I.S., 2014, Modelling CO2 –Water Thermodynamics Using SPUNG Equation of State (EoS) concept with Various Reference Fluids, *Energy Procedia* 51 (2014) 353 – 362. Available online at www.sciencedirect.com

Klinthong, W., Yang, Y-H., Huang, C-H., and Tan, C-S., 2015, A Review: Microalgae and Their Applications in CO2 Capture and Renewable Energy, *Aerosol and Air Quality Research*, 15: 712–742, 2015. Copyright © Taiwan Association for Aerosol Research, ISSN: 1680-8584 print / 2071-1409 online. doi: 10.4209/aaqr.2014.11.0299

Knizley, A.; Mago Pedro, M.J., 2014, Potential Emissions Reductions from the Use of Dual-Combined Heat and Power Systems in Different Climate Conditions. *Int J Ambient Energy*, 35(1): 20–29, https://www.tandfonline.com/doi/abs/10.1080/01430750.2012.759151.

Puliafito S. E., Puliafito, J. L., Grand, M. C., 2006, Modeling Population Dynamics and Economic Growth as Competing Species: An Application to CO2 Global Emissions, *Elsevier.Science Direct. Ecological Economics* (2006), 602-615. https://ideas.repec.org/p/cem/doctra/334.html

Singh, S.P. and Singh, P., 2014, Effect of CO2 Concentration on Algal Growth: A Review, *Renewable and Sustainable Energy Reviews*, 38 (2014) 172–179. www.elsevier.com/locate/rser.

Simelane, S.M.; S. Abelman, S.; Duncan, F.D., 2014, Dynamics of the Oxygen, Carbon Dioxide, and Water Interaction across the Insect Spiracle, *Hindawi Publishing Corporation Abstract and Applied Analysis*, Volume 2014, Article ID 157573, 11 pages. http://dx.doi.org/10.1155/2014/157573

Sundar, S., 2015, Effect of Elevated Carbon Dioxide Concentration on Plant Growth: A Mathematical Model, *American Journal of Applied Mathematics and Statistics,* Vol. 3, No. 2, 59-67, Available online at http://pubs.sciepub.com/ajams/3/2/3, © Science and Education Publishing. DOI:10.12691/ajams-3-2-3.

Sundar, S.; Naresh, R.; Mishra, A.K.; Tripathi, A., 2014, Modeling the Dynamics of Carbon Dioxide Removal in the Atmosphere, Computational Ecology and Software, 4(4): 248-268. http://www.iaees.org/publications/ journals/ces/articles/2014-4(4)/-of-carbon modeling-dynamics -dioxide-removal.pdf

Trevisan, L.A., 2008, Prey-Predator Modeling Of CO2 Atmospheric Concentration, *Working Paper,* Departamento de Matemática e Estatística, Universidade Estadual de Ponta Grossa, Avenida Carlos Cavalcante 4748.

Biographies

Ruly Budiono is a lecturer in the Department of Biology, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran. He is focusing his research topic on Environmental Modelling.

Muhamad Nurzaman is a lecturer in the Department of Biology, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran. He is also Manager of Academic and Student Affairs of Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran.

Mustafa Mamat is Professor at Universiti Sultan Zainal Abidin (UniSZA), Malaysia since 2013. His research interests include conjugate gradient methods, steepest descent methods, Broyden's family and quasi-Newton methods.

Sukono is a lecturer in the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran. Chair of the Research Collaboration Community (RCC), the field of applied mathematics, with a field of concentration of financial mathematics and actuarial sciences.

Adam Sukma Putra is a lecturer at the Department of Physics, Faculty of Mathematics and Natural Sciences, Universitas Gajah Mada, the field of Geophysisc, with a field of concentration of Seismic.

Abdul Talib Bon is a professor of Production and Operations Management in the Faculty of Technology Management and Business at the Universiti Tun Hussein Onn Malaysia since 1999. He has a PhD in Computer Science, which he obtained from the Universite de La Rochelle, France in the year 2008. His doctoral thesis was on topic Process Quality Improvement on Beltline Moulding Manufacturing. He studied Business Administration in the Universiti Kebangsaan Malaysia for which he was awarded the MBA in the year 1998. He's bachelor degree and diploma in Mechanical Engineering which his obtained from the Universiti Teknologi Malaysia. He received his postgraduate certificate in Mechatronics and Robotics from Carlisle, United Kingdom in 1997. He had published more 150 International Proceedings and International Journals and 8 books. He is a member of MSORSM, IIF, IEOM, IIE, INFORMS, TAM and MIM.