Dynamical Analysis of Carbon Concentration Model Due to the Interaction with Biomass Production Based on Predator-Prey

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Abstract

The high energy demands and the development of technology produce transcendent concentration of atmospheric Carbon dioxide gas emission. Many studies have been conducted to develop a model of the carbon dioxide cycle to understand its characters. Here, we proposed the model of the interaction between Carbon consumption due to the biomass production. The model is currently developed to characterize the dynamical behaviour of the climate vector of the gas to predict its stability. We simulated the Runge-Kutta method to analyze the bifurcation and define the Jacobian matrix to see the eigen value. The result gave the idea that the gas concentration can be controlled by manipulating its uncertain parameters and could be a provisional prediction for several years ahead.

Keywords:  
Carbon dioxide, biomass production, dynamical behavior, Runge-Kutta method

1. Introduction

The carbon emission has been widely a serious problem during the last decades. Many efforts has been conducted by the people all over the world. Many scientist has modeled the carbon emission in order to predict the atmospheric concentration. Moreover to reduce the carbon emission, One of the safest and sustainable way is to use the biomass as the 'reductor', so that carbon dioxide (CO2) wastes can be converted into useful compounds for the living life (Knizley et al., 2014; Puliafito et al., 2006).
Global warming (global warming) is basically a phenomenon of global temperature increase from year to year due to the greenhouse effect caused by increasing greenhouse gas emissions. One way to avoid and overcome the increase of carbon dioxide emissions is the existence of an effort capable of fixation of carbon dioxide emission gas in the atmosphere using natural and safest biomass agent, phytoplankton (Trevisan, 2008; Klinthong et al., 2015).

Associated with the dynamic analysis of carbon dioxide concentration models, several studies have been conducted. Sundar (2015), examined the mathematical model of the effect of high concentrations of carbon dioxide on plant growth. Mathematical models were analyzed using the stability theory of ordinary differential equations and numerical simulations. The results show that plant biomass density increases as carbon dioxide concentration increases. Simelane et al. (2014), explores the dynamics of respiratory gas interactions accompanied by water loss through insect spirals equilibrium. Sundar et al. (2014), modeling the dynamics of carbon dioxide depletion in the atmosphere. The goal is to determine the limits for gas volume, and to discuss complexity and stability. Here a nonlinear mathematical model is used to study the reduction of carbon dioxide by using an appropriate absorbent near the emission source. The results show that the concentration of carbon dioxide is reduced by a large amount if adequate amounts of absorbent are used near emission sources. Budiono et al. (2018), modeling the interaction of CO2 and algae biomass concentrations due to anthropogenic carbon reductions based on predator-prey models. This study, the observations made to increase the production of cell biomass Nannochloropsis oculata, by providing some concentration of carbon dioxide, and setting different light intensity can be judged most effective in the growth of algae biomass.

Here, in this research, we continue the study conducted by Budiono et al. (2018) proposing a mathematical model of the interaction between the carbon concentration and the phytoplankton. He assumed that the carbon emission can be consumed as the fuel for the biomass production. The model is derived from the predator-prey model. Here, we will study more about the dynamical behaviour of the model. This work focus to analyse the equation proposed by Budiono et al. (2018) to see the dynamical behaviour of the model. Further study, is to optimize the model to fit with the experimental data.

2. Materials and Methods
In this section, first discussed about the material that is the model and data used in this study. The second discussed the methods used in the analysis of carbon concentration model due to the interaction with biomass production.

2.1. Materials
The experimental setting to collect the biomass abundance using specific growth rate the and the carbon concentration (Ibrahima et al, 2014).

\[ \mu = \frac{(\ln N_t - \ln N_0)}{T_t - T_0} \]  

Where the N_0 and N_t is the biomass density in t=0 and t=T. While the carbon concentration follow Boyd (1982) Formulation.

\[ [CO_2] = \frac{(m \times N \times p \times l)}{V} \]

Where the concentration is in mg/dm³, m is the titrant volume (NaOH), N is constant (0.0227), p is the molarity of CO₂, l is the ratio per liter of water, and V is the solvent volume. By setting the parameters for the experiment during 25 days, we extracted the optimum data for 5 samples of 5 controlled treatments. The initial density of the phytoplankton is 10⁶ cell/ml in 2 liters of culture volume and the CO₂ concentration is set at 60 gr/days for 25 days (Singh, 2014; Hernandez-Mireles et al., 2014).

2.2. Methods
This study is based on the experimental research conducted by Chiu et al. (2008) which used the Nannochloropsis oculata as a biomass phytoplankton for the carbon absorber in order to reduce the carbon emission in the atmosphere. The sample data which is used for the model is limited to the 1 experimental result for early 25 days of data collection. This data is then to be used for built the mathematical model based on modification of Lotka-
Volterra predator-prey model. This research is to draws a provision of the interaction between the biomass plant and the anthropogenic carbon in the atmosphere.

2.2.1. Mathematical Model

Here, we take into account to build a simple mathematical model to illustrate the dynamic relation between the carbon concentration \( C \) and the growth rate of the biomass \( N \). We proposed our model (Becker, 1994) (ARS Model) as following equation:

\[
\frac{dC}{dt} = -vN + \alpha C \\
\frac{dN}{dt} = -eC + fNA(L, T)
\]  

(3)  
(4)

In our model, we adapted the predator-prey model for the CO\(_2\) interaction problem. We consider the carbon concentration as prey, while the biomass density is predator. The carbon concentration (mg/l) is denoted by \( C \) which depends on adduction rate \( a \) (mg/days). The vegetal biomass distribution \( v \) is proportional to the density of produced phytoplankton \( N \) (ppm). \( A(t) \) is the parameter related to the in vitro treatment such as the light density \( L \) and the temperature \( T. \) (\( T_0 \) is room temperature):

\[
A(L, T) = \frac{bLT}{126 + T_0^T}
\]  

(5)

The \( T_0 \) is the room temperature, while \( b \) and 126 is measured constant during the simulation.

2.2.2. Dynamical Analysis

For further study, we also analyze the dynamical characteristics of the model which is to find the type of bifurcation of the system by evaluating the jacobian matrix and the eigen value. First we need to define the stability point which occurs when \( dC/dt \) and \( dH/dt \) is equal to zero, hence:

\[
0 = -vN + \alpha C \\
C^* = \frac{vN}{\alpha} \\
0 = -eC + fNA(L, T) \\
N^* = \frac{eC}{fA(L, T)}
\]  

(6)  
(7)

The equations (3) and (4) becomes equations (6) and (7). By substituting the number of all parameters (confidential). We obtain the \( C_{null} = -4.529 \) and the \( N = 0.183 \). (we set the temperature at 25\(^\circ\)C and the light density at 6000 Lux.

Next we build a jacobian matrix for the system which can be formulated as follow:

\[
J = \begin{pmatrix}
\frac{\partial f(C, N)}{\partial C} & \frac{\partial f(C, N)}{\partial N} \\
\frac{\partial g(C, N)}{\partial C} & \frac{\partial g(C, N)}{\partial N}
\end{pmatrix}
\]

\[
J = \begin{pmatrix}
a & -v \\
-e & fA
\end{pmatrix}
\]  

(8)

Where function \( f \) and \( g \) represents the equation (1) and (2) and by defining all of the parameters, we obtain the eigen value for each \( C \) and \( N \), are -8.371 and 0.113

3. Results and Discussion

This section may be divided by subheadings. It should provide a concise and precise description of the experimental results, their interpretation as well as the experimental conclusions that can be drawn.
3.1. Fitted Model and Experimental Data

In this section, we used the sampled data of the experiment conducted by Ruly et al. (2017) with the condition of 15% CO2 concentration with CO2 gas flow rate (0.0199 g/l/hr) at 180 bubble/hour and 35% carbon dioxide injection with CO2 gas flow rate (0.0599 g/l/hr), speed 480 bubble/hour, it is applied nutrition formula walne every 2 days once as much as 1 ml, during Growth of nannochloropsis oculata performed for 25 days. (controlled light density of 9000 Lux at room temperature (25°C).

The experimental data and modeled data are fitted based on the equation (3) and (4) as it is shown in Figure 1.

![Figure 1. The experimental data fitted with the model data.](image)

Based on the results obtained from the simulation, the proposed model may represent the results of a carbon conversion experiment into biomass with plankton media. Can be seen in experimental data of carbon concentration, the fit simulation model is near perfect. On the other hand, biomass density simulation faces little problem in emulating experimental data. This occurs because the biomass experimental data is slightly anomalous in the modeling. There may be several in vitro factors during the chemical treatment, in the experimental sample. However, in this real case it is within reasonable limits.

3.2. Dynamical Analysis

Based on the results obtained from the calculation, the eigen value for $C$ is negative, while $N$ has positive sign. Since the eigen value has a negative sign, it means that $C$ has the stable node point. While $N$ is unstable with the positive sign of eigen value. According to this result, we can conclude that the system has the saddle-Node stability point.

![Figure 2. The saddle-Node Stability point of the system of Class 1 Excitability](image)

The Node stability point is stable. In contrary the saddle point is not stable. However, this combination of the stability points can define the type of the bifurcation. For this case, the system has the saddle-node which is not stable.
In order to see another behaviour of the model, we change the parameters (confidential) somehow to be able to produce the focus stability denoted by the class 2 excitability system that can be compared to the class 1 excitability system.

Figure 2. The saddle-Node Bifurcation with stable node point and unstable saddle point.

To see further about the characteristic of the system, we also modified the value for each variable to see the behavior of the system. One of the type of the bifurcation which is to characterize a chaotic state of the system is the focus bifurcation or called *adronov-hopf* bifurcation.

Figure 2. Focus Bifurcation with unstable focus point.

The eigen value of the system for the focus point is -0.0525+0.04871i and -0.0525-0.04871i. Since the real parts have the negative sign, hence the system is stable.

3.3 Discussion

Since the model is a preliminary formulation, it is important to know its dynamical characteristics. According to the result, the system is stable for the chaotic state which is relevant for the experimental study which has many uncertainty and errors while collecting the data. However, this model needs more validation of data which is still continuing for 1 years instead of 25 days.

4. Conclusions

In this paper we have conducted a dynamical analysis of carbon concentration model due to the interaction with biomass production based on predator-prey. Based on the result of research, it can be concluded that: We proposed the mathematical model based on modification of predator-prey model to the experimental data of CO$_2$ consumption by the phytoplankton due to reduce the carbon emission. The result give us the idea that the model is well-matched...
to the data. Nevertheless, this represents only for a small data. The model may need a larger data set in order to validate its robustness so that it can be a provision for the CO₂ emission in the atmosphere. We have done the dynamical analysis of the model proposed by Ruly (2017) which draws the information about the interaction between the Carbon concentration and the biomass production. The model is stable and may be used for the forecasting of the experimental studies.

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