Mathematical Modeling of Friction In The Cutting Zone During Orthogonal Machining

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Abstract

In this work, effects of friction on the machining process during orthogonal cutting are theoretically investigated. Mathematical models were developed to predict the effects of friction and cutting force on the machining process. The results showed that the coefficient of friction increased with increased cutting forces, cutting stress and temperature. The frictional stress decreased as the exponent (P) increased. However, the frictional stress increased as the coefficient of friction increased. Also, study revealed that as the temperature increased, the flow stress decreased. Furthermore, it is shown that as the feed speed increased, there is an increase in the flow stress of the material used. The cutting force increased with increasing the feed rate, depths of cut but decreased with increasing cutting speed. This work will enhance the influence of friction on the cutting process and also assist in the development of good products with good surface finish.

Keywords: Friction, Temperature, Force, Machining

1. Introduction

Tool-chip friction and work material properties have long been recognized as two unsolved problems in fundamental machining research. Previous studies on the tool chip friction have been primarily focused in machining with a positive rake angle tool, with various theories and experimental techniques have been proposed. In order to improve metal cutting processes, it is necessary to model metal cutting processes at a system level. A necessary requirement of such is the ability to model interactions at the tool chip interface and thus, predict cutter performance. Many approaches such as empirical, mechanistic, analytical and numerical have been proposed. Some level of testing for model development, either material, machining, or both is required for all. However, the ability to model cutting tool performance with a minimum amount of testing is of great value, reducing costly process and tooling iterations. In the cutting process, friction is mainly present in the rake face and the flank of the tool. The friction that acts on the rake face has a major influence, the other one can become also important and could take part in the stability of the system. Therefore, it is very necessary to set mathematical models for the determination of the friction on the flank.

Molinari et al (2011) observed that the heating of the chip is essential in the cutting zone during orthogonal cutting. The heats from the chip have a lot of influence in the sticking contact of the material as it leads to delay of the flow stress. They also focused on the roles of cutting speed and feed. As the cutting speed increases, the feed rate also increases thereby influencing the sticking contact.

Shi et al. (2002) studied the effect of friction on thermo-mechanical quantities in a metal cutting process. They observed that the shear strain is in the primary shear zone of the cutting process while the material near the tool tip undergoes the largest plastic strain rate. They observed that the maximum temperature rises in the cutting process occurs along the tool-chip interface and not in the primary zone. In orthogonal cutting the maximum temperature, the contact length, the shear angle and the cutting force is influenced strongly by the friction at the cutting zone.

Marusich (2001) compared cutting forces and chip morphologies for the Al6061-T6 where he made use of Finite element simulations. It was observed that in the orthogonal cutting process, the increase in speed leads to
decrease in chip thickness and significant increase in temperature at the tool-chip interface in the cutting zone, while the temperature in the primary zone rise only modestly and have little effect in the change in cutting force.

Ojolo et al (2013) observed that during orthogonal cutting, the temperature of the cutting tool used in cutting zone plays an important role in thermal distortion and the machined part’s dimensional accuracy, as well as in tool life in machining. They used the Finite Element Analysis (FEA) to simulate the thermal behaviour of a carbide cutting tool in three-dimensional dry machining. Also a model was developed to determine the temperature rise at the shear plane and this was used to determine the effect of various parameters on temperature rise.

Salem et al (2012) studied the mathematical models for the cutting force which facilitates the choice of cutting conditions in machining of the tool steel. They studied how the cutting parameters influence the type of chip that is produced during cutting at the tool-chip interface. It was observed that the removal of material from tool increases the plastic deformation and also led to more heat generation.

The aim of this study is to theoretically investigate the effects of friction at the cutting zone during orthogonal cutting. The objective is to develop and simulate mathematical model for friction at the cutting zone during orthogonal cutting.

2. Merchant’s Force Model

The development of force model is based on Merchant force circle in Fig. 1.

![Force equilibrium in orthogonal cutting](image)

According to Merchant’s force model, the cutting \( F_c \) and thrust \( F_t \) force components can be transformed to normal \( N \) and friction \( F_f \) force components applied on the tool face as follows:

\[
F = F_c \sin \alpha + F_t \cos \alpha \\
N = F_c \cos \alpha - F_t \sin \alpha \\
\]

Where, \( \alpha \) is the normal rake angle.

According to Coulomb’s law, the apparent coefficient of friction on tool-chip interface becomes:

\[
\mu = \frac{\tau}{\sigma} = \frac{F_f}{N} = \frac{F_c \sin \alpha + F_t \cos \alpha}{F_c \cos \alpha - F_t \sin \alpha} \\
\]

Based on the Fig. 1, the forces cannot be measured but expressed as:

\[
F_s = F_c \cos \phi - F_t \sin \phi \\
F_n = F_c \sin \phi + F_t \cos \phi \\
\]

The total tangential, radial and axial forces can be obtained by summing the force elements in the same direction. The coefficient of friction is thus written as

\[
\mu = \frac{F_f}{F_c - F_t \tan \alpha} \\
\]

The shear stress in the sticking and sliding regions can be model as
Also, from Fig. 1, the following can be written
\[ F_c = R \cos(\lambda - \alpha) \]
\[ F_l = R \sin(\lambda - \alpha) \]
\[ F_s = K_{AB} l_w \]
\[ N = R \cos \lambda \]
\[ F = R \sin \lambda \]

where
\[ R = \frac{F_s}{\cos \theta} = \frac{K_{AB} l_w}{\sin \phi \cos \theta} \]

Following Eqns. (9) - (13), an expression for the friction coefficient is
\[ \mu = \frac{F_c}{F_r} = \frac{R \sin(\lambda - \alpha) + R \cos(\lambda - \alpha) \tan \alpha}{R \cos(\lambda - \alpha) - R \sin(\lambda - \alpha) \tan \alpha} \]
which gives
\[ \mu = \frac{\sin(\lambda - \alpha) + \cos(\lambda - \alpha) \tan \alpha}{\cos(\lambda - \alpha) - \sin(\lambda - \alpha) \tan \alpha} \]

In this work, an improved model of Johnson-Cook flow stress is developed, the shear force can be written as
\[ \sigma_{AB} = \sigma_m + \left(\sigma_0 + B \dot{\varepsilon}_{AB}^m\right)\left(1 + C \log \frac{\dot{\varepsilon}_{AB}}{\dot{\varepsilon}_0}\right)\left(1 - \left(\frac{T - T_r}{T_m - T_r}\right)^m\right) \]

Recall that the frictional force is given as
\[ F_{AB} = F_s = \mu \sigma_{AB} \]

Therefore, from Eqns. (15), (16) and (17), the frictional force is
\[ F_{AB} = \left[\frac{\sin(\lambda - \alpha) + \cos(\lambda - \alpha) \tan \alpha}{\cos(\lambda - \alpha) - \sin(\lambda - \alpha) \tan \alpha}\right] \left[\sigma_m + \left(\sigma_0 + B \dot{\varepsilon}_{AB}^m\right)\left(1 + C \log \frac{\dot{\varepsilon}_{AB}}{\dot{\varepsilon}_0}\right)\left(1 - \left(\frac{T - T_r}{T_m - T_r}\right)^m\right)\right] \]

Where, \( T \) determined from the temperature rise along the tool-chip interface is calculated as:
\[ T_{chip-shear}(X, Z) = \frac{q_{shear}}{2\pi k_{chip}} \int_0^{l_i} e^{\frac{(x-x_i)\nu}{2a_{chip}}} \left[ k_0 \left( \frac{V_{chip}}{2a_{chip}} \sqrt{(X-X_i)^2 + (Z-Z_i)^2} \right)^2 \right] + \left[ k_0 \left( \frac{V_{chip}}{2a_{chip}} \sqrt{\left(2t_{chip} - Z - Z_i\right)^2} \right)^2 \right] + \left[ k_0 \left( \frac{V_{chip}}{2a_{chip}} \sqrt{(X-X_i)^2 + (Z-Z_i)^2} \right)^2 \right] \]
where
\[ R_i = \sqrt{(X - x_i)^2 + (Y - y_i)^2 + Z^2} \]
\[ R_i' = \sqrt{(X - 2h + x_i)^2 + (Y - y_i)^2 + Z'^2} \]
\[ q_{shear} = \frac{F_s V_s}{l w} \]
\[ q_{friction} = \frac{F V_{chip}}{h w} \]

Based on the temperature rise profiles along the primary shear zone and tool-chip interface, the average temperatures are categorized as
\[ T_{shear-average} = T_0 + \frac{\int_0^l T_{chip-shear} (X, Z) \, dl}{l} \]  
(21)

and
\[ T_{tool-chip} = T_0 + \frac{\int_0^h T_{chip-shear} (X, 0, 0) \, dX}{h} \]  
(22)

The calculated average temperature is used to compute the flow stress along the primary shear zone and at the tool-chip interface.

It should be noted that the normal stress acting on the tool-chip interface is
\[ \sigma_N = \tau_{AB} \left( 1 + 2 \left( \frac{\pi}{4} - \alpha \right) - \frac{2c \gamma_{At} I}{K_{AB}} \right) \]  
(23)

where
\[ I = \left( \frac{\partial \tau}{\partial \gamma} \right)_{AB} = \frac{\partial \tau}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial \gamma} + \frac{\partial \tau}{\partial T} \frac{\partial T}{\partial \gamma} \]  
(24)

where
\[ \tau = \left[ \sin (\lambda - \alpha) + \cos (\lambda - \alpha) \tan \alpha \right] \cos (\lambda - \alpha) - \sin (\lambda - \alpha) \tan \alpha \]  
(25)

From Eqns. (24) and (25), I become
\[ I = \left[ \sin (\lambda - \alpha) + \cos (\lambda - \alpha) \tan \alpha \right] \left\{ n \dot{\varepsilon}_{AB}^{n-1} \left( 1 + C \log \frac{\dot{\varepsilon}_{AB}}{\dot{\varepsilon}_0} \right) \left( 1 - \frac{T - T_m}{T_m - T_r} \right)^m \right\} \]  
(26)

Maximum shear and shear strain rate at the tool-chip interface are given respectively as
\[ \dot{\gamma} = \frac{1}{2} \left( \frac{\cos \alpha}{\sin \phi \cos (\phi - \alpha)} \right) \]  
(27)
\[ \dot{\gamma}_{int} = \frac{V_{chip}}{\delta t_{in}} \]  
(28)

It should be noted that
\[ \dot{\gamma}_{AB} = C \frac{V_s}{l} \]  
(29)

where
\[ V_s = \frac{V_c \cos \alpha}{\sin \phi \cos (\phi - \alpha)} \]  
(30)

Therefore,
\[ \dot{Y}_{AB} = \frac{C \cdot V_c \cos \alpha}{I \cos(\phi - \alpha)} \]  

(31)

Where, \( \phi \) can be numerically determined using Oxley’s model (1961)

\[ \phi = \alpha \tan \left[ 1 + \frac{\pi}{2} - 2\phi + \frac{\cos(2(\phi - \gamma))}{\tan \rho} - \sin 2(\phi - \gamma) \right] - (\rho - \gamma) \]  

(32)

And the friction angle as

\[ \tan \theta = 1 + 2 \left( \frac{\pi}{4} - \phi \right) - \frac{c\gamma_I \mu I}{K_{AB}} \]  

(33)

3. Results and Discussion

Figures 2a-c show the comparison of the experimental results of Xu et al. (2010) with the present developed models. The model results cutting and tangential forces are compared with the experimental results as shown in Fig. 2. Fig. 2b presents the comparison of the experimental results of cutting stress with the results of the cutting stress in the present developed models while Fig. 1c displays the comparison of the experimental results of cutting temperature with the results of the cutting temperature in the present developed models. Also, the effects of coefficient of friction on the forces, cutting stress and temperature are presented. It is depicted that the coefficient of friction has direct relationship on the forces, cutting stress and temperature in a way that as coefficient of friction increases, the forces, cutting stress and temperature increase. This is because as the coefficient of friction increase, the frictional force that enhances the cutting process increases which consequently increase the cutting stress and more heat are generated in the material and the cutting zone thereby the cutting temperature is increased.

![Figure 2. Model results of (a) cutting and tangential forces; (b) cutting stress; (c) cutting temperature](image)

Figure 3 shows the effect of temperature on the flow stress. It is shown that as the temperature increases, the flow stress decreases. It is shown that increase in temperature results in decrease in flow stress due to softening effects on the material. Also, it is established that the cutting action and related friction at cutting surfaces increase the temperature of the tool material, which further accelerates the physical and chemical processes associated with tool wear. In order to remove the unwanted material as chips, these forces and motions are necessary; therefore, cutting tool wear is an economic penalty that must be accounted for in order to machine the part. The magnitude of this economic penalty can be minimized if the cutting process is planned and controlled based on sound knowledge of the cutting engagement, wear process and its dependency on the selection of cutting conditions. The cutting conditions normally controlled are the engagement of the cutting edge with the workpiece, the relative velocity of the cutting edge with the workpiece, and the feed velocity used to keep the tool engaged into uncut material. During the process planning stage, an assessment must be made concerning how difficult the material is to machine, the correct tool for the surface to be created must be selected, the appropriate tool material must be selected, and the type of cutting fluid needed must be determined. To make such choices, the wear environment in metal cutting must be understood and the related friction must be analyzed as carried in the present study.
Figure 3. Effects of Temperature on the flow stress

Fig. 4 illustrates the variation of the tool rake angle with the flow stress. It shows that as the rake angle moves from the negative angle to zero, the flow stress reduces. It is clearly seen that the increase of cutting speed, feed rate and axial depth causes the friction stress increase dramatically. Meanwhile, the friction coefficient, angle and force increases with decreases of feed rate and increases of cutting speed and axial depth. It is established in literature that that the feed rate was the most dominant cutting condition on the cutting force, followed by the axial depth, radial depth of cut and then by the cutting speed. The cutting force increases with increasing the feed rate, depths of cut but decreases with increasing cutting speed.

Figure 4. Effects of tool rake angle on the flow stress for the two types on aluminum

4. Conclusion

In this work, effects of friction on the machining process during orthogonal cutting has been theoretically investigated. Mathematical models were developed and parametric studies of the effects of friction and cutting force parameters on the machining process were carried out. The developed models were validated. It was established that as coefficient of friction increases, the forces, cutting stress and temperature increase. The frictional shear stress directly proportional to the normal stress increases. However, the frictional stress increases as the coefficient of friction increases. Also, the present work revealed that as the temperature increases, the flow stress decreases. Furthermore, it is shown that as the feed speed increases, there is an increase in the flow stress of the material used. The cutting force increases with increasing the feed rate, depths of cut but decreases with increasing cutting speed. This work will enhance the
understanding of the influences of friction coefficient on the cutting process and also assist in the development of good products with good surface finish.

References

Biographies

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