A MODIFIED FIREFLY ALGORITHM FOR GLOBAL OPTIMIZATION OF COLLABORATIVE SUPPLY CHAIN NETWORKS UNDER STOCHASTIC DEMAND

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Abstract

The optimality of supply chain networks (SCN), associated with inventory management, is one of the mandatory objectives for industries. The main purpose of this research is to minimize the total expected costs of collaborative supply chain networks. This research investigates the applicability of the modified firefly algorithm for minimizing the total cost of collaborative supply chain networks based on large-sized problems, in which the vendor uses VMI policy for collaborations and direct-delivery policy for non-collaborations. It is assumed that industrial plants with assigned warehouses are willing to share their existing capacities with other industrial plants with no assigned warehouses to reduce the undesirable costs associated with shortage and extra inventory. Since this problem is one of the larger problems related to industrial supply chain networks, commercial software cannot obtain the optimal results for these problems considered in this research. To achieve better findings, we applied a modified firefly algorithm to solve the problem. The computational time of the proposed algorithm was also reduced as compared with commercial software and standard firefly algorithm.

Keywords

Firefly Algorithm, Supply Chain Networks, Global Optimization, Stochastic Demand, Vendor-Managed Inventory

1. Background

Vendor-managed inventory (VMI) plays an essential role in the optimality of supply chain networks associated with location-inventory assignment problem. Regarding supply chain networks (SCN), designing an effective and suitable network associated with location-inventory assignment problem is one of the most important aspects that assures profitability for companies in a competitive environment under uncertainty. In this paper, we consider that the vendor plays a significant role in the collaboration strategy between the industrial plants with assigned warehouses and those without. Some industrial plants with no assigned warehouses may have demands more than other plants with assigned

warehouses under uncertainty. So, they borrow products from those industrial plants that reach a high capacity of inventory.

Chopra (2003) developed a model for the supply chain distribution network and described a variety of factors affecting distribution network choices. Their aim was to minimize the total cost in inventory, transportation, and facility location. The model used in (Schmitt et al. 2015) presented risks pooling, which affect a multi-location system associated with stochastic demand and deterministic supply. They used a decentralized inventory to reduce the expected costs related to supply chain networks. Shahabi et al. (2013) developed the problem as a mixed integer non-linear programming model for integrated inventory control and location of facility using two simultaneous decisions, including the location of plants/facilities and allocation. They also formulated the problem as a non-linear integer programming to minimize the total expected costs associated with inventory systems. Another study done by Shen et al. (2003) investigated different joint location-inventory problems characterized by stochastic demand and lead times. They developed the problem as a non-linear integer program to identify the retailers who can serve as a distribution center. They solved the problem using column generation algorithm. Other researchers like Vidyarthi et al. (2007) used a multi-product production-inventory distribution system design problem characterized by uncertain demand. The researchers aimed at establishing plants and distribution center locations and the shipment sizes from industrial facilities to distribution centers, which serve as distribution centers for retailers. The objective was to minimize the total expected cost associated with facility location and transportation. They developed the problem as a mixed-integer non-linear programming model. In this research, the meta-heuristic algorithms based on the firefly algorithm will be used to solve our proposed model, which is considered one of the NP-hard problems.

Researchers such as Diabat and Deskoores (2016) considered joint location-inventory problems, including multiple retailers, multiple warehouses, and capacity limitations for the model's warehouses. They presented a heuristic approach that relies on a genetic algorithm. Saragih et al. (2019) investigated a location-inventory problem, including single supplier and multi retailers. They used a heuristic method based on simulated annealing to enhance the solution. They improved the heuristic method by using two phases, including improvement and construction. Additionally, they compared the solution obtained by a heuristic method with one obtained by mixed-integer non-linear programming. In this paper, our model integrates the location assignment inventory problem under uncertainty with vendor-managed inventory consideration using a modified meta-heuristic algorithm based on firefly algorithm, which can solve the larger instances related to supply chain networks effectively as compared with the exact optimization software, such as GAMS and Lingo, as most previous research has focused on. In this research, we solve our model associated with the location-inventory assignment problem using a modified meta-heuristic algorithm based on firefly algorithm, standard Firefly algorithm, GAMS, and Lingo.

In this study, we demonstrate the efficiency of our model using a meta-heuristic algorithm based on a modified firefly algorithm by implementing our model at a pipe manufacturing company, which manages the supply of three main different pipes: Reinforced Concrete Pipes (RCP), Glass Reinforced Epoxy Pipes (GREP), and Glass Reinforced Plastic Pipes (GRPP). VMI policy is implemented by the vendor for the whole manufacturing network by combining the location-inventory assignment with the allocation of the entire replenishment policy. In this case study, the vendor implements two policies for the supply of goods, including the direct-delivery policy and VMI policy under demand uncertainty. The role of the vendor is to be focused on the inventory level to overcome the expected shortage that may occur in industrial plants without warehouses. Once shortages occur, the industrial plants with assigned warehouses collaborates with industrial plants without warehouses under VMI policy. Moreover, we present a comparison of the results using commercial software and meta-heuristic algorithms.

2. Related Work

VMI strategies is an effective tool in improving customer satisfaction, solving demand uncertainties, and minimizing the total expected cost of systems, as highlighted in many studies (Yao and Dresner 2008); (Abuhilal et al. 2006) and (Arora et al. 2010). Moreover, as compared to traditional systems, the VMI system can reduce total expected inventory cost and supply goods through allowable backorder situations. Vendor-managed inventory is proposed to minimize the expected costs associated with ordering and transportation (Yao and Dresner 2008). The system under VMI strategies is considered an effective tool to minimize the total expected cost more efficiently than the traditional system without VMI policy (Abuhilal et al. 2006). Some researchers have already discussed the benefits of VMI under stochastic demand (Arora, Chan, & Tiwari, 2010). They have proven the ability of VMI to be considered as one of the effective collaborative strategies between industrial plants.

Zhang et al. (2007) showed that only a single-buyer can be faced by a single-supplier under demand uncertainty, and backorders will be allowable in the system. They formulated the problem for two systems, including VMI and traditional systems, to get the total cost of each system and used the results for the comparison, which indicates that the system that has the lower cost is the VMI system. In this regard, the VMI system is considered a powerful tool to minimize the total cost of inventory. On the other hand, Pasandideh et al. (2010) declared that the VMI system will not always have better performance in every situation when compared to the traditional system. For instance, when the ordering cost of retailers is lower than the ordering cost of suppliers, the traditional system performs better. Other researchers, like Nia et al. (2015), developed economic order quantity (EOQ) for a two-echelon supply chain, including single-supplier and single-retailer under the VMI system. They applied a hybrid genetic algorithm to find the optimum solution of the model. The results indicate that meta-heuristic algorithms are a powerful tool for validation. Mateen et al. (2015) expanded the same concept (ÖZGEN 2010) with different consideration of single-vendor and multi-retailer under stochastic demand. The resultant situation is indicative of higher cost-saving in VMI than in traditional supply chain.

According to Govindan and Fattahi (2017), stochastic demand is more practical despite numerous studies on deterministic demand. In an allowed backorder, Bookbinder et al. (2010) investigated the study system with a single manufacturer-retailer network in stochastic demand and developed models for both traditional system and VMI intending to compare their performances.

The determinants of VMI adoption have not been widely studied empirically under demand uncertainty. Irungu et al. (2011) investigated the underlying factors that may affect VMI program. Their findings indicate that VMI is able to fulfill a collaborative supply chain and improve stock management. Dorling et al. (2007) developed survey scales for the competition of supplier and buyer and demand of product under uncertainty environment. They investigated how environmental factors act as determinants in the adoption of VMI and discovered that competition between vendor market and retailer-vendor cooperation correlates positively with the VMI system, which indicates that retailer operational uncertainty negatively correlates with the VMI system. Govindan et al. (2013) distinctly highlighted the significance of proper metrics selection when evaluating both retailer and vendor performances. They showed the effectiveness of VMI based on five dimensions, including transportation, information sharing, inventory, collaboration, and manufacturing. Kim and Park (2010) also investigated the use of decision support systems in VMI and their effects on service levels.

Most researchers conducted the effectiveness and benefits of vendor-managed inventory strategy under demand uncertainty. Glock and Kim (2015) studied the benefit of VMI with a two-echelon system, including a single vendor and multi-retailers. Li et al. (2018) presented the VMI system for advertising and considered the demand stochastic. Also, Pramudyo and Luong (2017) proved the ability of VMI to reduce the total cost of the system with a single-vendor and a retailer. Most of this research has focused on the solution regarding the system under VMI policy using commercial software and exact methods of optimization problems. These optimization problems can be solved by commercial software for small-sized instances. Therefore, our paper has expanded to solve a complicated model regarding large-sized instances using meta-heuristic algorithm based on the firefly algorithm.

3. The Modified Version of Firefly Algorithm

The firefly algorithm stands out among the best new bio-inspired optimization techniques. The modified firefly algorithm is introduced to solve supply network optimization problems. The need for modifying the algorithm is that the standard firefly algorithm has limitations. We implemented the standard firefly algorithm to compare it with the new modified version. The modified firefly algorithm is employed and used for validation and verification. The flow chart of firefly algorithm is shown in Figure 1. The mathematical articulation used to improve the movement of the firefly is done in Sababha et al. (2018). See Equation (1):

$$x_{i+1} = x_j + \beta(t)(x_i - x_j) + \alpha(t)\varepsilon_i \tag{1}$$

Where $\alpha(t)$ is the randomness coefficient at time t, ε_i is the random number, and $\beta(t)$ is the attractiveness coefficient at time t. x_i, x_j are the initial random population with less brightness (i) and more brightness (j). Sababha et al. (2018) considered this an exploitation technique represented by the first two terms to demonstrate a superior answer for the remainder of the operators while exploration procedures are depicted by the last term for inquiry space exploration in Equation (1). To enhance the precision of the algorithm of extraordinary worth, at that point, their movements should be focused on. Sababha et al. (2018) showed and expressed the randomness coefficient (α) in

Equation (2) as:

$$\alpha(Itr_i) = \exp\left(1 - \left(\frac{Itr_{max}}{Itr_{max} - Itr_i}\right)^c\right) \tag{2}$$

C is considered as the whole or integer number to decide the speed of rotting of the irregularity, while Itr_{max} is considered as the maximum iteration number, and Itr_i is considered as the current number of iterations. In addition, parameter ϒ is a crucial viewpoint in describing the distinctions of the speed of the assembly and attractiveness. The result is that when a steady (c) is connected in taking care of the issues of optimization, the firefly algorithm execution will be detectably obliged, as done in a customary firefly algorithm. Sababha et al. (2018) showed and expressed the ϒ equation as follows:

$$\Upsilon(Itr_i) = 1 - \exp\left(1 - \left(\frac{Itr_{max}}{Itr_{max} - Itr_i}\right)^c\right) \tag{3}$$

Because of the modification, the proposed and modified firefly algorithm will not get caught in the neighborhood extraordinary, and it will heighten the combination speed, improving the arrangement at ideal a superior since there will be a balance among nearby and worldwide hunts. In the inquiry procedure, two refreshing formulas are clarified, picked arbitrarily, and introduced in equation (4) by Sababha et al. (2018):

$$\boldsymbol{x_{i+1}} = \begin{cases} \beta(i)x_i + x_j\big(1 - \beta(i)\big) + \alpha(i)\varepsilon_i & rand > 0.5 \\ \dfrac{NG - i}{NG}(1 - \delta)x_i + \delta x_{best} & Elsewhere \end{cases} \tag{4}$$

Parameter δ is considered as the gray coefficient, and NG is considered as the generation number. The fireflies can secure increasingly valuable data from others and change the bearings of light adaptively. At the last point, the progression estimation is intended to cause the algorithm to have a balance between exploitation and exploration procedures and to improve the algorithm on the off chance that the pursuit conditions are in modern and plentiful space and high terms.

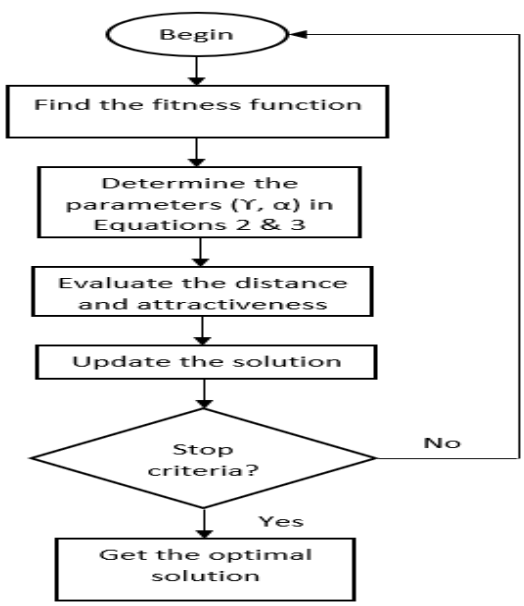

Figure 1. Flow chart of proposed algorithm

4. Problem Definition and Mathematical Modeling

4.1 Motivation and Problem Definition

VMI policy is implemented by the vendor for the whole manufacturing network by combining the location-inventory assignment with the allocation of the entire replenishment policy. In this research, we consider the vendor implements two policies for the supply of goods, direct-delivery policy and VMI policy under demand uncertainty. The role of the vendor is to be focused on the inventory level to overcome the expected shortage that may occur at the industrial plants without warehouses. Once shortages occur, the industrial plants with assigned warehouses collaborate with industrial plants without warehouses under VMI policy. The direct-delivery system is applied in industrial plants with no assigned warehouse as a non-collaborative strategy, while the VMI policy is used for industrial plants with designated warehouses as a collaborative strategy. The vendor also applies a direct-delivery policy for the supply of goods to customers by courier. Figure 2 presents the integrated supply chain networks of a case study for a pipe manufacturing company to demonstrate the efficiency of the model. This company produces three different types of pipes in different sizes: Glass Reinforced Plastic pipes (GRP), Glass Reinforced Epoxy pipes (GRE), and Reinforced Concrete pipes (RC). The role of vendors is to supply these products to different industrial plants for use in their projects.

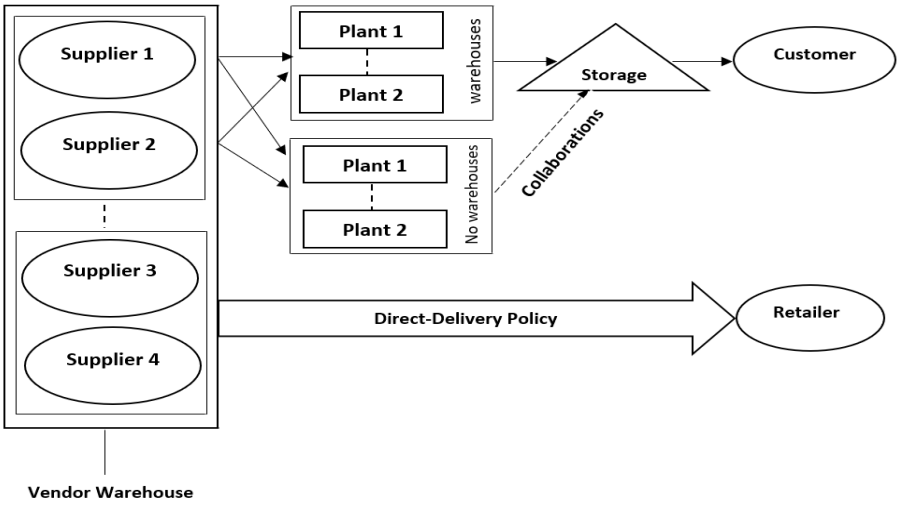

Figure 2. Proposed supply chain networks for case study

As commercial software, such as GAMS and Lingo, cannot solve the larger problems in a reasonable run time to achieve better outcomes, we apply a modified firefly algorithm to solve the problem. The vendor implements VMI for the whole manufacturing network by combining the storage facility location with the allocation of the entire replenishment policy. The demand is a random variable and follows a normal distribution regardless the demand size. The objective function of the system is to minimize the total expected costs associated with vendors. In the case when we consider the system under VMI policy, the vendor is responsible for all expected costs associated with the inventory operations under demand uncertainty.

Table 1. Indexes, notations, and decision variables used in this paper

Indexes	**Decisions Variables**
$\boldsymbol{i}$, $\boldsymbol{j}$, **k, and l** indicate plants, products, vendors and retailers respectively	$\boldsymbol{Z_{ij}}$ Number of order quantity of product j delivered to plant i with designated warehouse
Parameters $\boldsymbol{TC_{ij}}$ The cost of transportation of product j delivered to plant i $\boldsymbol{OC_{ij}}$ The ordering cost per order $\boldsymbol{V_j}$ The volume of item per product j $\boldsymbol{HC_{ij}}$ The holding cost for item j $\boldsymbol{R_{ij}}$ Average of demand rate for product j at industrial plant i $\boldsymbol{\mu_{ij}}$ Average of the demand rate per units per lead time for product j at industrial plant i $\boldsymbol{SC_i}$ Cost of setting up a warehouse at plant i $\boldsymbol{K_j^v}$ Safety factor of product j associated with warehouse of vendor $\boldsymbol{\sigma_{ij}}$ Standard deviation of the rate of demand for product j at industrial plant i $\boldsymbol{\rho}$ Cost of penalty associated with shortage for products $\boldsymbol{\rho_{kl}}$ Cost of penalty associated with shortage for vendor $\boldsymbol{S_i}$ Required space of warehouse used at industrial plant i $\boldsymbol{U_i}$ Transportation cost of orders delivered to plant i $\boldsymbol{HC_j^v}$ Holding cost of product j associated with the warehouse of vendor $\boldsymbol{T}$ Lead times $\boldsymbol{S_T}$ Standard deviation of lead time for product j $\boldsymbol{OP}$ is the price offering for receiving external capacity $\boldsymbol{PP_i}$ is the price of product of industrial plant i with assigned warehouse $\boldsymbol{U_i}$ is the penalty for unmet demand $\boldsymbol{\alpha, \beta}$ are the factors of price adjustments	$(\boldsymbol{NS})_i$ Number of shipments to plant i with designated warehouse $\boldsymbol{L_{ij}}$ Safety stock level at the plant i with allocated warehouse for product j $\boldsymbol{P_{dl}}$ 1 if shipment for vendor is delivered directly to retailers, otherwise 0 $\boldsymbol{P_{wi}}$ 1 if vendor set a warehouse facility for product, otherwise 0 $\boldsymbol{S_k^v}$ Safety stock level of product j associated with warehouse of vendor $\boldsymbol{SS_{ij}}$ Safety factor of product j at plant i $\boldsymbol{SC_i}$ is the capacity shared from industrial plant with assigned warehouse to industrial plant with no warehouse $\boldsymbol{I_i}$ is the inventory at industrial plant with assigned warehouse $\boldsymbol{UD_i}$ is the unmet demand at industrial plant with no warehouse $\boldsymbol{Q_{kl}}$ Retailer's order quantity from vendor $\boldsymbol{S_{kl}}$ The safety stock factor of vendor for retailer

$\boldsymbol{AV_i}$ is value allocated for extra capacity at industrial plant i with assigned warehouse $\boldsymbol{DC_i}$ is time of delivery from industrial plant with assigned warehouse to industrial plant with no warehouse $\boldsymbol{H_i}$ is cost of holding at industrial plant i with assigned warehouse $\boldsymbol{OC_l}$ The ordering cost of retailer from vendor $\boldsymbol{TC_l}$ The cost of transportation for orders to retailers $\boldsymbol{HC_l}$ The holding cost of retailer $\boldsymbol{SC^v}$ The setup cost of vendor $\boldsymbol{D_k}$ The vendor's demand rate $\boldsymbol{D_{kl}}$ Retailer's demand rate from vendor $\boldsymbol{R_k}$ The rate of production for vendor $\boldsymbol{RS_k}$ Space requires for vendor for products $\boldsymbol{W_k}$ Space of warehouse for vendor	

The assumptions considered for this model are as follows:

- The demand of all industrial plants for products is assumed unknown and following normal distribution.
- We applied VMI policy for the orders delivered to the industrial plants with designated warehouses and a direct-delivery policy (DDP) for the orders delivered to retailers from vendors.
- There are collaborations between the industrial plants with designated warehouses and those without to reduce the expected cost associated with inventory operations.
- We applied VMI and direct-delivery policies simultaneously for the inventory operations of industrial plants with designated warehouses and industrial plants without warehouses, and a DDP only for orders delivered directly to retailers.
- The shortage is allowed for the model.

4.2 Mathematical Modeling

In this research, the objective function is to minimize the total cost of vendor under VMI policy and direct-delivery policy (DDP) with demand uncertainty. The vendor used VMI and DDP policies with the industrial plants with assigned warehouse and the ones with no assigned warehouse under demand uncertainty. Equation (5) is used to the objective function of the proposed supply chain networks under different policies.

$$\begin{aligned} Min \sum_{i=1}^{n}\sum_{j=1}^{m}(TC_{ij}+OC_{ij})\,R_{ij}\,P_{dl} &+ \sum_{k=1}^{v}\sum_{l=1}^{r}\frac{D_k}{Q_k}\,(TC_l+OC_l) + \sum_{k=1}^{v}\sum_{l=1}^{r}HC_l\left(\frac{Q_{kl}}{2}+S_{kl}\right) \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{m}HC_{ij}\left(\frac{Z_{ij}}{2}+L_{ij}\right) + \sum_{j=1}^{m}HC_j^v\,S_j^v + \sum_{k=1}^{v}SC^v\left(\frac{D_k}{Q_k}\right) + \sum_{i=1}^{n}SC_i\,P_{wi} \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{m}\rho\,NS_i + \sum_{k=1}^{v}\sum_{l=1}^{r}\rho_{kl}\left(\frac{D_{kl}}{Q_{kl}}\right)*\left(\frac{\sigma_{ij}}{2}*\left(\sqrt{1+S_{kl}^2}-S_{kl}\right)\right) \end{aligned} \tag{5}$$

The first term stands for the courier transportation delivery costs per item for all demanded products and each plant with no designated warehouse. The second term indicates the ordering cost of retailer and the cost of transportation for orders delivered from vendor to retailer under direct delivery policy. The third term stands for holding cost of retailers from vendors added to the average order quality and the safety stock level of all vendors for all retailers. The fourth term stands for holding cost per item of the added average order quality and the safety stock level for all the products at each industrial plant with designated warehouses. The fifth term stands for the cost of holding for the safety stock of products associated with the warehouse of vendor. The sixth stands for setup cost of vendors multiplied by the rate of demand for vendors divided by the order quantity of vendors. The seventh term stands for the fixed cost of setting up the warehouses at each industrial plant for the assigned space by each plant for the vendors' warehousing purposes. The eighth and ninth term stand for cost of shortage associated with the number of shipments and the demand rate from vendors for retailers divided by retailers' order quantity.

In this research, commercial software, such as GAMS and Lingo, cannot solve large-sized instances in a reasonable run time, so we solved this problem with meta-heuristic algorithms.

We assume the demand is unknown and following a normal distribution. The expected shortage per replenishment cycle as a function of S_{kl} can be written as follows (Kundu and Chakrabarti 2012):

$$ESPR_{S_{kl}} = \frac{\sigma_{ij}}{2} * \left(\sqrt{1 + S_{kl}^2} - S_{kl}\right) \tag{6}$$

Subjected to:

$$\sum_{j=1}^{m} Z_{ij}\ V_j \leq P_{wi}\, S_i \quad for\ \forall\ i = 1,2,3, \dots, n \tag{7}$$

$$\sum_{k=1}^{v} Q_{kl} \leq P_{wi} \qquad for\ \forall\ j = 1,2,..,m \quad and \quad i = 1,2,..,n \tag{8}$$

$$NS_i \geq R_{ij} P_{wi} \qquad for\ \ \forall\ i = 1,2, \dots, n\ \ and\ \ j = 1,2, \dots, m \tag{9}$$

$$Z_{ij} \leq P_{wi} \qquad for\ \forall\ i = 1,2,..,n\ \ and\ j = 1,2,..,m \tag{10}$$

$$(P_{wi}, P_{dl}) = \begin{cases} 1\ if\ vendors\ set\ a\ warehouse\ and\ use\ DDP\ for\ shipments \\ 0 \qquad\qquad Otherwise \end{cases} \tag{11}$$

$$\sigma = \sqrt{\sum_{l=1}^{r} \sigma^2 * P_{dl}} \tag{12}$$

$$S_j^v = S_{kl} * S_T \tag{13}$$

$$S_k^v \geq m_{kl}\left(\frac{\sigma_{ij}}{2} * \left(\sqrt{1+S_{kl}^2} - S_{kl}\right)\right) \ for \ \forall \ k = 1,2,..,v \ and \ l = 1,2,..,r \tag{14}$$

$$RS_k \sum_{k=1}^{v}\sum_{l=1}^{r} \frac{Q_{kl}}{2}\left(1 - \frac{D_{kl}}{R_k}\right) \leq W_k \ for \ \forall \ k = 1,2,..v \ and \ l = 1,2,..r \tag{15}$$

$$OP = (U_i + PP_i)\,(1 - \alpha) \tag{16}$$

$$AV_i = (DC_i - H_i)(1 + \beta) \tag{17}$$

$$SC_i = min\,(I_i, UD_i) \tag{18}$$

$$Z_{ij}, NS_i, S_j^v, S_{kl} \geq 0 \qquad for \ \forall \ i = 1,2,..,n \ and \ j = 1,2,..,m \tag{19}$$

Equation (7) indicates that the safety stock level for products and the total size of order quantities per order should be equal to or less than the designated area for each industrial plant of i with designated warehouse. Equation (8) indicates that each order of quantity of retailer from vendor should be determined if the vendor set a warehouse for products at the industrial plant. Equation (9) is the constraint of demand satisfaction. It states that the total number of order quantity of product j delivered to plant i with allocated warehouse is greater than or equal to the demand of product j of plant i with allocated warehouse. Equation (10) indicates that the maximum order numbers shipped to plant i with designated warehouse are less than or equal to the $bigM$ value. Equation (11) indicates the binary constraint and shows that each vendor k could have a warehouse with VMI delivery or directly delivered products to retailers under direct delivery policy. Equation (12) represents that the standard deviation at the vendor is equal to the square root of the squared standard deviation of demand rate and the number of shipments received by retailers. Equation (13) indicates that the safety stock level associated with the warehouse of the vendor is the safety factor multiplied by the combined standard deviation of demand and lead time ($S_{d,T}$). Equation (14) indicates that safety stock level of product j associated with warehouse of vendor should be greater than or equal to the shortage of all number of shipments to industrial plant i. The available space for warehouse of vendor is determined in Equation (15). Equation (16) indicates that the cost of unmet demand should be more than the price offered for receiving external capacity for demanding industrial plant. Equation (17) represents that the cost spent for product delivery should be less than the value allocated for extra capacity at the industrial plant with assigned warehouse. Equation (18) indicates that the shared capacity from the industrial plant with assigned warehouse and the industrial plant with no warehouse will be the minimum of available inventory and demanded inventory. All parameters in Equation (19) should be greater than or equal to zero, which indicates a non-negativity constraint. Safety stock level associated with industrial plants with designated warehouses is defined as follows:

$$L_{ij} = SS_{ij} * \left(\sqrt{T * \sigma_{ij}^2 + R_{ij}^2 * S_T}\right) \tag{20}$$

5. Results and Discussion

5.1 Tuning Parameters of the Modified Firefly Algorithm

The result performance of the modified firefly algorithm depends on its parameters, including $Itr_{max}, \beta, \gamma, \alpha \text{ and } population$. So, its parameters need to be calibrated to obtain the effectiveness of the proposed algorithm. We used the Taguchi method for tuning the parameters of the modified firefly algorithm. Based on the Taguchi method, the signal-to-noise (S_N) ratio is used to determine the amount of variation in the response variable (Y_i). The ratio of signal-to-noise is determined in Equation (21) (Zandieh et al. 2009):

$$S_N = -10 \log_{10}\left(\frac{1}{N_{experiment}} \sum_{i=1}^{N} Y_i^2\right) \tag{21}$$

Where $N_{experiment}$ refers to the experiment number of each set of problem sizes. We applied a meta-heuristic algorithm based on the modified firefly algorithm and standard firefly algorithm in MATLAB, R2019a and run on AMD Ryzen 5 2500U 2.00 GHz processor with 8.00 GB memory. In this paper, we assume level 1, level 2, and level 3 for Itr_{max} are 230, 280, and 330, respectively. Level 1, level 2, and level 3 for β are 0.5, 0.6, and 0.7, respectively. Level 1, level 2, and level 3 for γ are 0.6, 0.7, and 1.07, respectively. Level 1, level 2, and level 3 for α are 0.3, 0.62, and 0.4, respectively. Level 1, level 2, and level 3 for $Number\ of\ population$ are 53, 78, and 103, respectively. Figure 3 shows the signal-to-noise average for each parameter at three different levels. The better signal-to-noise is smaller. Figure *3* represents that each parameter of the proposed algorithm has a main effect on the response level. For example, the attractiveness coefficient parameter (β) at the level 3 has a higher mean of response level than the first and second levels of the parameter.

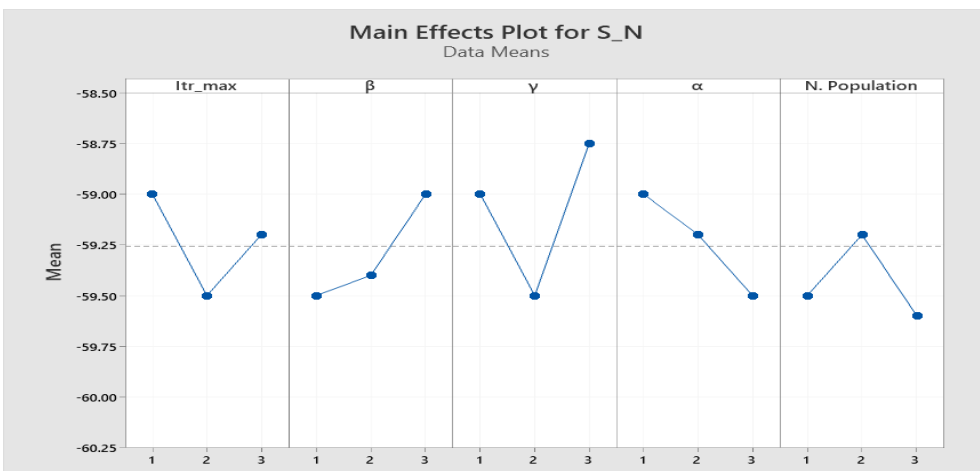

Figure 3. Plot of S_N at each level of proposed algorithm parameters

As commercial software such as, GAMS and Lindo, cannot solve the large-sized problems in a reasonable run time in order to achieve better outcomes, we apply a modified firefly algorithm to solve the problem. We also applied a standard firefly algorithm to test the efficiency of the proposed algorithm. In addition, we solved the model using commercial software, namely GAMS and Lindo software. Then, we compared the results of the commercial software with the results of the modified firefly algorithm and standard firefly algorithm, which indicate that commercial software cannot solve the model on a large scale as does a meta-heuristic algorithm based on a modified firefly algorithm. We presented the integrated supply chain networks of a case study for a pipe manufacturing company to demonstrate the efficiency of the model, as shown in Figure 2. As previously stated, this company produces three different types of pipes in different sizes. The role of vendors is the supply of these products to different industrial plants for use in their projects.

5.2 Discussion

The vendor is responsible for all costs associated with shortages for all industrial plants with no assigned warehouses and the safety stock levels at industrial plants with assigned warehouses. Commercial software, such as GAMS and Lingo, cannot solve the large-sized problems in a reasonable run time, so to achieve better outcomes, we applied a modified firefly algorithm to solve the problem. We also applied a standard firefly algorithm to test the efficiency of the proposed algorithm. In addition, we solved the model using commercial software, namely GAMS and Lingo software. Then, we compared the results of the commercial software with the results of the modified firefly algorithm and standard firefly algorithm, which indicate that commercial software cannot solve the model on a large scale as a meta-heuristic algorithm based on a modified firefly algorithm does. We analyzed the effectiveness of the modified firefly algorithm with small and large problems.

S1, S2, S3, S4, and S5 for small-sized problems are bounded by one-item and one-industrial, one-item and one-industrial plant, one-item and two industrial plants, two-items and two-industrial plants, and two-items and one-industrial plant, respectively. L1, L2, L3, L4, and L5 for large-sized problems are bounded by two-items and two-industrial plants, two-items and three-industrial plants, two-items and two-industrial plants, three-items and two-industrial plants, and three-items and three-industrial plants, respectively, as shown in Table 2 and Table *3*. The chart of convergence for large-sized problems is presented in Figure 5.

We solved the optimization problem using different commercial software to test the efficiency of the meta-heuristic algorithms based on firefly for solving large-sized problems.

Table 2 shows that commercial software, Lingo and GAMS, is not able to obtain the optimal solution. However, meta-heuristic algorithms based on firefly can solve large-sized problem and can obtain the optimal solution for different sized problems. We applied the analysis of variance (ANOVA) to show the results statistically.

Table 2. The best solution obtained by GAMS, Lingo, and meta-heuristic algorithms with collaboration

Problems		**Best Solution of the objective function with collaboration ($)**			
		GAMS	**Lingo**	**Standard FA (SFA)**	**Modified FA (MFA)**
Small	**S1**	864,490	843,293	802,392	783,345
	S2	996,492	914,392	848,393	832,356
	S3	1,061,304	994,345	882,922	864,985
	S4	-	1,012,234	996,392	932,392
	S5	-	1,209,466	1,029,345	996,904
Large	**L1**	-	-	1,203,334	1,085,932
	L2	-	-	1,594,493	1,209,903
	L3	-	-	1,893,495	1,585,934
	L4	-	-	2,034,303	1,803,834
	L5	-	-	2,584,334	2,102,395

Table 2 and Figure 4 represent the best solution of the second objective function obtained by commercial software and meta-heuristic algorithms. As the objective function of the model is to minimize the total expected cost of supply chain networks, we run the model as a minimization problem to get the optimal solution. Table 2 and Figure 4 show that the modified firefly algorithm has the best performance of solution when compared with the standard firefly algorithm and commercial software.

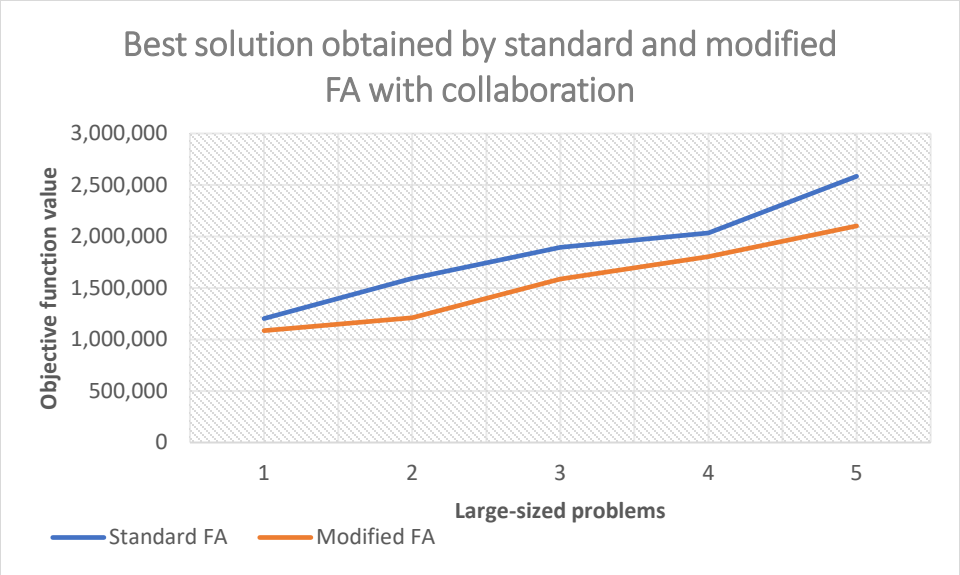

Figure 4. Comparison of best solution with collaboration based on large-sized problems

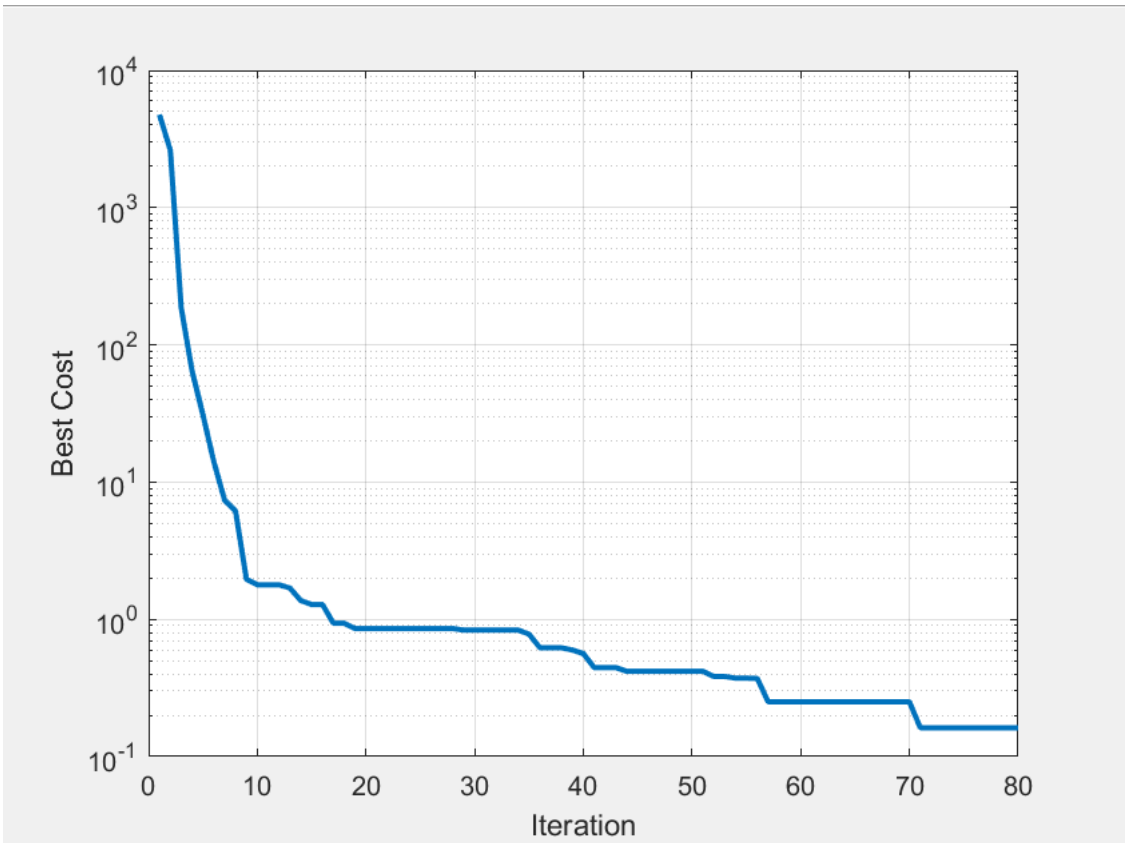

Figure 5. Chart of convergence for large-sized problems

The results show that there is a significant difference between the best solution of large-sized problems using the proposed algorithm with and without collaborations, as shown in Figure 6. This figure shows the interval plot of the objective function of large-sized problems with and without collaborations using the modified firefly algorithm.

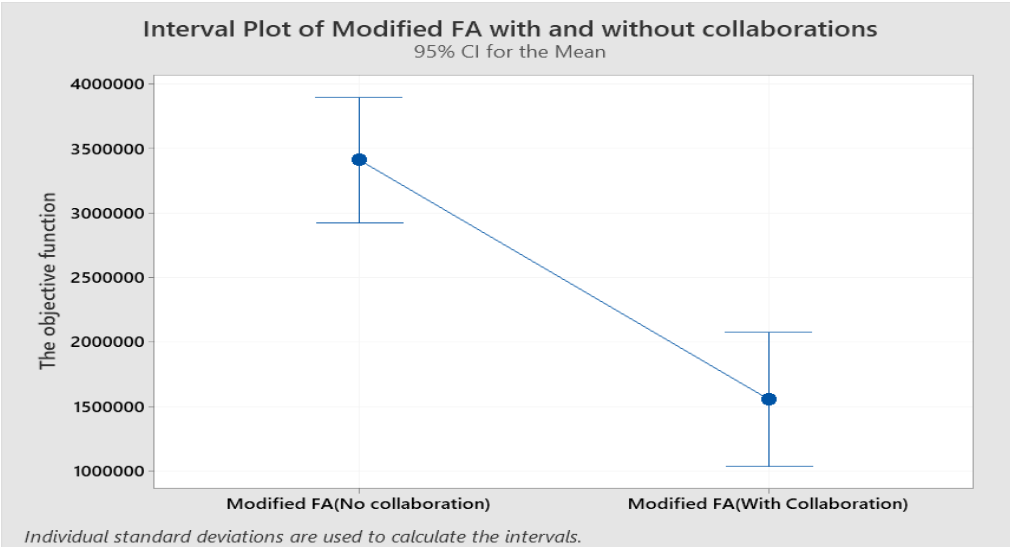

Figure 6. Interval plot of proposed algorithms based on large-sized problems with and without collaboration

5.3 Computational time (CPU)

After running the meta-heuristic algorithms based on the firefly algorithm and commercial software, the results of CPU time with collaboration are obtained, as shown in
Table 3.

Table 3. CPU time of commercial software and meta-heuristic algorithms with collaboration

Problems		CPU time of algorithms with collaboration ($)			
		GAMS	Lingo	Standard FA (SFA)	Modified FA (MFA)
Small	S1	585.45	720.23	347.23	204.34
	S2	693.43	813.98	593.40	293.12
	S3	943.23	1234.57	893.34	338.34
	S4	-	1483.95	997.76	402.45
	S5	-		1009.47	593.78
Large	L1	-	-	1093.85	682.45
	L2	-	-	1709.34	845.09
	L3	-	-	1923.56	920.34
	L4	-	-	2492.43	993.03
	L5	-	-	2843.90	1007.54

Table 4 and Figure 7 show the ANOVA table and interval plot of the CPU time for algorithms with collaborations based on the large-sized problems. Table 4 shows that P-value is less than the significance level ($\alpha = 0.05$), which indicates that there is a substantial difference between the CPU time of algorithms with collaboration. It shows that the standard firefly algorithm needs more time to obtain the optimal solution as compared with the proposed algorithm.

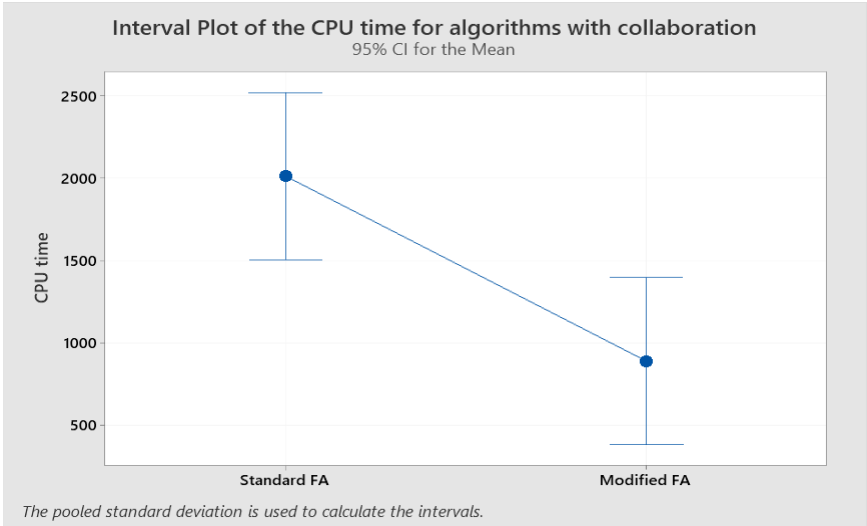

Figure 7. CPU time of algorithms with collaboration

Table 4. ANOVA table of CPU time for algorithms with collaboration

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Algorithm	1	3152407	3152407	13.03	0.007
Error	8	1935738	241967		
Total	9	5088145			

The vendor is also responsible for picking the desired service level to determine the safety stock level at the industrial plants with assigned warehouses and prevent stockout. In this case, the inventory level depends on the service level picked by the company. The safety stock level depends on the service level picked by the industrial plant with assigned warehouse. So, the amount of inventory depends on the service level. Figure 8 , Figure 9, and Figure 10 show the safety stock level for Glass Reinforced Plastic pipes (GRP), Glass Reinforced Epoxy pipes (GRE), and Reinforced Concrete pipes (RC), respectively. They indicate that as the service level increases, the more inventory the company needs to carry to prevent a stockout.

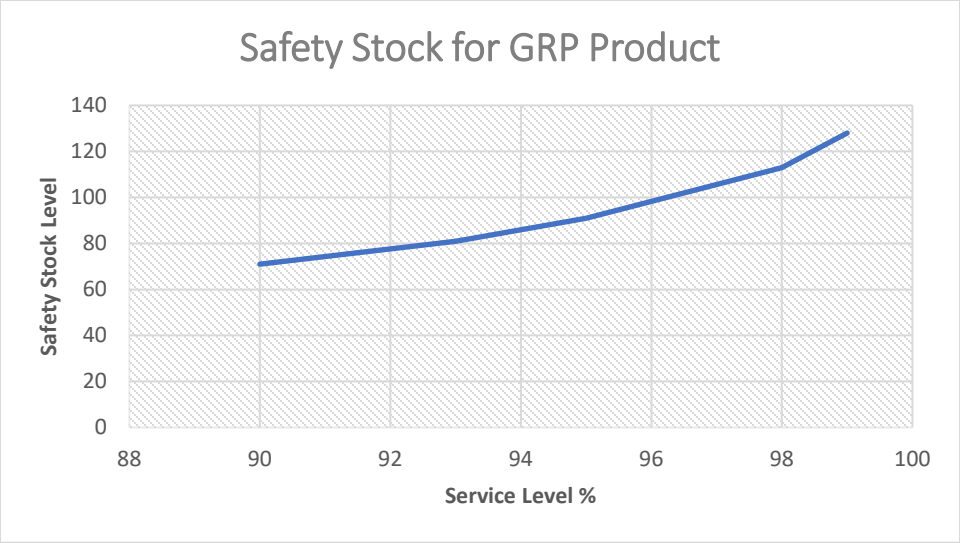

Figure 8. Safety stock level for glass reinforced plastic pipes (GRP) based on service level

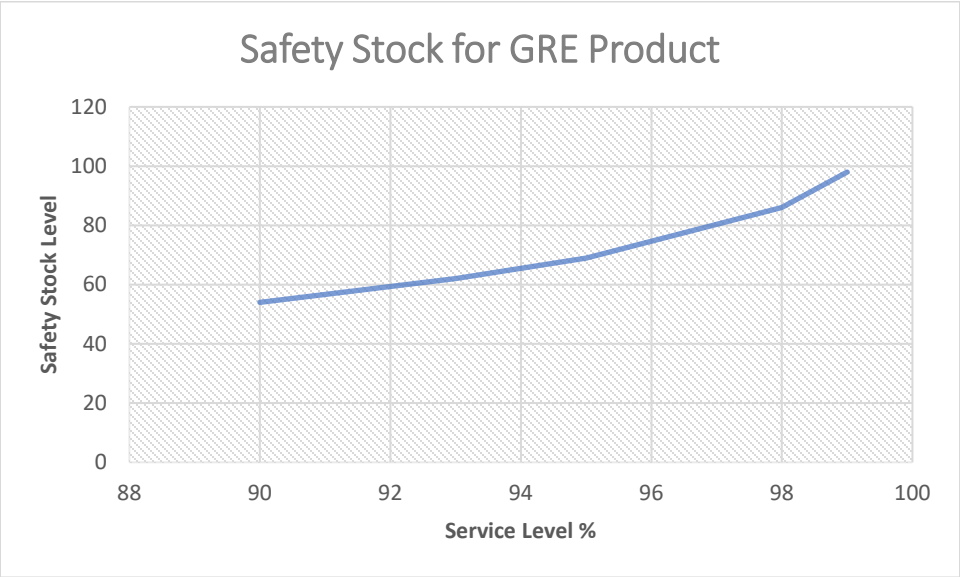

Figure 9. Safety stock level for glass reinforced epoxy pipes (GRE) based on service level

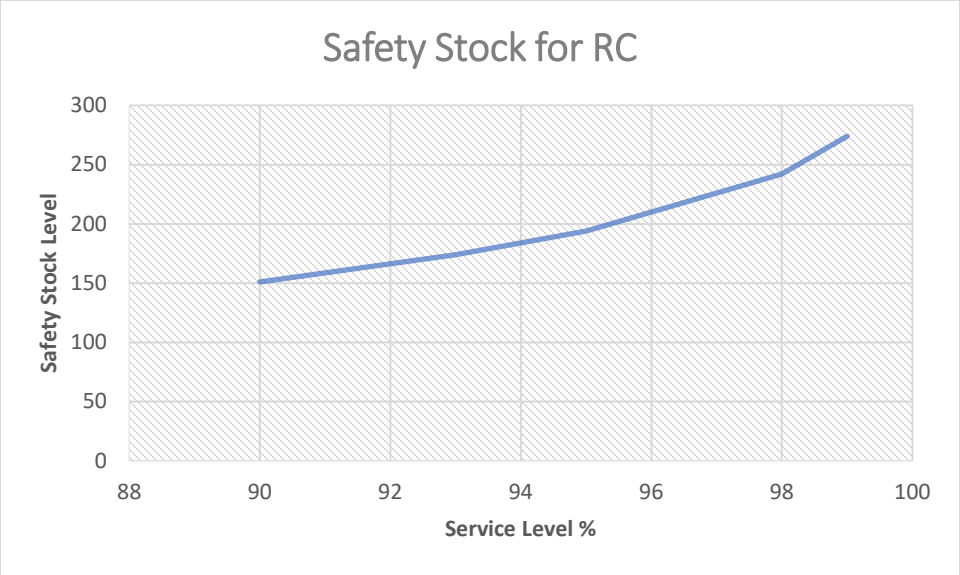

Figure 10. Safety stock level for reinforced concrete pipes based on service level

5.4 Sensitivity Analysis

In this paper, we present the sensitivity analysis for the essential parameters in the proposed model for objective function determined by Equations (5). These important parameters consist of transportation and ordering costs and holding cost of vendor. We changed the values of these important parameters while keeping the other parameters with their original values to identify the effectiveness of these parameters on the model. The effect of vendor holding cost is shown in Figure 11. It shows the effect of the holding cost on the total expected cost of the vendor under VMI and direct-delivery policy with demand uncertainty. It shows that the total expected cost of vendor under VMI policy and direct-delivery policy increases slowly as the cost of holding increases. It also shows that collaborations between industrial plants with assigned warehouses and industrial plants with no assigned warehouse minimize the total expected cost of the vendor. Figure 11 indicates that the collaborations between the industrial plants minimize the total expected costs when compared with the results of non-collaborations.

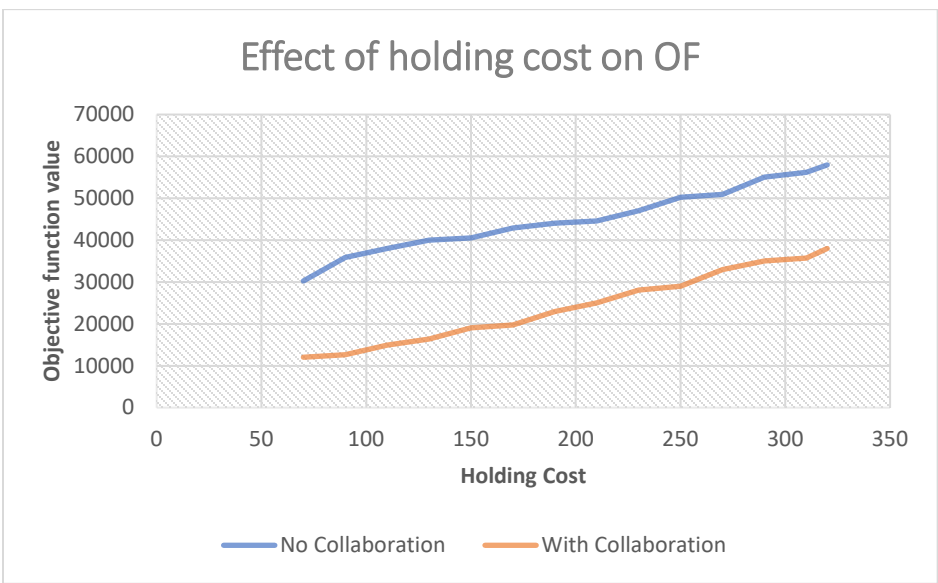

Figure 11. Effect of holding cost HC_{ij} on the total expected cost

Figure 12 shows the effect of transportation cost on the total expected cost of the vendor under VMI and direct-delivery policy with demand uncertainty. It shows that the total expected cost of the vendor under VMI policy and direct-delivery policy increases slowly as the cost of transportation increases. It also shows that collaborations between industrial plants with assigned warehouses and the industrial plants with no assigned warehouses minimize the total expected cost of the vendor.

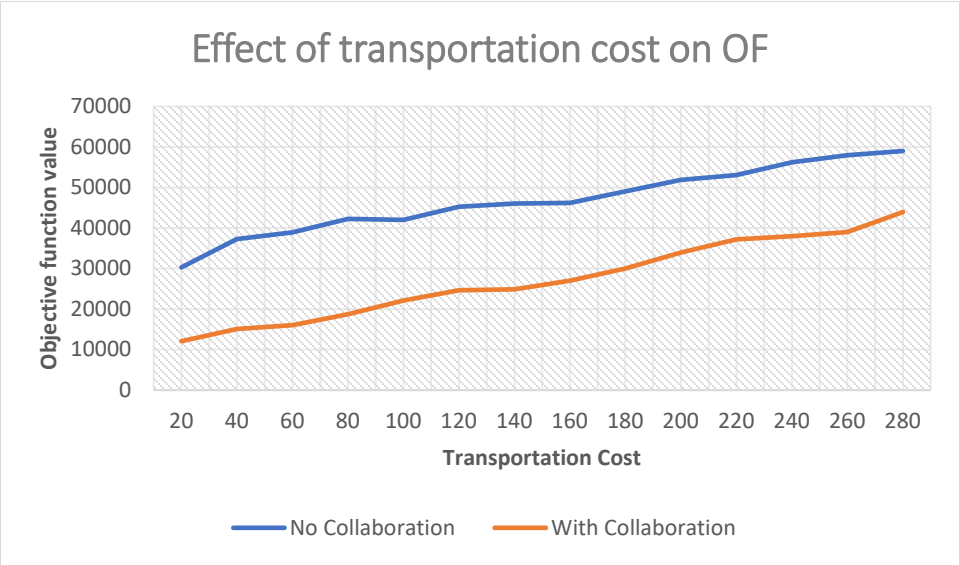

Figure 12. Effect of transportation cost TC_{ij} on the total cost

6. Conclusion

This research explores the effects of collaboration between industrial plants with assigned warehouse and the industrial plants with no assigned warehouses, in which the vendor uses VMI policy for collaboration and direct-delivery policy for non-collaborations. In this research, commercial software, GAMS and Lingo, and meta-heuristic algorithms based on the firefly algorithm, were applied to solve the model based on small and large-sized problems with and without collaboration. The parameters of algorithms are calibrated to obtain the better outcomes of algorithms. In this regard, we used the Taguchi method. The aim of objective function of the model is to minimize the total expected cost of supply chain networks. The findings for different scenarios, including collaboration and non-collaboration, are tested. They showed that collaboration between the industrial plants with and without assigned warehouses can save money and minimize the expected costs associated with shortage and inventory. The results also showed that collaboration can be affected by the performance of industrial plants in a highly reliable environment. The sensitivity analysis was applied by changing the values of significant parameters and keeping the others. The computational time for the proposed algorithm is reduced as compared with standard firefly algorithm and commercial software. This research can be extended by using other probability distributions and determining the impact of the parameter setting of the proposed algorithm.

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