Reserve of Life Insurance Prospective Dwiguna Joint Life and Last Survivor with Gompertz Law

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Abstract

In this paper, the double life insurance premium stipulated with the combined status of last survivor and join life involving two insurance participants who have a kinship such as husband and wife, brother and sister, for which they work in the same institution. In determining the policy to be made by the life insurance party does not require two policies to be made, but only has one policy. So by having one policy it is expected that the premium paid by life insurance participants to the life insurance company will be smaller than if they had to pay in two policies. Determination of the bilingual life insurance premium that participants will pay to the insurer based on the chance of death of the two life insurance participants, stating a condition that will remain for at least one surviving member and will cease after the death of the last person of its members (all its members die), and is also a condition that lasts as long as all members of the combined several people can survive and will cease after one of its members first dies. To determine a single premium and an annual premium using the cash value of an early life annuity from dual-use life insurance. Meanwhile, the cash value of the initial living annuity was influenced by interest rates and discounted vaktor and also affected the combined life chances of the two insurance participants. Furthermore, prospective premium reserves are obtained by determining a single premium and an annual premium. In formulating the chances of dying insurance participants used the distribution of Gompertz, which in this distribution there are constants used to determine life chances and chances of dying.

Keywords
Premium reserves, last survivor, joint life, Gompertz distribution

1. Introduction

The old days led to an inability to earn income and resulted in economic hardship for the workers themselves and their families. One of the efforts to anticipate the risks of the old days is to include each worker in a life insurance program. Where life insurance is an insurance that provides payment of a certain amount of money for the death of the insured to the heir or the person who is entitled to receive it in accordance with the provisions and agreements that have been approved by the insured in the life insurance policy. Married couples working at the same agency make it possible to take out one insurance program with the same policy to facilitate premium payments and sum-insured payments (Sukono et al., 2017; Sidi et al., 2018).

Life insurance based on the number of insured is divided into two types of single life insurance (individual) and combined life insurance. In individual life insurance is limited to one participant only, while in combined life insurance there are two or more insurance participants, for example husband and wife, brother and sister, children and parents (Bain and Engelhardt, 1992; Saputra et al., 2018). In this article discussed is a combined insurance consisting of people by taking the example of husband and wife working in the same agency. According to Bowers et al. (1997) combined life insurance is distinguished into two, namely joint life and life insurance last survivor. Joint life insurance is a condition that lasts as long as all members of the combined multiple people can survive and will cease after one of its members first dies. While last survivor life insurance is a condition that will remain for at least one surviving member and will cease after the death of the last person of its members (all its members die).

Based on the coverage period is divided into pure dual life insurance, life insurance for life, term life insurance and bipurpose life insurance. According to Futami (1993), dual-use life insurance is a type of insurance that is a combination of pure dual-use life insurance and term life insurance which means in and at the end of the coverage period to the policyholder, both dead and alive will be paid the sum insured. For the payment of the sum insured on the insurance dual use joint life states the sum insured will be paid if one of the husbands and wives dies first, while on the insurance of the last survivor is if the insurance participant is the husband and wife at the most recent time of death or at the end of the coverage period is still alive then the sum insured will be paid.

On dual life insurance and last survivor Jhon and Albert (2016) and Matvejevs and Matvejevs (2001), single premium stipulated and annual premium. The determination of premiums can be calculated through the value of living annuities from insurance participants. In this paper will be used annual premiums. Annual premiums are a series of payments made by insurance participants to the insurance company once a year within the deadline throughout the agreed insurance contract, while the single premium is the payment of insurance premiums made by participants only once during the time of the contract that has been approved upon entry into the insurance participant. Premium payments are also affected by discount factors, life chances and the chances of dying from insurance participants. Hasriati (2019) in previous research has discussed determining last survivor insurance premiums using pareto distribution i.e. by estimating θ parameters for age x and y years. One of the distributions that can be used is attributable to Gompertz or often also called gompertz law (Rytgaard, 1990). The determination of Gompertz distribution and survival function is carried out by stating in the form of acceleration of Gompertz mortality (Lenart, 2014). Which in this case the acceleration of mortality is influenced by gompertz constants. Constant Gompertz has a role in determining the life chances and chances of dying from insurance participants, so that it will ultimately affect the amount of premiums that will be paid by the insurance participants and the sum insured that the insurance participant will receive.

2. Method

2.1 Gompertz Survival and Distribution Function

Based on Bowers et al. (1997) the survival function is related to the distribution function and the opportunity density function. The distribution function of the X continuous random variable that is notified F(x) is a function related to the opportunity density function f(x).

Definition 2.1 (Bain and Engelhardt, 1992**).** Random X variables are said to be continuous random variables if there is a $f(x)$ function, so the cumulative distribution function is declared as

$$F(x) = \Pr(X \leq x) = \int_{-\infty}^{x} f(u)du.$$

There is a term of survival function called survival function. The survival function is notified with S(x), expressed as:

$$S(x) = \Pr(X > x).$$

So the relationship between survival function and distribution function is as follows:

$$S(x) = 1 - F(x).$$

In 1825, an English mathematician named Benjamin Gompertz declared a distribution called the Gompertz distribution.

Definision 2.2 (Willemse and Koppelaar, 2000). Gopertz Distribution $(x|\mu,\sigma)$ with average μ and standard deviation σ is referred to:

$$G(x|\mu,\sigma) = W\left(\frac{x-a}{b}\right)$$

with $W(x) = 1 - e^{-e^x}$ and Constanta a and b as:

$$\sigma = \frac{\pi}{\sqrt{6}} b \text{ and } \mu = a - b\gamma.$$

$G(x|\mu,\sigma)$ called Gompertz distribution cause:

$$G(x \mid \mu, \sigma) = 1 - g^{c^x}.$$

Where $g^{c^x} = S(x)$ is survival function based on Gompertz:

$$g = e^{-e^{\frac{-a}{b}}} \text{ and } c = e^{\frac{1}{b}}.$$

Gompertz's law is a form of mortality law, which is stated as follows:

$$\mu_x = Bc^x, \text{ with } B > 0,\ c > 1,\ x > 0.$$

where constant B represents the general mortality rate and constant c is the specific growth of the mortality rate. The acceleration of mortality of an x+s person is expressed by:

$$\mu_{x+s} = Bc^{x+s}, \text{ with } B > 0,\ c > 1,\ x > 0.$$

That obtained,

$$_tp_x = e^{-\int_0^t Bc^{x+s} ds}$$
$$_tp_x = e^{\frac{-B}{\ln c} c^x (c^t - 1)}.$$

If $\frac{-B}{\ln c} = \ln g$, the equation become (2) as follow:

$$_tp_x = e^{(\ln g) c^x (c^t - 1)},$$
$$_tp_x = g^{c^x (c^t - 1)}. \quad (1)$$
$$_tp_y = g^{c^y (c^t - 1)}. \quad (2)$$

2.2 Annuity lives on individual status with Gompertz Law

An annuity is a payment of a certain amount made each particular interval of time and length on an ongoing basis. The cash value of a living annuity whose payment is made once at the beginning of the year for a certain period of time is called an early life annuity futures is (Dickson et al., 2009):

$$\ddot{a}_{x:\overline{n}|} = 1 + v \quad p_x + v^2 \ {}_2p_x + v^3 \ {}_3p_x + \cdots + v^{n-1} \ {}_{n-1}p_x.$$
$$\ddot{a}_{x:\overline{n}|} = \sum_{t=0}^{n-1} v^t \ {}_tp_x.$$

Furthermore, an early life annuity is obtained for x-year life insurance participants with a period of n years under Gompertz law as follows (Finan, 2011):

$$\ddot{a}_{x:\overline{n}|} = \sum_{t=0}^{n-1} v^t g^{c^x (c^t - 1)}.$$

Then in the same way can be obtained the value of the initial life annuity of life insurance participants aged y years with a period of n years according to Gompertz's law namely (Bebbington et al., 2014):

$$\ddot{a}_{y:\overline{n}|} = \sum_{t=0}^{n-1} v^t g^{c^y (c^t - 1)}.$$

2.3 Change to Die Joint Life and Last Survivor with Gompertz Law

Life insurance not only provides coverage for one insured person, but the life insurance company also provides coverage for two or more insured people. Thus, insurance participants can include other family members in one policy e.g. spouse, mother and child, sister and sister and so on in the hope that the premium paid is smaller than paying two life insurance policies. The number of insured is limited to two people (Youn et al., 2002).

Combined life insurance is divided into two parts namely joint life insurance and last survivor life insurance. Joint life insurance is a condition that lasts as long as all members of the combined multiple people can survive and will cease after the first death of their members. While last survivor life insurance is a condition that will remain for at least one surviving member and will cease after the last death of its members or all its members die. The difference between joint life insurance and last survivor life insurance lies in the time of payment of premiums. Joint life insurance is a combined life insurance that payments premiums until the first death of its participants, while the last survivor life insurance is a combined life insurance whose premium payment is made until the last death of the participant (Youn et al., 2001).

In the combined life insurance in determining the amount of premium paid required survival function for the combined status. The survival function for the combined status is derived from the relationship of the survival function to the individual status. Suppose T(x) states a continuous random variable for x-year-old life insurance participants and T(y) declares a continuous random variable for life insurance participants who are y years old. Continuous random variables for joint life insurance T(x) and T(y) are mutually free i.e. T(xy) = min[T(x),T(y)] with (Futami, 1994):

$$\begin{aligned} F_{T(xy)}(t) &= P\,(\min\,[T(x),T(y)] \le t) \\ &= 1 - P\,(\min\,[T(x),T(y)] > t) \\ &= 1 - P\,(T(x) > t \text{ and } T(y) > t), \end{aligned}$$

$$F_{T(xy)}(t) = 1 - S_{T(xy)}(t).$$

Obtained cumulative distribution function for combined life insurance i.e.

$$F_{T(xy)}(t) = {}_tq_{xy}.$$

$$1 - S_{T(xy)}(t) = {}_tq_{xy}.$$

So obtained survival function from joint life insurance. Life chances of joint life insurance are expressed in the form of:

$${}_tp_{xy} = {}_tp_x \; {}_tp_y. \tag{3}$$

By substantiation of equations (1) and equations (2) into equations (3) obtained life chances of joint life insurance participants based on Gompertz law as follows (Roger et al., 2012):

$${}_tp_{xy} = g^{c^x(c^t-1)} g^{c^y(c^t-1)},$$

$${}_tp_{xy} = g^{(c^x+c^y)(c^t-1)}. \tag{4}$$

Bowers et al. (1997) also states the continuous random variable for last survivor life insurance with T(x) and T(y) is T((xy) - = max[T(x),T(y)] with its cumulative distribution function as follows:

$$F_{T(\overline{xy})}(t) = P\,(\max\,[T(x),T(y)] \le t)\,,$$

$$F_{T(\overline{xy})}(t) = P\,(T(x) \le t \text{ and } T(y) \le t),$$

$$F_{T(\overline{xy})}(t) = P\,(T(x) \le t) P(T(y) \le t).$$

That obtained,

$$F_{T(\overline{xy})}(t) = {}_tq_x \; {}_tq_y\,,$$

$${}_tq_{\overline{xy}} = {}_tq_x \; {}_tq_y\,.$$

Then,

$$1 - {}_tp_{\overline{xy}} = {}_tq_x \; {}_tq_y\,.$$

$$\begin{aligned} 1 - {}_tp_{\overline{xy}} &= (1 - {}_tp_x)(1 - {}_tp_y) \\ &= 1 - {}_tp_x - {}_tp_y + {}_tp_x \; {}_tp_y \end{aligned}$$

$${}_tp_{\overline{xy}} = {}_tp_x + {}_tp_y - {}_tp_x \; {}_tp_y\,.$$

Furthermore, joint life insurance relationship with last survivor life insurance is obtained as follows (Musian, 2003):

$${}_tp_{\overline{xy}} = {}_tp_x + {}_tp_y - {}_tp_{xy}\,.$$

Thus obtained life chances of last survivor life insurance participants based on Gompertz law as follows (Walpole, 2007; Walpole and Myers.(2012):

$${}_tp_{\overline{xy}} = g^{c^x(c^t-1)} + g^{c^y(c^t-1)} - g^{(c^x+c^y)(c^t-1)}. \tag{5}$$

3. Result and Discussion

3.1 Joint Life Annuity and Last survivor with Gompertz Law

Based on Musian (2003), early life annuity futures for combined status there are two, namely the initial life annuity of the joint life term and the early life annuity of the last survivor futures.

The cash value of an early life annuity futures is annotated with a $\ddot{a}_{xy:\overline{n}|}$ is the cash value of an annuity early life futures. The cash value of an annuity early life futures is the cash value influenced by the discount factor v and the life chances for the combined status ${}_tp_{xy}$ over a period of n years are

$$\ddot{a}_{xy:\overline{n}|} = 1 + v \quad p_{xy} + v^2 \; {}_2p_{xy} + v^3 \; {}_3p_{xy} + \cdots + v^{n-1} \; {}_{n-1}p_{xy}.$$

$$\ddot{a}_{xy:\overline{n}|} = \sum_{t=0}^{n-1} v^t \ {}_tp_{xy}. \tag{6}$$

By substantiate the equation (4) into the equation (6) obtained the cash value of the initial life annuity of the joint life term from life insurance participants aged x and y years during the period n years under Gompertz's law, namely:

$$\ddot{a}_{xy:\overline{n}|} = \sum_{t=0}^{n-1} v^t g^{(c^x+c^y)(c^t-1)}.$$

The cash value of the initial annuity futures is notified with $\ddot{a}_{\overline{xy}:\overline{n}|}$ is the cash value of the annuity of the early life term last survivor. The cash value of the last survivor's early life annuity is the cash value affected by the discount factor v and the life chances for the combined status ${}_tp_{\overline{xy}}$ for a period of n years are:

$$\ddot{a}_{\overline{xy}:\overline{n}|} = 1 + v \quad p_{\overline{xy}} + v^2 \ {}_2p_{\overline{xy}} + v^3 \ {}_3p_{\overline{xy}} + \cdots + v^{n-1} \ {}_{n-1}p_{\overline{xy}}.$$

$$\ddot{a}_{\overline{xy}:\overline{n}|} = \sum_{t=0}^{n-1} v^t \ {}_tp_{\overline{xy}}. \tag{7}$$

By substantiate the equation (5) into the equation (7) obtained the cash value of the initial life annuity futures last survivor from life insurance participants aged x and y years over a period of n years under Gompertz law:

$$\ddot{a}_{xy:\overline{n}|} = \sum_{t=0}^{n-1} v^t \left(g^{c^x(c^t-1)} + g^{c^y(c^t-1)} - g^{(c^x+c^y)(c^t-1)}\right).$$

3.2 Joint Life and Last Survivor Single Premium

River to Roger et al. (2012), the single premium of dual-use life insurance is the sum of the single premium of pure dual-use life insurance and term life insurance that is notified by:

$$A_{xy:\overline{n}|} = A_{xy:\overset{\ 1}{\overline{n}|}} + A^{1}_{xy:\overline{n}|}.$$

The single premium of pure dual-use combined life insurance for insurance participants aged x and y years with an insurance coverage period of n years and a sum insured of 1 unit of payment is stated as:

$$A_{xy:\overset{\ 1}{\overline{n}|}} = v^n \ {}_n p_{xy}.$$

and the single premium of future life insurance for the combined status of life is:

$$A^{1}_{xy:\overline{n}|} = \sum_{t=0}^{n-1} v^{t+1} \ {}_{t|} q_{xy}.$$

That obtained:

$$A_{xy:\overline{n}|} = v^n \ {}_n p_{xy} + \sum_{t=0}^{n-1} v^{t+1} \ {}_{t|} q_{xy}.$$

$$A_{xy:\overline{n}|} = v^n \ {}_n p_{xy} + v \sum_{t=0}^{n-1} v^t \ {}_t p_{xy} - \sum_{t=0}^{n-1} v^{t+1} \ {}_{t+1} p_{xy}. \tag{8}$$

Then the substitution of equations (6) to equations (8) is obtained:

$$A_{xy:\overline{n}|} = v^n \ {}_np_{xy} + v\ddot{a}_{xy:\overline{n}|} - (\ddot{a}_{xy:\overline{n}|} - (1 - v^n \ {}_np_{xy})).$$

$$= 1 + v\ddot{a}_{xy:\overline{n}|} - \ddot{a}_{xy:\overline{n}|}$$

$$A_{xy:\overline{n}|} = 1 - d\ddot{a}_{xy:\overline{n}|}. \tag{9}$$

That obtained a single premium of dual life insurance assuming Gompertz is:

$$A_{xy:\overline{n}|} = 1 - d \sum_{t=0}^{n-1} v^t g^{(c^x+c^y)(c^t-1)}. \tag{10}$$

The single premium of life insurance last survivor dwiguna from participants aged x and y years with insurance coverage period for n years is notified with $A_{\overline{xy}:\overline{n}|}$.

Suppose v states the discount factor,, ${}_np_{\overline{xy}}$ states life chances and ${}_{t|}q_{\overline{xy}}$ states the chance of dying delayed for the status of last survivor, then the single premium of life insurance last survivor is purely dual:

$$A_{\overline{xy}:\overset{\ 1}{\overline{n}|}} = v^n \ {}_n p_{\overline{xy}}, \tag{11}$$

and the single premium of last survivor life insurance futures is declared with:

$$A^{\;1}_{\overline{xy}:\overline{n}|} = \sum_{t=0}^{n-1} v^{t+1}\,{}_{t|}q_{\overline{xy}}. \tag{12}$$

Single premium of life insurance last survivor dwiguna is the sum of the single premium of pure dwiguna last survivor life insurance and life insurance last survivor futures namely:

$$A_{\overline{xy}:\overline{n}|} = A_{\overline{xy}:\overline{n}|}^{\;\;\;\;1} + A^{\;1}_{\overline{xy}:\overline{n}|}. \tag{13}$$

By substituting equations (11) and (12) to equations (13) it is obtained:

$$A_{\overline{xy}:\overline{n}|} = v^{n}\,{}_{n}p_{\overline{xy}} + \sum_{t=0}^{n-1} v^{t+1}\,{}_{t|}q_{\overline{xy}}.$$

$$A_{\overline{xy}:\overline{n}|} = v^{n}\,{}_{n}p_{\overline{xy}} + \sum_{t=0}^{n-1} v^{t+1}\left({}_{t}p_{\overline{xy}} - {}_{t+1}p_{\overline{xy}}\right).$$

$$A_{\overline{xy}:\overline{n}|} = (1-d\ddot{a}_{x:\overline{n}|}) + (1-d\ddot{a}_{y:\overline{n}|}) - (1-d\ddot{a}_{xy:\overline{n}|})$$

$$A_{\overline{xy}:\overline{n}|} = 1-d\ddot{a}_{\overline{xy}:\overline{n}|}. \tag{14}$$

By using the Law of Gomperzt obtained:

$$A_{\overline{xy}:\overline{n}|} = 1-d\sum_{t=0}^{n-1} v^{t}\left(g^{c^{x}(c^{t}-1)} + g^{c^{y}(c^{t}-1)} - g^{(c^{x}+c^{y})(c^{t}-1)}\right). \tag{15}$$

3.2 Annual Premium of Joint Life Insurance and Last Survivor

The annual premium of dual-use insurance for the combined living status of participants aged x and y years with the period of insurance coverage for n years and the sum insured paid at the end of the policy year is stated in as (Futami, 1994):

$$P_{xy:\overline{n}|} = \frac{A_{xy:\overline{n}|}}{\ddot{a}_{xy:\overline{n}|}}. \tag{16}$$

Substitution equation (14) to equation (16) that obtained:

$$P_{xy:\overline{n}|} = \frac{1-d\ddot{a}_{xy:\overline{n}|}}{\ddot{a}_{xy:\overline{n}|}},$$

$$P_{xy:\overline{n}|} = \frac{1}{\ddot{a}_{xy:\overline{n}|}} - d.$$

Then obtained the annual premium of dual life insurance joint life assuming Gompertz:

$$P_{xy:\overline{n}|} = \frac{1}{\sum_{t=0}^{n-1} v^{t} g^{(c^{x}+c^{y})(c^{t}-1)}} - d.$$

The annual premium of life insurance last survivor is double the period of insurance coverage for n years, notified by $P_{\overline{xy}:\overline{n}|}$. The amount of annual premium on life insurance last survivor can be stated by Jhon and Albert (2016):

$$P_{\overline{xy}:\overline{n}|} = \frac{A_{\overline{xy}:\overline{n}|}}{\ddot{a}_{\overline{xy}:\overline{n}|}}.$$

$$P_{\overline{xy}:\overline{n}|} = \frac{1}{\ddot{a}_{\overline{xy}:\overline{n}|}} - d.$$

Using Gompertz law, obtained:

$$P_{\overline{xy}:\overline{n}|} = \frac{1}{\sum_{t=0}^{n-1} v^{t}\left(g^{c^{x}(c^{t}-1)} + g^{c^{y}(c^{t}-1)} - g^{(c^{x}+c^{y})(c^{t}-1)}\right)} - d.$$

3.4 Prospective Reserves of Joint Life and Last Survivor

To pay the amount of coverage money is required reserves in an insurance company. The reserve is the amount of coverage that exists in the insurance company within the period of coverage. The reserves used are prospective reserves that are large reserves oriented towards future expenditures. The amount of prospective reserves of joint life with $P_{xy:\overline{n}|}$ the amount of premiums paid by insurance participants, $A_{x+j,y+j:\overline{n-j}|}$ single premium and $\ddot{a}_{x+j,y+j:\overline{n-j}|}$ the cash value of an early term annuity aged $(x+j)$ and $(y+j)$ years with a period of coverage for (n-j) years under the following circumstances.

The reserves are as follows:

1. Both live until the end of the policy year

$$ {}_jV_{xy:\overline{n}|} = A_{x+j,y+j:\overline{n-j}|} - P_{xy:\overline{n}|}\ddot{a}_{x+j,y+j:\overline{n-j}|}. $$

$$ {}_jV_{xy:\overline{n}|} = \left(1 - d\ddot{a}_{x+j,y+j:\overline{n-j}|}\right) - \left(\frac{1}{\ddot{a}_{xy:\overline{n}|}} - d\right)\ddot{a}_{x+j,y+j:\overline{n-j}|} $$

$$ {}_jV_{xy:\overline{n}|} = 1 - \frac{\ddot{a}_{x+j,y+j:\overline{n-j}|}}{\ddot{a}_{xy:\overline{n}|}}. $$

Using Gompertz law that obtained:

$$ {}_jV_{xy:\overline{n}|} = 1 - \frac{\sum_{t=0}^{n-j-1} v^t\left(g^{c^{x+j}(c^t-1)} + g^{c^{y+j}(c^t-1)} - g^{(c^{x+j}+c^{y+j})(c^t-1)}\right)}{\sum_{t=0}^{n-1} v^t\left(g^{c^x(c^t-1)} + g^{c^y(c^t-1)} - g^{(c^x+c^y)(c^t-1)}\right)}. $$

2. At the time x life, and y died

$$ {}_jV_{x:\overline{n}|} = A_{x+j:\overline{n-j}|} - P_{xy:\overline{n}|}\ddot{a}_{x+j:\overline{n-j}|}. $$

$$ {}_jV_{x:\overline{n}|} = \left(1 - d\ddot{a}_{x+j:\overline{n-j}|}\right) - \left(\frac{1}{\ddot{a}_{xy:\overline{n}|}} - d\right)\ddot{a}_{x+j:\overline{n-j}|} $$

$$ {}_jV_{x:\overline{n}|} = 1 - \frac{\ddot{a}_{x+j:\overline{n-j}|}}{\ddot{a}_{xy:\overline{n}|}}. $$

Using Gompertz law that obtained:

$$ {}_jV_{x:\overline{n}|} = 1 - \frac{\sum_{t=0}^{n-j-1} v^t g^{c^{x+j}(c^t-1)}}{\sum_{t=0}^{n-1} v^t\left(g^{c^x(c^t-1)} + g^{c^y(c^t-1)} - g^{(c^x+c^y)(c^t-1)}\right)}. $$

3. At the time y life and x died

$$ {}_jV_{y:\overline{n}|} = A_{y+j:\overline{n-j}|} - P_{xy:\overline{n}|}\ddot{a}_{y+j:\overline{n-j}|} $$

$$ {}_jV_{y:\overline{n}|} = \left(1 - d\ddot{a}_{y+j:\overline{n-j}|}\right) - \left(\frac{1}{\ddot{a}_{xy:\overline{n}|}} - d\right)\ddot{a}_{y+j:\overline{n-j}|} $$

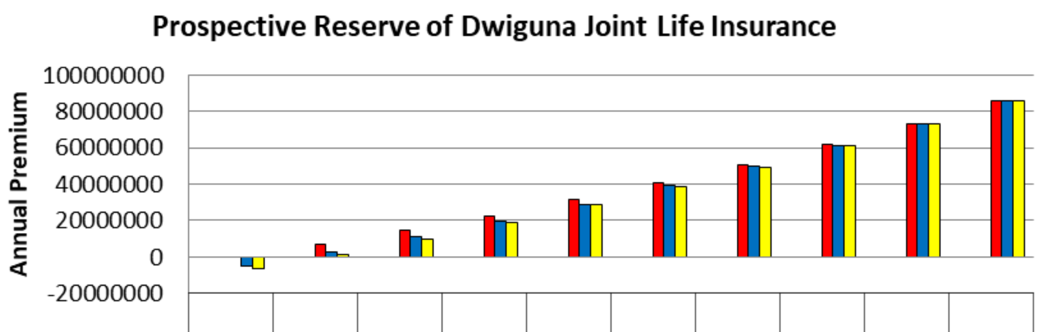
$$ {}_jV_{y:\overline{n}|} = 1 - \frac{\ddot{a}_{y+j:\overline{n-j}|}}{\ddot{a}_{xy:\overline{n}|}}. $$

Using Gompertz law that obtained:

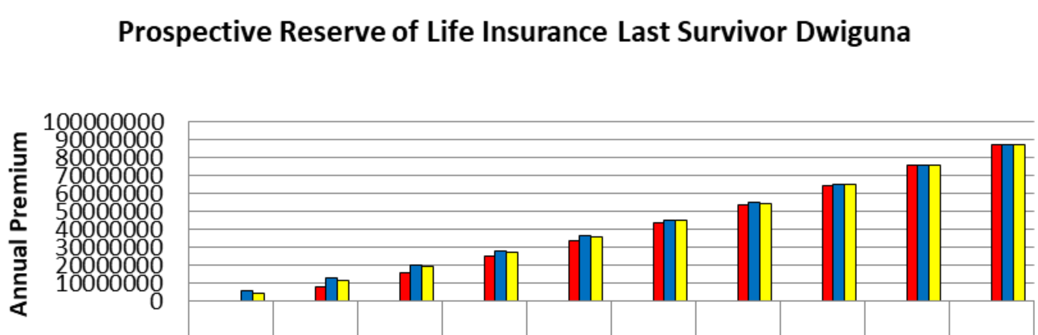
$$ {}_jV_{y:\overline{n}|} = 1 - \frac{\sum_{t=0}^{n-j-1} v^t g^{c^{y+j}(c^t-1)}}{\sum_{t=0}^{n-1} v^t\left(g^{c^y(c^t-1)} + g^{c^y(c^t-1)} - g^{(c^x+c^y)(c^t-1)}\right)}. $$

Participants x husbands who are 45 years old and participants y his wife who is 40 years old want to participate in a life insurance program with a coverage period of 10 years and UP of IDR.100,000,000.00 which will be given to the

heirs with an interest rate of 0.05, then determine the prospective reserve of dual life insurance at the end of each year assuming Gompertz with the following cases:
1. Both participants live insurance
2. When the husband is alive, and his wife dies,
3. At the time the wife was still alive, and the husband passed away.



x	45	45	45	45	45	45	45	45	45	45
y	40	40	40	40	40	40	40	40	40	40
j	0	1	2	3	4	5	6	7	8	9
C1	0	7.118	14.70	22.80	31.48	40.80	50.83	61.66	73.39	86.12
C2	(5.33	2.543	10.88	19.73	29.12	39.10	49.74	61.07	73.18	86.12
C3	(6.56	1.497	10.01	19.03	28.59	38.72	49.49	60.94	73.13	86.12



x	45	45	45	45	45	45	45	45	45	45
y	40	40	40	40	40	40	40	40	40	40
j	0	1	2	3	4	5	6	7	8	9
C1	0	7.882	16.17	24.89	34.07	43.73	53.88	64.57	75.80	87.60
C2	5.863	12.90	20.36	28.26	36.66	45.58	55.08	65.21	76.03	87.60
C3	4.769	11.97	19.58	27.64	36.18	45.24	54.86	65.09	75.99	87.60

Figure 1. Prospective Reserve of dwiguna joint life insurance and life insurance last survivor.

4. Conclusion

The last survivor's life insurance coverage is paid to the heir until the last death of the insurance participant.. The cash value of annuities for life insurance for the last survivor is influenced by discounted factor and life chances of insurance participants. Using Gompertz mortality acceleration, reserves increased until the last death of the insurance participant in the case of the last survivor. So for married couples working in the same agency it would be better to use last survivor insurance.

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