The Caracteristics of Alpha Cut on Fuzzy Graphs and Its Application in Scheduling System

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Abstract

This article discusses the characteristics of alpha-cut on fuzzy graph. Alpha-cut on fuzzy graph was introduced by Munoz (2005) for vertex coloring on fuzzy graph with crips vertices. It was continued by Arindam Dey, et all (2013) for the fuzzy set of vertices and edges. Alpha-cut on fuzzy graph is crips graphs G_α that are induced from fuzzy graph by removing all vertex and edges in fuzzy graphs that have a degree of membership less than α, $\alpha \in [0,1]$. Therefore, the caracteristics of alpha-cut on fuzzy graph is related to the crips graph produced. Furthermorer alpha cut on fuzzy graph will produce many ways of scheduling.

Key words:
Alpha-cut, fuzzy graph, degree of membership, crips graph, scheduling

1. Introduction

Fuzzy graphs were introduced by Azriel Rosenfeld in his book entitled Fuzzy Graph (1975). The theoretical concept of fuzzy graph is based on fuzzy logic in fuzzy sets introduced by Zadeh L. A. (1965). Since the introduction of fuzzy graphs, many studies in classical graphs have been generalized into fuzzy graphs. One of the generalized studies is the vertex coloring. The problem of vertex coloring on a graph is a problem of finding the smallest integer k ∈ N, so two adjacent vertices have different labels. This smallest number is called as chromatic number. Vertex coloring on fuzzy graphs was introduced by Munoz, S., et al (2005).

One of the vertex coloring concepts on fuzzy graphs introduced by Munoz, S., et al. (2005) is the use of alpha-cut on fuzzy graphs based on the definition of aplha-cut on fuzzy sets. The fuzzy graph used in the research of Munoz, S., et al. (2005) is a fuzzy graph with a set of crips vertex. Furthermore, Dey, A., et al. (2013) developed a definition of alpha-cut on fuzzy graphs for a fuzzy set of vertex and edge. This article discusses some of the α-cut characteristics in fuzzy graphs using definition from Dey, A., et al (2013) and it is applied on scheduling system.

2. Definitions

Definition 1. Let V be non empty and finite set. A fuzzy graph G_F is a pair of functions (σ, μ) with σ is the fuzzy set on V and μ is a symmetric fuzzy relation on σ, in such a way that:

i. $\sigma: V \rightarrow [0, 1]$
ii. $\mu : V \times V \rightarrow [0,1]$ that fulfil $\mu(v_i, v_j) \leq min\{\sigma(v_i),\ \sigma(v_j)\}$, for any $v_i,\ v_j \in V$.

Furthermore, σ it is called the set of fuzzy vertices and μ is called the set of fuzzy edges. A fuzzy graph with a set of fuzzy vertices and a set of fuzzy edges is then denoted by $G_F = (\sigma, \mu)$. The notation $\sigma(v_i)$ on a fuzzy graph states the degree of membership of a vertex and $\mu(v_i, v_j)$ states the degree of membership of a edge. The fuzzy graph discussed in this study is a simple fuzzy graph, so $\mu(v_i, v_i) = 0$, for any $v_i \in V$.

Fuzzy graphs can be represented in an image just like classical graphs. Furthermore, in the figure, Henceforth if $\sigma(v_i) = 0$, then the fuzzy vertexs are not drawn. Likewise, if $\mu(v_i, v_j) = 0$, the fuzzy side is not drawn. If all of vertices and edges on fuzzy graph have degree of membership 1, then the fuzzy graph can be called a crips graph.

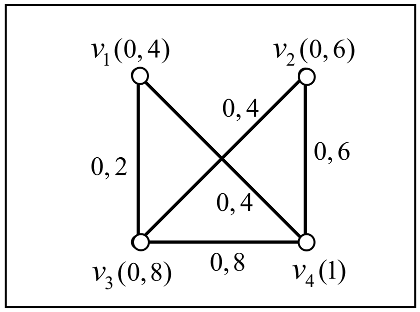


Figure 1. Fuzzy Graph $G_F = (\sigma, \mu)$

Definition 2. The basic graph of a fuzzy graph $G_F = (\sigma, \mu)$ is a graph $G = (V, E)$ with a set of vertices and edges whose membership degree is more than zero. In other words, the basic graph of a fuzzy graph G is a graph $G = (V, E)$ where $V = (v;\ \sigma(v) > 0\}$ and $E = \{e;\ \mu(e) > 0\}$.

The coloring on fuzzy graphs is an development of the coloring problem on classical graphs. One of vertex coloring on fuzzy graphs is by using the alpha cut on fuzzy graph. The following is given the definition of alpha cut on fuzzy graph and fuzzy chromatic number according to Dey, A., et al (2013).

Definition 3. Let V be an empty set and finite set. For $\alpha = [0,1]$, the alpha cut on the fuzzy graph $G_F = (\sigma, \mu)$ notated by *α-cut* is an ordered pair (V_α, E_α) where $V_\alpha = \{v \in V \mid \sigma(v) \geq \alpha\}$ and $E_\alpha = \{e \in E \mid \mu(e) \geq \alpha\}$.

Based on the definition 3., the *σ-cut* on fuzzy graph is a crips graph that is induced from a fuzzy graph G_F by deleting all vertices and edges on the fuzzy graph that have membership degrees less than α. Furthermore *α-cut* on fuzzy graph is denoted by $G_\alpha = (V_\alpha, E_\alpha)$.

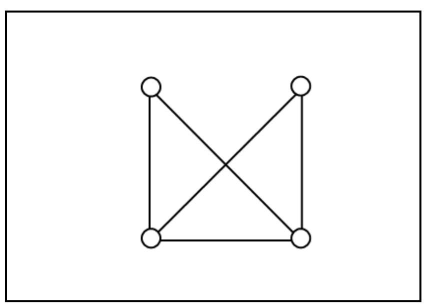


Figure 2. $G_{0,2} = (V_{0,2}, E_{0,2})$ for $G_F = (\sigma, \mu)$

Definition 4. The fuzzy chromatic number of the fuzzy graph $G_F = (\sigma, \mu)$ is the fuzzy number, $\chi^f(G_F) = ((\chi(G_\alpha), \alpha))$ where $\chi(G_\alpha)$ is the chromatic number of G_α and α is the degree of membership of the vertex or edge which is different from the fuzzy graph G_F.

This research applies study literature method. The study resulted in the characteristics of *σ-cut* on fuzzy graph.

3. The Caracteristics of alpha cut on fuzzy graph

Some of the σ-*cut* characteristics on a fuzzy graph are described as follows:

Lemma 1. Let V be an non empty and finite set. $G_F = (\sigma, \mu)$ is an arbitrary fuzzy graph with a set of fuzzy vertex σ and a set of fuzzy edge μ. If $\alpha = 0$, then α-*cut* of G_F is a complete graph $G_0 = (V_0, E_0)$.

Proof. Based on the definition 2., for $\alpha = 0$, the *0-cut* of G_F is $G_0 = (V_0, E_0)$ where $V_0 = \{v \in V \mid \sigma(v) \geq 0\}$ and $E_0 = \{e \in E \mid \mu(e) \geq 0\}$. In other words V_0 is the set of all vertices on crips graph $G = (V, E)$ and E_0 is the set of all edges that connected to every two vertices on crips graph $G = (V, E)$. Consequently, $G_0 = (V_0, E_0)$ is a complete graph. ■

Lemma 2. Let V be an non empty and finite set and $G_F = (\sigma, \mu)$ is an arbitrary fuzzy graph with a set of fuzzy vertex σ, and a set of fuzzy edge μ. If $\mu(e) < \sigma(v)$ and $\alpha = min\{(\mu(e)\}$, for any $v \in V$, $e \in V \times V$, then α-*cut* of G_F is the basic graph of fuzzy graph G_F.

Proof. Since $\alpha = min\{\mu(e)\}$ and $\mu(e) < \sigma(v)$, it is based on the definition of 3. neither the vertex nor the edges are removed from the G_F. As a result, $G_\alpha = (V_\alpha, E_\alpha)$ is a graph where the set of vertices is all vertices in G_F and the set of edges is all edges in G_F. So it is proven that $G_\alpha = (V_\alpha, E_\alpha)$ is the basic graph of the fuzzy graph G_F. ■

Lemma 3. If given a fuzzy graph $G_F = (\sigma, \mu)$ with $\sigma(v) = 1$, for any $v \in V$ and $\mu(e) < 1$ for any $e \in V \times V$, then the 1-*cut* of G_F is a null graph $G_1 = (V_1, E_1)$.

Proof. For $\alpha = 1$, based on the definition of 3, $G_1 = (V_1, E_1)$ is a graph with the set of vertices $V_1 = \{v \in V \mid \sigma(v) = 1\}$ and the set of edges $E_1 = \{e \in E \mid \mu(e) \geq 1\}$. Since $\mu(e) < 1$ for all $e \in V \times V$, all edge in G_F is deleted. So there are no edges at $G_1 = (V_1, E_1)$. In other words $E_1 = \emptyset$. As a result, $G_1 = (V_1, E_1)$ is a null graph. So the α-*cut* of G_F is a null graph. ■

Lemma 4. If given a fuzzy graph $G_F = (\sigma, \mu)$ with $\sigma(v)$, $\mu(e) < 1$ for any $v \in V$ and $e \in V \times V$, then $G_1 = \emptyset$.

Proof. Since $\sigma(v) < 1$ and $\mu(e) < 1$, for any $v \in V$ and $e \in V \times V$, then $G_1 = (V_1, E_1)$ is an empty graph with $V = \emptyset$ and $E = \emptyset$. As a result, $G = \emptyset$. ■

4. Applied Alpha Cut on fuzzy graph in scheduling system

Given a fuzzy graph with a set of crips vertices and a set of fuzzy edges. This fuzzy graph can represent the following things:

1. A vertex expressly states a subject that is held in one semester;
2. A edge that connects the two vertex clearly states that there are one or more of the same students taking the two subject;
3. Each subject is held once in a certain period (for example one week). If the subject is held twice a week, then in a fuzzy graph the subject is represented by two vertices;
4. The degree of edge membership states the degree of participation of students who take subject that can be formulated by

$$\mu(v_i, v_j) = \frac{x}{\min\{m_i, m_j\}}, \quad 0 \leq x \leq min\ \{m_i, m_j\} \qquad (1)$$

with
x represents the same number of students taking subject i and j; $i, j = 1, 2, ..., n$. (n = number of subject)
m_i states the number of students taking subject i.
m_j states the number of students taking subject j.

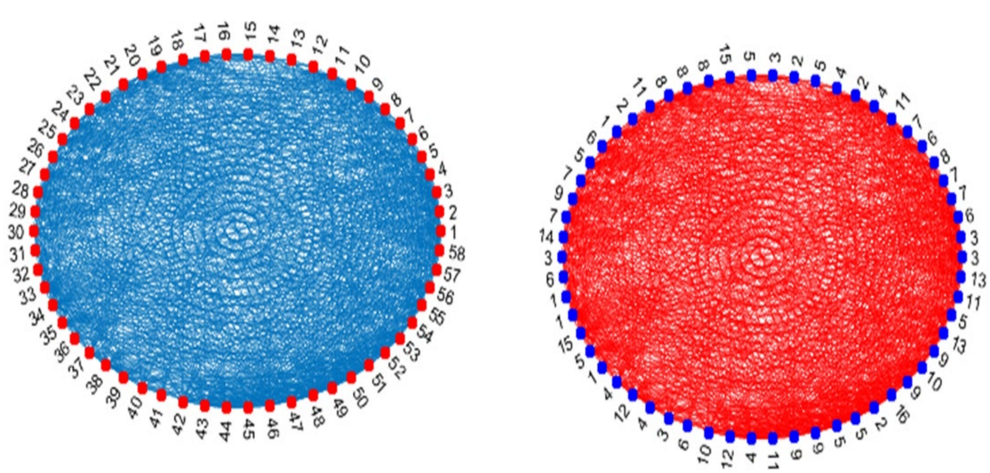
A fuzzy graph with this assumption has a degree of membership for each vertex equal to 1 and the degree of edge membership can be expressed as a matrix μ as follows

$$
M_\mu = \begin{pmatrix}
0 & \mu_{12} & \mu_{13} & \cdots & & & \cdots & \mu_{1n} \\
\mu_{21} & 0 & \mu_{23} & \cdots & & & & \mu_{2n} \\
\mu_{31} & \mu_{32} & 0 & \mu_{34} & \cdots & & & \mu_{3n} \\
\vdots & \vdots & \mu_{41} & 0 & \mu_{45} & \cdots & & \vdots \\
& & \vdots & & 0 & \mu_{56} & & \mu_{(n-3)n} \\
\mu_{(n-2)1} & \mu_{(n-2)2} & & \cdots & \mu_{(n-1)n} & & \cdots & \mu_{(n-2)n} \\
\mu_{(n-1)1} & \mu_{(n-1)2} & & & & & \ddots & \mu_{(n-1)n} \\
\mu_{n1} & \mu_{n2} & \mu_{n3} & \cdots & & \mu_{n(n-2)} & \mu_{n(n-1)} & 0
\end{pmatrix}
$$

This matrix μ is a symmetry matrix with the main diagonals being zero and $\mu_{ij} \in [0,1]$. If $\mu_{ij} = 0$, there is not one or more of the same students taking subjects i and j. Conversely if $\mu_{ij} = 1$, there is one of these possibilities occurs, namely: $P_i = P_j$; $P_i \subset P_j$; or $P_i \supset P_j$, where P_i is the group of students taking course i. For example, given M_1, M_2,…, M_{58} which represent 58 subjects and the relationship between subjects that represents whether or not the same student takes two different subjects. The degree of edge membership μ_{ij} is calculated based on equation (1) which can be obtained from the student study plan data. Based on this calculation, we can choose the value $\alpha \in [0,1]$ which corresponds to the scheduling facilities such as the number of lecture halls, and the time slots arranged in the scheduling (Suyudi et al., 2016; 2017; 2018).

For $\alpha = 0$, then G_0 is a complete graph with 58 vertices. This means that for every two subjects there is at least 1 same student taking two different subjects. If G_0 is given to vertex coloring, then the chromatic number $\chi(G_0) = 58$. This means that to compile a schedule, 58 different time periods are needed so that each student follows each subject he takes. If the number of these periods is greater than the number of available time slots, it takes too long time to schedule with 58 periods. Therefore we have to choose $\alpha > 0$ so that the chromatic number is smaller or equal than the number of available time slots.

Base on the form of student study plan, here is one solution for vertex coloring on a fuzzy graph with $\alpha = 0.02$. By using MATLAB program, for $\alpha = 0.02$, it is obtained $G_{0.02}$ and $\chi(G_{0.02}) = 16$. In scheduling, it means that the value of student participation which is less than 2% is ignored.



Crips Graph $G_{0.02}$ Vertex Coloring on $G_{0.02}$

Figure 2. Crips Graphs and vertex Coloring

If vertex label on crips graph $G_{0.02}$ on figure 2a., consecutively represent M_1, M_2, …, M_{58}, then based on figure 2b, these subject divided into 16 group as the following table3.

Tabel 3. Classification of subjcet based on $\alpha = 0.02$

Periods	Subject					
1	M11	M36	M37	M50		
2	M12	M16	M22	M33	M46	
3	M10	M13	M14	M41	M44	
4	M15	M23	M27	M32	M48	M49
5	M1	M2	M26	M30	M40	
6	M9	M39	M43	M45	M47	
7	M7	M18	M29	M42	M56	M58
8	M3	M8	M24	M31	M52	M57
9	M20	M25	M34	M38		
10	M54	M55				
11	M4	M5	M28			
12	M6	M19				
13	M35	M53				
14	M21					
15	M17					
16	M51					

In Table 3, it can be seen that 58 subjects are divided into 16 groups. Each group is a set of subjects whose lectures can be held simultaneously.

5. Conclusion

α-cut on the fuzzy graph $G_F = (\sigma, \mu)$ is the set family of crips graphs $G_\alpha = (V_\alpha, E_\alpha)$. Vertex coloring on a fuzzy graph using *α-cut* can be used to design a fuzzy scheduling system. Vertex coloring algorithm on a fuzzy graph using *α-cut* produces a fuzzy chromatic number that represents the set of several pairs of the number of time intervals in scheduling with the value α.

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