Two-dimensional Maintenance Service Contract Ensuring Availability using Non-periodic Imperfect Preventive Maintenance

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Abstract

This paper deals with a two-dimensional maintenance service contract (MSC) characterized by time and usage limits. We consider that an agent offers the two dimensional MSC and promises a high availability of the equipment. The subsequent preventive maintenance (PM) actions are carried out to ensure the availability target. The optimal non-periodic PM policy is obtained to maximize the availability. Numerical examples are presented to illustrate the subsequent PM intervals meeting the availability target.

Keywords
Maintenance service contract; Two-dimensional; Availability; Imperfect preventive maintenance.

1. Introduction

In many companies, maintenance activities are considered as non-core business activities, and hence outsourced to an external agent. The other reason to outsource the maintenance activities is that the facilities to do the maintenance activities require a large capital investment and it is not economical to carry out in house. As the demand of maintenance services tends to increase rapidly in the last two decades, agents respond proactively with offering various MSC (maintenance service contract) to meet a variety requirement of the companies. A MSC defines maintenance (PM and/or CM) actions covered, period agreed and the payment to be made by the owner of the equipment to the service provider. Jack and Murthy (2017) provide a comprehensive review of MSCs studied in the literature. The MSCs can be classified into one-dimensional (1-D) and two-dimensional (2-D) MSCs.

For one-dimensional (1-D) MSCs, Murthy and Yeung (1995), Iskandar et al. (2005), and Iskandar et. al. (2014) to name a few belong to this group. The study of a 2-D MSC has not received much attention, and only three papers - i.e., Murthy and Yeung (1995), Husniah, et.al (2016) and Hamidi, et.al (2014) deal with the 2-D MSC.

In this paper, we study a two-dimensional maintenance service contract (MSC) in which the contract is characterized by time (age) and usage limits. Furthermore, we propose a 2-D MSC which ensures the performance of the equipment in term of availability. This work can be viewed as the extension of Yeh and Chang (2007) to the case of a 2-D MSC. The motivation to prioritize availability as the important performance measure is that the cost incurred downtime of the equipment is very large. For example, the breakdown of the equipment such as dragline, dump truck, etc. (in mining business) and trains, buses, etc. (in transportation business) will result in a big loss to a company.

PM policy with considering availability can be found in Ceschini and Saccardi (2002), Marseguerra, et.al (2002), Yeh and Chang (2007), and Husniah, et al. (2019) whilst Lie and Chun (1986), Yeh, et.al (2009), Khatab, et.al (2014), Mabrouk, et al. (2017) and Iskandar and Husniah (2017) investigated a PM policy ensuring reliability of the equipment. These all models do not ensure a certain availability level. In contrast, this proposed PM model aims to ensure an availability target of the equipment which is of interest to the company.

This paper is organized as followed. The model formulation is given in Section 2, this includes 2-D MSC formulation, failure modelling and PM impact modelling. Section 3 deals with optimization to find the suboptimal solution that ensures the availability target. Section 4 provides numerical example and discussion of results. Finally, we present the conclusion and future research topics in Section 5.

2. Model Formulation

This section will describe the 2-D MSC studied, failure modelling, and PM impact modelling.

2.1. 2-D MSC

We consider a 2-D MSC characterized by two dimensions as in Iskandar, et al. (2014), where one dimension represents age and the other dimension is usage (e.g. the maximum coverage for L (e.g. 1 year) and U (e.g. 50.000 km). Hence, the 2-D MSC forms a rectangle region Ω_{W+W_1}(see Figure 1).

Let y_0 (=U/L) be the nominal usage rate. For $y \leq y_0$ (called a low usage) the 2D-MSC region is given by $[(W, \mathrm{W}+W_1)\mathrm{x}(yW, y(\mathrm{W}+W_1))]$ and $[U/y, U/y+W_1]\mathrm{x}[U, \mathrm{U}+U_1]$ for $y > y_0$ (called a high usage).

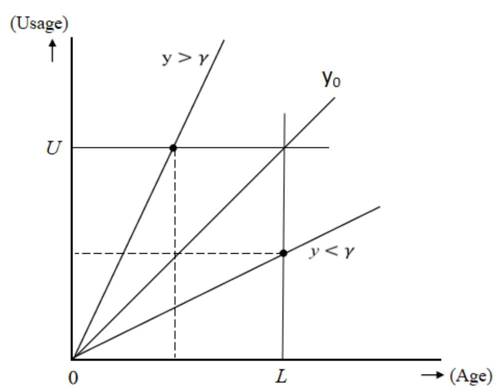

Figure 1. MSC region Ω_{W+W_1} for (a) $y \leq y_0$ and (b) $y > y_0$. This formulation and graph are referred to Iskandar, et al. (2014).

2.2. Failure Modelling

Item failures can be viewed as random points occurring over the MSC region. Three approaches have been proposed for modelling item failures in two-dimensional MSC (Murthy and Jack, 2014). We apply the one-approach as in Iskandar et. al. (2005) to modeling failure under a two-dimensional MSC.

Let Y be the equipment's usage rate (e.g. distance travelled per unit time, machine hours per day). In case of a dump truck used for hauling mining materials, it represents the distance travelled per day (e.g. 300 km/day). It is assumed that each equipment has a constant usage rate over the MSC period.

For a given Y=y, failures can be modelled by a one-dimensional point process. Let $r(t|y)$ denote the failure rate of the equipment for a given usage rate $Y = y$. If every failure under the MSC period is restored by a minimal repair, then failures occur according to a Poisson process with an intensity function $r(t|y), t > 0$.

Let T_y be the time to the first failure of the equipment. The distribution function of T_y is given by $F_y(t)$. We consider that $F_y(t)$ is given by Weibull distribution. Now, we describe the impact of a usage pattern to modeling the equipment failure.

Let y_0 denotes the nominal usage rate of the equipment. We consider that the usage rate is less than y_0 (y<y_0) classified as a low usage and the one is greater than y_0 $(y > y_0)$ classified as a high usage. The equipment used with high usage is considered to deteriorate faster than the equipment with low usage. Here, the AFT formulation is applied to model the impact of the usage pattern.

If $T_0[T]$ denotes the time to first failure under usage rate $y_0[y]$ then we have $T_y/T_0 = (y_0/y)^\rho$.

Furthermore, if the distribution function for T_0 is given by $F_0(t, \alpha_0)$, where α_0 is the scale parameter, then the distribution function for T_y is the same as that for T_0 but with a scale parameter given by:

$$\alpha_y = (y_0/y)^\rho \alpha_0, with\ \rho \geq 1$$

Hence, we have $F_y(t, \alpha_y) = F_0((y_0/y)^\rho t, \alpha_y)$.

The intensity function and the cumulative intensity function associated with $F_y(t, \alpha_y)$ are given by

$$r_y(t|Y = y) = f(t, \alpha_y)/(1 - F(t, \alpha_y) = \frac{\beta}{\alpha_y}\left(\frac{t}{\alpha_y}\right)^{\beta-1}$$

$$R_y(t|Y = y) = \int_0^t r_y(x|y)dx$$

where $f(t, \alpha_y)$ is the density function associated with $F_y(t, \alpha_y)$.

2.3. PM Policy

The agent will perform PM to maintain a high performance of the equipment. We consider the situation where a down time of the equipment will result in a big loss to the owner of the equipment. For examples, equipment (such as dragline, excavators, dump trucks) used in mining industry play a major role in supporting production activities, and hence a high availability of the equipment is needed to minimize a breakdown that will disrupt the production activities.

As a result, the availability of the equipment is the appropriate performance measure that needs to be controlled.

An appropriate PM policy is required to minimize downtimes due to failures, and this in turn will increase the availability. We propose a preventive maintenance policy which minimizes the number of failures causing downtimes. The PM policy is defined as follows.

For a given usage rate (or a given equipment), a PM is carried out sequentially at $T_i, i = 1,2,3, \dots, N$ with $T_0 = 0$, $T_N = L$.

Let A$(T_i, T_{i+1} \mid y)$ be the availability of the equipment in (T_i, T_{i+1}) conditional on $Y = y$. A$(T_i, T_{i+1} \mid y)$ is given by

$$\mathrm{A}(T_i, T_{i+1} \mid y) = \frac{(T_{i+1}-T_i)-t_{pm}-t_{cm}\int_{T_i}^{T_{i+1}} r_y(t)\,\mathrm{d}t}{(T_{i+1}-T_i)} \tag{1}$$

where t_{pm} and t_{cm} are the times to do a PM, and a CM, respectively.

In practice, the owner wants a high availability of the equipment to achieve a production target (e.g. $\widetilde{\mathrm{A}} = 0.98$). To ensure the availability target ($\widetilde{\mathrm{A}}$) over the MSC period, one way to achieve that is to determine $(T_i, T_{i+1}), i = 1,2,3, \ldots$ such that the availability of each interval at least $\widetilde{\mathrm{A}}$. The algorithm for finding the sequent of PM actions will be described in Section 3.

As mentioned before that the sequential PM actions are carried out at $T_i, i = 1,2,3, \ldots, N$. We, now, define the approach for finding the value of T_i ensuring $\widetilde{\mathrm{A}}$. Suppose that the first PM is performed at T_1, and the next PM action will be at $T_2 = T_1 + \Delta t_1$. Then, the time instance of the *(i+1)th* PM can be written as $T_{i+1} = T_i + \Delta t_i$. As for a given usage rate, the failure rate is increasing with the age of the equipment, then we have $\Delta t_{N-1} < \cdots < \Delta t_2 < \Delta t_1$. In other words, the interval between two consecutive PMs, $(T_i, T_i + \Delta t_i)$ decreases as *i* increases.

As a result, the optimal value of Δt_i^* will be searched so that it meets the following criteria:

$$\Delta t_i^* = \sup_{\Delta t_i} \{\mathrm{A}(T_i,\ T_i + \Delta t_i | Y = y) \geq \tilde{A}\} \tag{2}$$

for $i = 0,\ 1,\ \ldots, N-1$.

2.4 PM Impact Modelling

The effect of the PM's actions can be modelled through (a) virtual age or (b) failure intensity function (Jiang and Murthy (2008)), Here, the sequential imperfect PM actions result in a reduction in the failure intensity function, and this is described as follows.

When a PM action is performed at time epoch T_i, the failures rate of the equipment in the interval $(T_i, T_{i+1}]$ becomes $r_y(T_i) - i\delta_y$, $\delta_y > 0$, δ_y is the reduction for each PM (See Fig. 2).

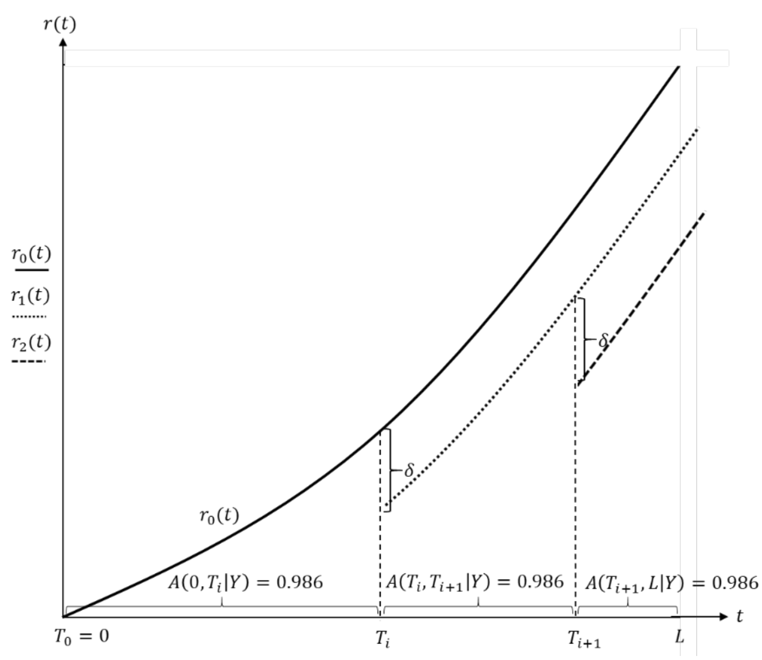

Figure 2. Non-periodic imperfect PM ensuring availability throughout L. Every time the OEM (or Agent) doing PM, the failure intensity function drops by δ (in case of Agent in $\delta_A \leq \delta$). $\mathrm{A}(T_i,\ T_{i+1}|Y = y)$ show availability value during $(T_i,\ T_{i+1})$ at usage $Y = y$.

3. Optimal Decision

As mentioned in Subsection 2.3 that the availability of the equipment is the essential performance measure that needs to be controlled by the Agent. In other words, the objective of the agent is to ensure the availability (which is at least $\widetilde{A}$) over the MSC period agreed. Hence, we find the sequent of PM epochs, $T_i, i = 1,2,3, \ldots, N-1$ with $T_0 = 0$, $T_N = L$ that fulfills equation (2) for a given δ_y (i.e., the reduction for each PM). We need to develop the algorithm to obtain the best values of T_i.

Step 1. Set values of parameters: $\widetilde{A}$, y, t_{pm}, t_{cm}, and δ_y ; $i = 1$ and $T_0 = 0$.

Step 2. Compute T_i satisfying the availability formula in (2) using bisection method, and $A_i = A(T_{i-1}, T_i)$.

Step 3. Set $i = i + 1$

Step 4. If $T_i < L$ then Go to Step 2, otherwise Go to Step 5.

Step 5. We have T_i that satisfies $A_i \geq \widetilde{A}$, STOP.

4. Results and Discussion

The time to the first failure of the equipment is assumed by Weibull distribution with $F(t, \alpha_y) = 1 - \exp\left(-\frac{t}{\alpha_y}\right)^{\beta}$, where $\alpha_y = (y_0/y)^{\rho}\alpha_0$. The parameter values are given as follows: $\beta = 1.5$, $t_{pm} = 0.03$, $t_{cm} = 0.015$, $L = 10$ $\alpha_0 = 10$,$b = 100$, $C_r = 70$, and $I_v = 100$. For comparisons, we consider the model of Yeh and Chang (2007) and extend it to the case of a two-dimensional MSC. Furthermore, three type of operating conditions are examined – i.e., light, medium or heavy operating conditions.

Table 1 shows the highest availability resulting from the proposed model ($\widetilde{A}_{\text{Proposed}}$) and Yeh and Chang model ($\widetilde{A}_{\text{Yeh}}$) for a light operating condition. We consider $\alpha_y = 6,7,..10$ and $\delta_y = 0.06, 0.08, 0.10$ (degree of failure rate reduction). Note that a large α_y means a higher reliability. In general, the model of Yeh and Chang is better for the case of a light operating condition. The similar results for moderate and heavy operating conditions are shown in Tables 2 and 3. In these operating conditions, The proposed model shows the superiority over the Yeh and Chang's model. In other words, the proposed model is appropriate to be applied when the equipment is operated in moderate to severe conditions (e.g., hilly contour or high inclined road for dump trucks used in mining industry), whilst the Yeh and Chang's model is better in a relatively flat contour road considered as a light operating condition.

Table 1 Increasing availability between the proposed model and Yeh and Chang model, Light ($\rho = \frac{y_0}{y} \leq 1$)

δ_y	0.06		%	0.08		%	0.10		%
α	$\widetilde{A}_{\text{Yeh}}$	$\widetilde{A}_{\text{Proposed}}$		$\widetilde{A}_{\text{Yeh}}$	$\widetilde{A}_{\text{Proposed}}$		$\widetilde{A}_{\text{Yeh}}$	$\widetilde{A}_{\text{Proposed}}$	
6	99.20%	98.90%	-0.30%	98.80%	98.90%	0.10%	97.40%	98.90%	1.54%
7	98.70%	98.90%	0.20%	96.90%	98.90%	2.06%	99.30%	99.10%	-0.20%
8	98.80%	98.90%	0.10%	99.30%	99.10%	-0.20%	99.40%	99.10%	-0.30%
9	99.20%	99.10%	-0.10%	99.40%	99.10%	-0.30%	99.00%	99.60%	0.61%
10	99.40%	99.10%	-0.30%	99.30%	99.10%	-0.20%	99.60%	99.70%	0.10%

Table 2 Increasing availability between the proposed model and Yeh and Chang model, Medium ($1 < \rho \leq 1.4$)

δ	0.06		%	0.08		%	0.10		%
α	$\tilde{A}$ Yeh	$\tilde{A}$ Proposed		$\tilde{A}$ Yeh	$\tilde{A}$ Proposed		$\tilde{A}$ Yeh	$\tilde{A}$ Proposed	
6	95.50%	98.40%	3.04%	96.80%	98.50%	1.76%	97.50%	98.60%	1.13%
7	95.90%	98.60%	2.82%	97.60%	98.60%	1.02%	98.10%	98.60%	0.51%
8	88.70%	98.60%	**11.16%**	98.10%	98.60%	0.51%	85.40%	98.60%	**15.46%**
9	97.50%	99.60%	1.13%	98.00%	98.60%	0.61%	98.70%	98.60%	-0.10%
10	98.30%	98.70%	0.41%	98.70%	98.60%	-0.10%	98.60%	98.60%	0.00%

Table 3 Increasing availability between the proposed model and Yeh and Chang model, Heavy ($1.4 < \rho \leq 2$)

δ	0.06		%	0.08		%	0.10		%
α	$\tilde{A}$ Yeh	$\tilde{A}$ Proposed		$\tilde{A}$ Yeh	$\tilde{A}$ Proposed		$\tilde{A}$ Yeh	$\tilde{A}$ Proposed	
6	90.80%	98.40%	**8.73%**	92.80%	98.50%	6.14%	94.10%	98.60%	4.78%
7	93.10%	98.60%	5.91%	91.00%	98.60%	8.35%	96.30%	98.60%	2.39%
8	94.60%	98.10%	3.70%	96.20%	98.60%	2.49%	96.90%	98.60%	1.75%
9	95.60%	99.40%	2.93%	96.90%	98.50%	1.65%	97.70%	98.60%	0.92%
10	96.30%	98.70%	2.49%	97.50%	98.60%	1.13%	98.10%	98.60%	0.51%

Figure 3 shows plots of Availability vs. δ_y for the case of a heavy (severe) operation condition. It can be seen In Fig. 3 that the proposed model is always better when δ_y increases (or the degree of PM is much better) for all levels of reliability considered ($\text{i.e.}, \alpha_y = 6,7,..10$). Also, the proposed model is not sensitive to the change of the PM as the best availability obtained is relatively stable over the three degrees of PM considered.

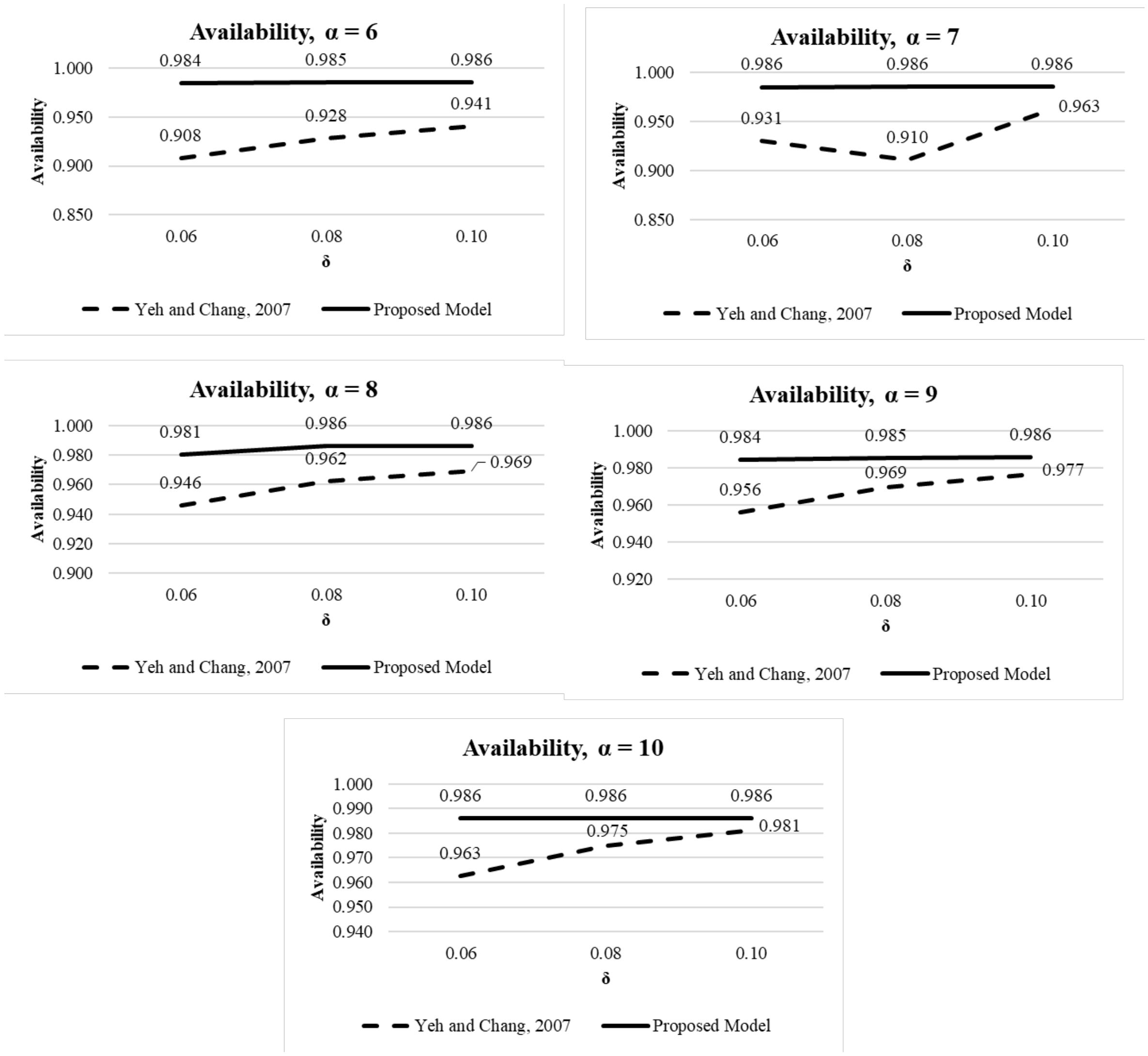

Figure 3 Plots of Availability vs. δ_y for the proposed model and Yeh and Chang's model in heavy usage rate $(1.4 < \rho \leq 2)$

5. Conclusions

In this paper we study 2-D MSC that ensuring availability using a sequential imperfect PM. The algorithm is proposed to obtain the best value of decision variable, T_i and its corresponding availability (A_i). Numerical results show that the proposed model is better for the case of moderate to severe conditions. The algorithm developed does not give the optimal solution – it is only the suboptimal solution, and hence the optimal method based the dynamic programming formulation is a challenging further research topic. Also, one needs to consider the total cost of maintenance as well as the availability to find the best PM policy, and this is another interesting topic for further research.

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