# **Optimisation of a Distribution System in the Retail Industry: An Australian Retail Industry**

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## Abstract

This paper develops a mathematical model based on inventory routing problem that aims to minimise transportation cost, inventory carrying cost and optimises delivery schedules in a retail Australian industry. A supply chain is considered which comprises of a single distribution centre, having homogenous fleet of vehicles, supplying a single product to multiple retailers having deterministic demand. The mathematical model takes into account varying level of road congestion.

## Keywords

Inventory Routing Problem, Retail Operation, Supply Chain Management, Distribution, Optimisation.

## 1. Introduction

The Australian logistics industry contributes to 8.6 per cent of the GDP, adding \$131.6 billion to the Australian economy in 2013. Productivity improvement of 1 per cent is estimated to increase the Australian GDP by \$2 billion (Allen 2014). However, increase in competition and reduction of profit margin has posed a significant challenge to the transport industry and optimisation of individual businesses no longer yields increased efficiency. Companies are now forced to locate efficiency improvements for entire supply chains rather than focusing on improvement at each successive echelon by sharing information beyond prices and tariffs (Andersson et al. 2010). Information shared includes consumption patterns, inventory status, sales forecast and profit margins. Thus the competition has shifted away from individual businesses to competition between entire supply chains, with each actors in the supply chain opting for profit sharing (Alaei & Setak 2015) rather than individual profit maximisation avoiding suboptimal solutions (Stadtler 2015).

Apart from cost of transportation, another significant cost incurred in the transport industry is the cost of inventory. According to Brown (2011), on average a company incurs an inventory carrying cost of 10 per cent of the value of the inventory.

Modern supply chains consist of one or more actors that extend their decision making beyond their realm to account for transportation and inventory management for their customers. This has enabled competitive prices, service quality and maximum utilisation of their fleet of vehicles. Furthermore, this has allowed certain actors to focus on their core competency while other actors handle the transport and inventory management for the entire chain.

In this paper we aim to employ research from supply chain and operations management to realise the goal of holistic optimisation of a supply chain in the Australian retail industry. We consider a distributor having a homogenous fleet of vehicles, providing a single product to multiple customers having deterministic demand. The delivery origins from a single distribution centre and the delivery must be made within a given time-window to cater for individual delivery times of each customer and to account for variation in road congestion, thus ensuring the model is as close as possible to real world constraints. In order to achieve the said objectives, we will employ the use of Inventory Routing Problem (IRP). Since all inventory problems originate from VRP, which is a NP-Hard problem itself, IRP is also a NP-Hard and thus finding exact solution is complex. Therefore, to solve the IRP, variable neighbourhood search (VNS) heuristic will be applied (Mjirda et al. 2014).

This paper adds to the literature related to inventory routing by incorporating varying level of road congestion and how that influences delivery scheduling. Having performed literature review of over 130 articles from 2010 to 2016, no previous research considers traffic as a constraint in the mathematical model of IRP.

The paper provides a literature review of IRP in section 2. This is followed by development of the mathematical model in section 3. Section 4 then evaluates the results of mathematical model. Section 5 provides a discussion and finally conclusion and recommendations are presented in section 6.

## 2. Literature Review

Inventory routing problem (IRP) is an integrated logistics and inventory management problem which aims to minimise cost of transportation, inventory handling cost and maximise fleet utilisation. IRP is an extension of vehicle routing problem (VRP) which was originally proposed by (Dantzig & Ramser 1959). The initial models of VRP focused primarily on optimum routing of the fleet. In the following years, many different variants of VRP emerged, each with own set of constraints and solutions, such as (Beltrami & Bodin 1974; Clarke & Wright 1964; Cooke & Halsey 1966; Schrage 1981). However, major breakthrough was achieved by Bell et al. (1983) as a component of inventory cost was incorporated into the VRP, bringing VRP closer to holistic optimisation of transportation industry. VRP was thus transformed into Inventory Routing Problem. Since then, extensive research has been done on IRP, each with incremental improvements, bringing IRP closer to real-world constraints. Table 1 highlights different variants of IRP available in the literature.

Criteria	Variants			
Time Horizon	Finite	Infinite	Instant	
Structure	One-to-one	One-to-many	Many-to-many	
Routing	Direct	Multiple	Continuous	
Inventory Policy	Maximum Level (ML)	Order-up-to-level (OU)		
Inventory Decisions	Lost Sales	Back-order	Non-negative	
Fleet composition	Homogenous	Heterogeneous		
Fleet size	Single	Multiple	Unconstrained	
Demand	Deterministic	Stochastic		

IRP emerges with regards to vendor managed inventory (VMI), a business strategy which aims to reduce cost of logistics and creates value for businesses. In VMI, the vendor or distributor take control of the inventory replenishment decision for its customer by monitoring the inventory levels and based on this information, taking decisions for when to supply, how much to supply and in what order. (Waller, Johnson & Davis 1999). This is described by Coelho, Cordeau & Laporte (2013) as a beneficial for all as vendors can minimise their distribution cost by coupling shipments made to multiple customers; while buyers benefit by saving resources which would have been allocated to inventory

control. However, in order to achieve the said objective, the vendor or distributor must take the following decisions simultaneously: When to make the delivery, what quantity to be delivered and how to combine multiple deliveries. (Coelho, Cordeau & Laporte 2013).

In recent years, the research has shifted away from traditional IRP models into more complex, application focused models. The pioneering paper by Bell et al. (1983) considered application of IRP in industrial gases. This was further explored by Golden, Assad & Dahl (1984) and Campbell & Savelsbergh (2004). Blumenfeld et al. (1987) and Alegre, Laguna & Pacheco (2007) as these papers applied IRP for automobile components; Miller (1987), Christiansen (1999), Persson & Göthe-Lundgren (2005) and Dauzère-Pérès et al. (2007) for the chemical industry; and Shaabani & Kamalabadi (2016), Park, Yoo & Park (2016) and Ketzenberg et al. (2013) in the retail industry.

Notable contributions to IRP in recent years include the paper by Armentano, Shiguemoto & Løkketangen (2011) which developed a model that further enhanced IRP to include production planning into the model. The objective function defined in their paper included production cost, inventory cost at both the production plant and customer and distribution cost. A similar paper by Nananukul (2013) also explored the possibility of implementing IRP along with production planning. A limitation to this type of model, as highlighted by Coelho & Laporte (2015), is the specific nature of these models. Although over generalisation of an IRP model poses difficulty in formulation of the model, models developed specifically for an industry limits their application to that industry.

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Considering supply chains have multiple echelons, recent IRP models are exploring the possibility of including multiechelons optimisation, truly realising holistic efficiency improvements and minimisation of costs. Govindan et al. (2014) formulated an IRP model that not only tackled multi-echelons, but also focused on improved supply chain sustainability. However, due to the complexity of the problem, the solution achieved relied on multi-stage heuristic application, which given the underlying properties of each heuristic led to increased number of sub-optimal solutions. A recent paper by Alhaj, Svetinovic & Diabat (2016) addressed this problem by formulating a hybrid heuristic to ensure sub-optimal solutions were avoided.

### 2.1 Use of heuristics

It must be highlighted that even though IRP has been applied to various industry, the method of formulation of the mathematical model, assumptions, constraints and method of solution have had significant variations, each with increasing complexities to achieve a model as close as possible to real world constraints.

The mathematical model is solved using mixed integer linear programming (MILP) and since it is a NP-Hard problem, obtaining an optimal solution may not be possible. Ideally a model for IRP should be solved using an exact method. However, development in this area has been limited due to the difficulty in formulation of an exact solution model (Luenberger & Ye 2008). Therefore, recent literature focuses on heuristics. Since the number of variables and constraints in IRP are large, finding an optimal solution may require significant amount of computing time. Furthermore, there are possibilities that the obtained solution does not reflect an optimal solution. Due to this, it is common to apply heuristics to decrease the computation time and in some cases even force the model to converge to a pre-defined solution.

Heuristics that have been applied in IRP include, but are not limited to, simulated annealing used by BañOs et al. (2013); Tabu search used by Bolduc et al. (2010) and Genetic algorithms used in model by Moin, Salhi & Aziz (2011). Although several heuristics exist, each have certain limitations and advantages. The selection of the heuristic to be used depends on carefully weighing the limitations against the advantages. The model proposed in this paper employs

the use of VNS for solution. The reason for selection of VNS is due to the layout of Sydney city, which has distinct north, south, east and west 'partitions'. This enables the model to easily identify the neighbourhoods and limits the optimal solution to confine a set of vehicles to a given neighbourhood, significantly reducing computation time. It must be highlighted at this stage that VNS is considered an inferior heuristic due to its iterative nature, which, if given a large set of variables, may take up to 3 hours to produce an optimal solution. Since the paper is focused primarily on development of the mathematical model, and not towards decreased computation time, selection of VNS is feasible. Furthermore, to test the models, the number of variables will be limited to no more than 20 customers and 5 vehicles. Nevertheless, the model may be used for as many 200 customers and 50 vehicles with a trade-off of increased computation time.

Variable neighborhood search (VNS) heuristic was originally formulated by Mladenović & Hansen (1997). The basic idea of VNS revolves around formulation of '*large search spaces*', which serve as a predefined location within which the optimal solution is to be found. Figure 1 shows a simplified example of how VNS functions. A set of locations are defined within a graph. Firstly, all locations are treated independently and the distance is noted between two closest locations. This process is repeated until the distance between a set of locations are determined to be within a certain 'radius' and are therefore grouped together. Each group is now a neighborhood,  $K_w$  where w is a set of total neighborhoods. Therefore, as can be seen, locations 1, 2, 4 and 6 belong to neighborhood  $K_1$ ; locations 3, 5 and 7 to  $K_3$  and locations 8, 9, 10 and 11 belong to neighborhood  $K_2$ .

Having done this segregation, the IRP model will begin by first identifying optimal inventory carrying costs for the entire graph, and then determine the optimal route between each location within the neighborhood. Therefore, inventory holding cost decision will be based on the entire set of customers, ensuring holistic optimisation of inventory carrying cost, while the vehicle routing decisions will be localised within neighborhoods.

## 3. Model Formulation

The proposed model is an expansion of Malandraki & Daskin (1992) paper in which vehicle routing problem with traffic constraints was considered. A similar component of time dependent variable will be incorporated into the IRP to consider the effect of traffic on the IRP. The list of symbols is shown in Figure 1 as follows.

NNumber of CustomersTNumber of time periodsKNumber of vehiclesMNumber of intervalstTime at each node $b_k$ Vehicle Capacity $C_i$ Capacity at retailer $i$ $d_{it}$ Demand at retailer $i$ during period $t$ $c_{ij}^{tm}$ Travel time from node $(i, j)$ at period $t$ during time interval $m$ $s_{it}$ Service time at node $i$ $T_{ij}^{tm}$ Upper bound for interval $m$ , at time period $t$ for link $(i, j)$ $h_{Ci}$ Inventory holding cost at $i$ per unit $h_{bi}$ Cost of backorder at $i$ per unit $c$ Routing cost per hour $f_{ct}$ Fixed cost of vehicle at period $t$
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c Routing cost per hour
f. Fixed cost of vehicle at period $t$
Jet inved cost of vehicle at period i
$I_{it}$ Inventory level at node <i>i</i> at period <i>t</i>
$B_{it}$ Inventory stock-outs at node <i>i</i> at period <i>t</i>
$t_{it}$ Departure time of any vehicle from node <i>i</i> at period <i>t</i>
$y_{ij}^{tm}$ Amount transported for time interval <i>m</i> for link ( <i>i</i> , <i>j</i> ) at period <i>t</i> – Decision Variable
$x_{ij}^{tm} \begin{cases} 1 \\ 0 \end{cases}$ 1 if vehicle travels from <i>i</i> to <i>j</i> during time interval <i>m</i> at period <i>t</i> , else 0

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The problem consists of a single distribution point with unlimited supply capacity, producing a single product, providing multiple customers with deterministic demand using homogenous set of vehicles (K). Set of customers located at different geographical locations are indexed(1, ..., N). The problem is formulated on a graph G(V, E) where  $V = \{0, 1, 2, ..., N\}$  is the set of nodes whereas E represents the links between the nodes. The direction is represented by (i, j) in which *i* is the starting point while *j* represents the destination.  $W(t) = [w_{ij}(t_i)]$  is an *n* x *n* time matrix which defines the travel time during a particular time interval,  $w_{ij}(t_i)$  is a step function for the time  $t_i$ . As proposed by Malandraki & Daskin (1992),  $W_{ij}$  is the number of distinct time intervals considered in the step function  $w_{ij}(t_i)$  representing the travel time for the link'. To simplify, a day is considered to have three or more time intervals, during which there is varying level of road congestion. For each interval, the value of  $W_{ij}$  changes to account for varying traffic levels. Figure 2 shows an example of travel time step function with three time intervals.

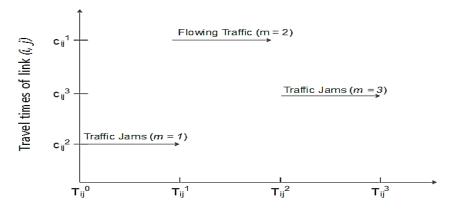


Figure 2. Example of step function at different time interval for link (i, j) (Adapted from Malandraki & Daskin 1992)

The time intervals during a day can be as many as required, however, increasing it more than three increases the computation time exponentially. Therefore, it is imperative to use the following assumptions to restrict the model (Malandraki & Daskin 1992):

1. The travel time is independent of vehicle type. It is illustrated in Figure 3.

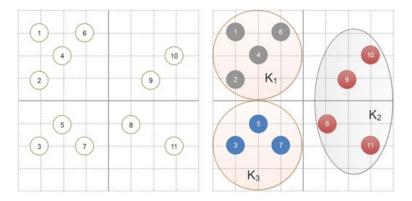


Figure 3. A simplified example of implementation of VNS

2. The collection (or delivery) time for each vehicle is independent of the type of vehicle and is entirely dependent on the customer.

Taking into account the IRP, it must be highlighted that inventory carrying cost,  $h_{ci}$ , is incurred at a constant rate per unit of time and per unit stock. This rate is assumed to be same for all customers. Cost of transportation, c, includes variable cost per hour based on the distance (i, j), whereas a fixed cost per vehicle at period t,  $f_{ct}$ , is also included.

The objective function is to minimise the holistic costs, which include transportation cost, backordering cost and inventory holding cost. Therefore, the IRP is formulated as follows.

$$\begin{split} &Min \sum_{t=1}^{T} \left[ \sum_{j=1}^{N} \sum_{m=1}^{M} f_{ct} x_{0}^{im} + \sum_{i=N+1}^{N+K} ct_{it} \\ &+ \sum_{l=1}^{N} (h_{cl} l_{it} + h_{bi} B_{it}) \right] \quad (1) \end{split}$$

$$\begin{aligned} &Subject to: \\ &\sum_{l=0}^{N} \sum_{m=1}^{M} x_{lj}^{im} = 1, \\ &i \neq j \\ j = 1, \dots, N + K; t = 1, \dots, T \quad (2) \end{aligned}$$

$$\begin{aligned} &\sum_{j=1}^{N+K} \sum_{m=1}^{M} x_{lj}^{im} = 1, \\ &j = 1 \\ j \neq i \\ i = 1, \dots, N; t = 1, \dots, T \quad (3) \end{aligned}$$

$$\begin{aligned} &\sum_{j=1}^{N} \sum_{m=1}^{M} x_{0j}^{im} = K, \\ &t = 1, \dots, T \quad (4) \end{aligned}$$

$$t_{0t} = t, \quad t = 1, \dots, T \quad (5) \end{aligned}$$

$$t_{jt} - t_{it} \geq w_{lm}^{im} + s_{jt} - Max_k b_k, \\ &i = 0, \dots, N; \ j = 1, \dots, M \quad (6) \end{aligned}$$

$$t + Bx_{lj}^{im} + Max_k b_k, i = 0, \dots, N; \\ &j = 1, \dots, T \quad (m + K; i \neq j; \\ &t = 1, \dots, T \quad (m + K; i \neq j; \\ &t = 1, \dots, T; \ m = 1, \dots, M \quad (7) \end{aligned}$$

$$t - T_{lm}^{im} \geq 0, i = 0, \dots, N; \\ &j = 1, \dots, N + K; i \neq j; \\ &t = 1, \dots, T; \ m = 1, \dots, M \quad (8) \end{aligned}$$

$$y_{lj}^{im} - b_k x_{lj}^{im} \leq 0, i = 0, \dots, N; \\ &j = 1, \dots, N + K; i \neq j; \\ &t = 1, \dots, T; \ m = 1, \dots, M \quad (9) \end{aligned}$$

$$\begin{split} & I_{it-1} - B_{it-1} - I_{it} + B_{it} \\ & + \left( \sum_{\substack{h=0\\h\neq i}}^{N} \sum_{\substack{m=1}}^{M} y_{hi}^{tm} - \sum_{\substack{l=1\\l\neq i}}^{N} \sum_{\substack{m=1}}^{M} y_{li}^{tm} \right) \\ & = d_{it}, i = 1, \dots, N; t = 1, \dots, T \quad (11) \\ & I_{it} \leq W_i, i = 1, \dots, N; t = 1, \dots, T \quad (12) \\ & I_{it} \geq 0, i = 1, \dots, N; t = 1, \dots, T \quad (13) \\ & B_{it} \geq 0, i = 1, \dots, N; t = 1, \dots, T \quad (14) \\ & x_{ij}^{tm} = \begin{cases} 0 \\ 1, & i = 0, \dots, N; \\ j = 1, \dots, N + K; i \neq j; \\ t = 1, \dots, T; & m = 1, \dots, M \quad (15) \\ & t_{it} \geq 0, i = 0, \dots, N; t = 1, \dots, T; \quad (16) \\ & y_{ij}^{tm} \geq 0, i = 0, \dots, N; \\ j = 1, \dots, N + K; i \neq j; \\ t = 1, \dots, T; & m = 1, \dots, M \quad (17) \\ & I_{0t} = 0, t = 1, \dots, T \quad (18) \\ & B_{0t} = 0, t = 1, \dots, T \quad (19) \end{split}$$

In Table 2 below, details regarding the equations in the model are provided.

Table 2. Details regarding the model equations

Eq.	Description
(1)	The objective function. Includes transportation
	cost (which has a component of fixed cost and
	variable cost), total delivery time (includes
	service time, travelling time), inventory
	carrying cost and cost of inventory shortage.
(2) &	A constraint which ensures one customer is
(3)	visited only once in a time period.
(4)	A constraint which ensures exactly K number
	of vehicles are utilised in a time period.
(5)	Constraint which sets the starting time to 1.
(6)	Constraint which determines the time of
	departure at every node <i>j</i>
(7) &	A constraint which ensures selected link $(i, j)$
(8)	corresponds to a specific time interval <i>m</i>
(9)	A constraint which ensures amount of product
	transferred does not exceed vehicle capacity
	and is not a negative value.
(10)	Eliminates sub-tour
(11)	A constraint that ensures balancing of
	inventory between customer and supplier.

(12)	A constraint which ensures amount transported
	to customer does not exceed maximum
	capacity at customer.
(13) to	A constraint which ensure non-negativity of
(19)	variables.

Since the number of constraints imposed on the mathematical model are large, the model was incorporated with the following pseudonym code for VNS heuristics, obtained from (Ali et al. 2014).

Vari	iable Neighborhood Search Algorithm (adapted from (Ali et al. 2014))
1:	<b>Define</b> a set of neighbourhood structure $N_k$ for $k = 1,, k_{max}$
2:	Set $x = x_0$ > Generate the initial solution
3:	repeat
4:	k = 1.
5:	repeat
6:	Generate a random neighbour x' from the $k^{th}$ neighbourhood $N_k(x)$ of x.
7:	x'' = local search(x')
8:	$ \inf f(x'') \leq f(x) $ then.
9:	x = x".
10:	Continue to search in N <sub>1</sub>
11:	k = 1.
12:	else
13:	k = k + 1
14:	Move to a new neighbourhood area.
15:	end if
16:	<b>until</b> $K = K_{max}$ > e.g. Number of neighbourhood structures
17:	<b>until</b> (stopping criteria satisfied) return $x > Best$ found solution

## 4. Experimental design

The model was programmed using IBM ILOG CPLEX optimisation studio (student promotional version) on an i5— 4200U CPU running at 1.6 GHz with 4 GB of RAM. Random test problems are generated, with customers randomly allocated in a 15 x 15 distance units. The distribution centre is assumed to be located in the centre of the 15 x 15 grid. Variable transportation cost is assumed to be 1, whereas the fixed cost per vehicle is assumed to be 15. Inventory holding cost is assumed to be 10 percent of inventory value as has previously been assumed by various IRP models (Bertazzi et al. 2013; Bertazzi, Bosco & Laganà 2015; Chitsaz, Divsalar & Vansteenwegen 2016). Each retailer is assumed to have a storage capacity of 100 units. Cost of stock out is assumed to be 3 units. The customer demand is generated randomly between 5 to 50 100 units per day. The starting inventory level at customer is assumed to be 0. The number of homogenous vehicles is assumed to be 2 with a carrying capacity of 900 each. A total of 18 test problems were generated with number of customers were selected to be 5, 10, 15, 20, 25 and 30. For each customer level, time periods of 1, 2 and 3 were used, whereas time period 1 assumed average speed of vehicle of 60, 2 assumed average speed of vehicle as 45 and 3 assumed speed of vehicle as 30. Each problem will be repeated 5 times and average of the 5 results will be taken to avoid the impact of randomly generated data on the results obtained.

To analyse the problem, we will take into account the impact on total cost by the number of customers, time periods and impact of different vehicle speeds as influenced by level of traffic.

Since the version of CPLEX used to obtain the solution was a promotional version, the maximum number of customers that could be programmed were limited to 30. The discussion of results, therefore, will be limited to 30 customers only.

After having obtained experimental results, the model will be applied to a real supply chain of retail industry, an Australian retail company with 15 customers. The results of the model will be compared to actual data obtained from the case study.

## 5. Results and discussion

Table 3 highlights the combination of results obtained by running the model. The first two columns highlight the variables that were kept independent. First column denotes number of customers, whereas the second column denotes the time period to include the impact of traffic into the experiment.

Customers	Period	Objective Solution	Mean Comp. Time (s)	Mean Objective Solution	VNS Iterations
5	1	236.26	15.52	236.26	87
10	1	545.30	17.00	1003.34	782
15	1	1129.19	20.47	1272.82	18309
20	1	1651.17	33.58	2066.01	27128
25	1	2329.28	88.09	2790.00	294518
30	1	3015.25	62.22	3389.75	179736
5	2	267.59	15.52	281.55	98
10	2	717.33	17.52	1059.33	824
15	2	1359.56	20.64	1506.46	20930
20	2	1920.62	34.18	2418.25	27995
25	2	2631.67	89.21	3274.77	299773
30	2	3041.16	68.12	3900.70	209247
5	3	303.22	15.61	319,45	111
10	3	755.80	17.61	1105.29	896
15	3	1435.99	20.79	1648.70	21229
20	3	2061.96	34.53	2594.53	27706
25	3	2701.06	90.08	3505.56	306478
30	3	3117.34	68.05	4200.67	260111

Table 3. Experimental results

Third and fourth columns list the respective objective solutions and mean objective solutions. Column 5 lists the computational time and column 6 lists the number of VNS iterations required to achieve the optimal solution.

Referring to the computation time required for each level of customer, the maximum time required to obtain a solution is 90.8 seconds which means this model can be used for tactical level supply chain decisions as intended. Comparison of objective solution and mean objective solution yields significant variations, therefore, the application of VNS as a heuristic for this model is not the optimal selection. This can further be verified based on the iterations required to obtain the result. However, since the purpose of this paper is to develop a model, rather than selection of heuristic, this limitation will be ignored

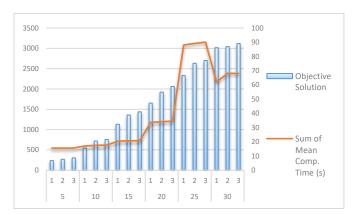


Figure 4. Relationship of number of customers and computational time

Referring to Figure 4 above, there is a significant rise in computation time for 25 customers, even though for 30 customers the computational time drops back down to 50 seconds. This can be an impact of using 2 vehicles for an odd number of customers, which results in higher number of iterations to determine which vehicle is better off in catering for higher number of customers. Since each result was repeated 5 times, any impact due to the randomly generated data is avoided.

The interaction between number of customers and total cost is also given in Figure 4. Any increase in the number of customer leads to an increase in the total costs. This is due to the fact that the vehicle now has to travel more distance, thus increase in variable vehicle traveling cost. The increased demand and inventory capacity at increased number of customer also rises, resulting in higher total cost.

The interaction between time interval and total cost is also presented in Figure 4. Although there is an increase in cost for time interval with higher traffic, hence lower average vehicle speed, the impact is minute. This can be a reflection of limitation of the model itself, or selection of assumed value. In order to verify the findings, the assumed values of average vehicle speed for each time interval 1, 2 and 3 is modified to 30, 50 and 70 respectively. Furthermore, the variable transportation cost is modified from 1 to 5. Figure 5, therefore, shows the results with modified average vehicle speeds and increased variable cost of transportation.

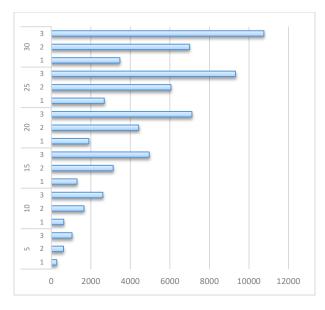


Figure 5. Impact of modified speed and variable transportation cost

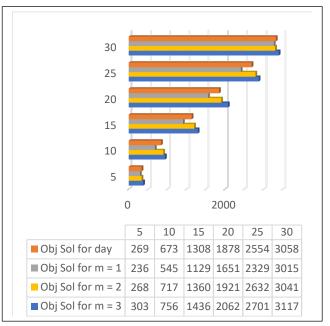


Figure 6. Comparison of objective solution between m = 1, 2 and 3 with objective solution of entire day

As can be seen from Figure 5, by modifying the assumed average speed at the three time intervals and increasing the assumed variable cost of transportation, the impact on total cost is apparent. This shows that the model adequately takes into account the impact of traffic on the total cost.

In Figure 6, a comparison is provided between objective solution obtained for each interval and objective solution obtained for the entire day, considering the day is divided into three intervals. It is evident from Figure 6 that incorporation of three time intervals yields an objective solution slightly less than that of m = 2 for all number of customers.

## 6. Conclusions and recommendation

Consideration of inventory cost along with transportation cost has led to improved supply chain efficiencies and thus competitive advantage. With increased utilisation of existing road networks and limitation of space for further development, a problem that most supply chains face is not being able to perform timely delivery due to increased

traffic congestion. In this paper, a model was developed that enabled division of a day into multiple intervals, each with different average speed of vehicle, to mimic the effect of traffic. This model was solved by incorporating the VNS heuristic. A computational time of under 95 seconds was achieved, allowing application of this model for tactical level decision making process. Impact of speed penalties was observed by comparing objective solution for each time interval with the objective solution for the entire day.

The model performed as per its requirement, however, given the increase in computational time for increased number of customers, the variable neighbourhood search heuristic applied can be deemed insufficient and possibility of incorporation of a hybrid heuristic is felt.

Application of IRP in the Australian logistics industry will have a significant impact in productivity improvements, while cutting down unnecessary costs. Australia already suffers increased prices of goods as compared to other countries due to its geographical location. By incorporating tools like IRP, local supply chains can achieve competitive prices of products and aim to compete in the international market.

Future variants of this model can incorporate real-time traffic conditions using API provided by Google Maps. Average speed for each time interval can be determined by taking average speed of vehicles travelling from the route in real-time, which can yield real-time routing of the vehicle via global positioning system (GPS). This can further be enhanced via use of radio frequency identification devices (RFID) which can be used to provide real-time information regarding inventory levels, and by incorporating data obtained from point of sale (POS) terminals, a live tool can be created, which will optimise every movement of inventory from supplier to retailer.

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