Selection and Scheduling of Interdependent Projects using a Modified Genetic Algorithm

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Abstract
This paper deals with the problem of selection and scheduling of interdependent projects in an integrated manner. Two types of interdependencies among projects - mutual exclusiveness and complementariness are considered which are found to be of great practical importance. The problem seeks to maximize the total expected benefit from the selected projects wherein the benefit from a project is considered to be time-dependent. A mathematical model for the problem along with an illustrative example is presented. A modified genetic algorithm (GA) is developed to solve the real life problems. The proposed algorithm is evaluated on a set of randomly generated test problems with varying complexity. The performance of the proposed GA is compared with TS algorithm. The results show that proposed GA outperforms the TS algorithm in 10 test problems out of the total 15 problems generated in this study.

Keywords
Project selection and scheduling, Meta-heuristics, Genetic algorithm, Interdependencies

1. Introduction
Project firms pursue multiple projects sharing a pool of resources. The success of these projects largely depends on the selection of right mix of the projects hence and project selection is a crucial decision in a firm. Project selection problem involves the selection of most profitable projects from a large set of available candidate projects to optimize the firm’s stated objective(s) respecting the resource, budget and other technical limitations. But models developed for project selection generally exclude project scheduling in the selection process (Coffin & Taylor, 1996a; 1996b). Traditionally, projects are selected first and then attempt is made to schedule these projects through various procedures in light of the available time and resources. If the selected projects cannot be scheduled, some of the projects may be dropped/changed, schedule may be relaxed or resource limit may be extended, ultimately resulting in a suboptimal portfolio. Thus project scheduling should be considered as an integral part of the project selection problem. This integrated problem of selection and scheduling of projects is known as project portfolio selection and scheduling problem (PPSSP). In recent years the PPSSP has gained attention of many researchers and has become an active field of research. Maximizing the total expected benefit has been the popular objective in project selection problems as well as in integrated project selection and scheduling problems (PPSSP). Moreover, the benefit from a project is considered to be time-dependent (Chen & Askin, 2009). This is especially true in the case of projects related to introducing new products to the market. Early market entry helps the firm to acquire a larger market share than the late entrants. This time-sensitive aspect of benefit in PPSSP is first considered by Chen & Askin (2009) and followed by Ghorbani & Rabbani (2009), Tofighian & Naderi (2015), Ganji et al. (2016), Amirian & Sahraeian (2017; 2018) and Kumar et al. (2018; 2019).
Further, there are inherent relationships between the projects. These interrelationships between the projects are known as interdependencies. There are three types of interdependencies defined in the literature namely: resource, benefit and technical (Aaker & Tyebjee, 1978; Fox et al., 1984) out of which technical interdependencies are most common which can be further divided into two types. One is mutual exclusiveness
in which only one project from the subset can be included in the portfolio. Many authors have considered mutual exclusiveness in modelling the PPSSP (Ghazemzadeh et al., 1999; Ghorbani & Rabbani, 2009; Carazo et al., 2010; Huang & Zhao, 2014; Tofighian & Naderi, 2015; Wang & Song, 2016; Huang et al., 2016; Ganji et al., 2016; Amirian & Sahraeian, 2018; and Kumar et al., 2018; Kumar et al., 2019). Another technical interdependency is complementariness which can be described as the selection and rejection of complimentary projects from the subset together. Kumar et al. (2018) have considered the project complementariness in PPSSP. Despite the fact, the complementariness has been considered exclusively in project selection problems, but it is scantily addressed in PPSSP.

Resource constrained project scheduling problems (RCPSP) belongs to a very large family of NP-hard problems (Demeulemeester & Herroelen, 2006). The PPSSP becomes even more complex due to consideration of interdependencies. Because of such complex nature of the problem, the exact approaches are not suitable to solve the large sized problems. In such cases, meta-heuristics are employed to find a near optimal solution to the problem. Several meta-heuristics have been developed for the PPSSP in the literature (Ghorbani and Rabbani, 2009; Carazo et al., 2010; Huang and Zhao, 2014; Tofighian and Naderi, 2015; Gahremani and Naderi, 2015; Amirian and Sahraeian, 2017; Kumar et al., 2018; Kumar et al., 2019). Evolutionary algorithms predominate in the research of applications of the meta-heuristics (Carazo et al., 2010). Genetic algorithm (GA) is an evolutionary algorithm and is widely applied to solve the complex engineering & optimization problems. In this paper, we consider the PPSSP with two types of interdependencies and formulate a 0-1 integer model for the problem. The objective of the problem is to maximize the total expected benefit from the portfolio. The benefit from the projects is considered to be time-dependent. A modified genetic algorithm (GA) is proposed to solve the problem. Problem is tested on 15 randomly generated test problems, and results are compared with tabu search (TS) also developed for the problem.

The rest of the paper is structured as follows. Section 2 presents the mathematical formulation of the problem with an illustrative example. The methodology for the proposed GA is provided in Section 3. Computational experiences are described in Section 4. Finally, Section 5 concludes the paper.

2. Problem Statement
In this section, the problem under study is formulated and described in detail through an illustrative example. Let us consider a set of N candidate projects available for selection. The planning horizon is assumed to be fixed and is taken as T time periods. Some project may have due dates before which they must be completed if selected. K types of resources are required for the implementation of the selected projects and are available in limited quantity. Technical interdependencies: mutual exclusiveness and complementariness are considered in the model. The objective is to maximize the total expected benefit from a selected portfolio of projects. Further, the expected benefit from a project is assumed to be time dependent. Early scheduling of the projects is desirable whose return is highly sensitive to the completion time.

2.1 Mathematical Model
Decision variables and coefficients of the mathematical model are listed as below:

**Technological coefficients and parameters:**

- \( P_i \) = expected profit if project \( i \) starts in period \( t \).
- \( d_i \) = duration of the project \( i \).
- \( r_{ik} \) = requirement of resource type \( k \) for project \( i \) in each time period.
- \( R_{it} \) = resource availability of type \( k \) in period \( t \).
- \( DD_i \) = Due date or milestone for the project \( i \).
- \( e \) = project mutually exclusive to project \( i \), \( \{ e \in E_i \} \)
- \( h \) = projects complementary to project \( i \), \( \{ h \in C_i \} \)

**Decision variable:**

- \( X_{it} \) = 1 if project \( i \) is selected and starts in period \( t \)
  = 0 otherwise.

**Formulation:**
Objective function:

\[
\text{Max } Z = \sum_{i=1}^{N} \sum_{t=1}^{T-d_i+1} P_{it} \cdot X_{it}
\]  \hspace{1cm} (1)

Constraints:

\[
\sum_{t=1}^{T-d_i+1} X_{it} \leq 1 \quad \forall i
\]  \hspace{1cm} (2)

\[
(t \cdot X_{it} + d_i - 1) \leq DD_i \quad \forall i
\]  \hspace{1cm} (3)

\[
\sum_{i=1}^{N} (\sum_{j=\max[1,t-d_i+1]}^{\min[t,T-d_i+1]} X_{ij}) \cdot r_{ik} \leq R_{kt} \quad \forall k,t
\]  \hspace{1cm} (4)

\[
\sum_{t=1}^{T-d_i+1} X_{it} = \sum_{t=1}^{T-d_h+1} X_{ht} \quad \forall i, h \in C_i
\]  \hspace{1cm} (5)

\[
\sum_{t=1}^{T-d_i+1} X_{it} + \sum_{t=1}^{T-d_e+1} X_{et} \leq 1 \quad \forall i, e \in E_i
\]  \hspace{1cm} (6)

\[
X_{it} = \{0,1\} \quad \forall i, t \leq T - d_i + 1
\]  \hspace{1cm} (7)

Equation (1) defines the objective function for the problem which is to maximize the total expected benefit from the portfolio. The selection status and start time of each project are ensured by constraint (2). Constraint (3) ensures that each selected project is completed before its due date. The resource limitations are imposed by the constraint (4). Constraint (5) enforces the selection of complementary projects from a subset together. The mutual exclusiveness is imposed by constraint (6). Constraint (7) defines the decision variables.

2.2 Illustrative Example

In this section, a small problem and its feasible solution are presented to get a basic understanding of the model. Example problem has 4 candidate projects with the planning horizon of 9 time periods. Two resources are needed to carry out the projects and availability of each resource is fixed at 25 units per period. Table 1 provides the durations, due dates, resource requirements and interdependencies of the projects. Expected profits of projects in each time period is given in Table 2.

<table>
<thead>
<tr>
<th>Projects</th>
<th>Duration</th>
<th>Due Date</th>
<th>Resource Requirements</th>
<th>Interdependencies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Type 1</td>
<td>Type 2</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>7</td>
<td>14</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>8</td>
<td>10</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>7</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>9</td>
<td>15</td>
<td>12</td>
</tr>
</tbody>
</table>

Table 1: Project durations, due dates, resource requirements and interdependencies

<table>
<thead>
<tr>
<th>Projects</th>
<th>Time Periods</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>935</td>
</tr>
<tr>
<td>2</td>
<td>832</td>
</tr>
<tr>
<td>3</td>
<td>932</td>
</tr>
</tbody>
</table>

Table 2: The expected profit in each time period

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Schedule of a feasible solution of the problem is shown in Table 3. Projects 1, 2 and 4 are selected and scheduled to start in periods 1, 1 and 5 respectively. Project 1 and 3 are mutually exclusive hence one of them (project 3) has not been selected. Projects 2 and 4 are selected together due to complementariness between them. The value of objective function equals 2387 units.

### Table 3: A feasible solution and schedule

<table>
<thead>
<tr>
<th>Projects</th>
<th>Time Periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>1</td>
</tr>
<tr>
<td>P2</td>
<td>1</td>
</tr>
<tr>
<td>P3</td>
<td>0</td>
</tr>
<tr>
<td>P4</td>
<td>1</td>
</tr>
</tbody>
</table>

3. Modified Genetic Algorithm

In this section, the procedure of the proposed genetic algorithm for PPSSP is described in detail. Genetic algorithm (GA) is an evolutionary algorithm inspired by natural evolution. This algorithm has been successfully applied to a variety of complex problems. In this study, maintaining the basic philosophy of the GA, the operators are modified and re-designed to suit the problem under study. Encoding scheme, population initialization and the operators of the proposed GA are explained as follows:

3.1 Encoding Scheme for proposed GA

The encoding scheme used in this paper is adapted from Ghorbani and Rabbani, (2009). A solution to the problem can be represented by a matrix of size \( N \times T \). There are \( N \) rows equal to the number of candidate projects and \( T \) columns equal to the number of time periods. Each row represents a project, and each column represents a time period. The element \( a_{ij} \) represents the selection status and start time of project \( i \) in period \( j \). The value ‘1’ corresponds to the selected project. Table 4 shows a chromosome for proposed GA.

### Table 4: Solution representation for proposed GA

<table>
<thead>
<tr>
<th>Projects (p)</th>
<th>Time Periods (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

3.2 Population Initialization

A population of the feasible chromosome with diversity is essential for the optimizing efficiency of the GA. The required chromosomes for the proposed GA are generated randomly to maintain the diversity. To obtain a feasible chromosome, projects are selected randomly one by one in light of the interdependencies and scheduled earliest possible satisfying the due date and resource limitations. All the chromosomes are obtained in the same manner.

3.3 Selection operator

A number of methods are available in the literature, but roulette wheel selection is the most efficient and widely applied method for selection. This method is used in the proposed GA to select the best chromosomes to be the part of the population for the next iteration.
3.4 Crossover operator

The crossover operator starts with defining a crossover probability \((p_c)\). Then, we generate \(M\) random numbers in the range \([0-1]\). If the random number \((r)\) is less than the crossover probability, the corresponding chromosome is selected for crossover. If odd numbers of parents are selected then one more chromosome is selected randomly from the remaining chromosome to make the pairing easy. The pairing of the selected chromosomes to be parents is done randomly. Now, a row is selected randomly from the parent 1 in the pair and is copied to the parent 2 at the same place in the same pair maintaining the feasibility of the chromosome. This process gives the child 1 for pair undergone crossover. To obtain the child 2 for the same pair, the row is selected from the parent 2 and is placed in the parent 1 in the same way as done for child 1. Similarly, crossover is applied to all the pairs selected for crossover.

3.5 Mutation operator

The mutation operator starts with defining a mutation probability \((p_m)\). Then, we generate random numbers equal to the number of children in the range \([0-1]\). If the random number \((r)\) is less than the mutation probability \((p_m)\) the corresponding chromosome is selected for mutation. Now, for mutation of a chromosome, a row is randomly selected and its start time is changed randomly maintaining the feasibility of the solution.

![Figure 1: Basic genetic algorithm scheme](image)

At the beginning, input parameters for GA are initialized. The proposed GA starts with initialization of input parameters (population size, crossover probability, mutation probability). The initial population of random
feasible chromosomes is generated with the population initialization scheme. The objective function is taken as the fitness function for the problem and is calculated for each chromosome. The chromosomes are then updated using the selection, crossover and mutation operators designed for this problem. New chromosomes are evaluated and compared with the old chromosomes. Chromosomes which are superior to the previous one are considered to be the part of the current population for the next iteration. The algorithm is iterated for a predefined number of iterations. The best chromosome found so far is reported as the solution to the problem. The scheme for the basic genetic algorithm is given in Figure 1.

4. Computational Experience

This section evaluates the performance of proposed GA based on the comparison with Tabu Search (TS) developed for the problem. The scheme of TS algorithm proposed by Kumar et al. (2018; 2019) for PPSSP is used to develop TS for this study. Both the algorithms are coded in MATLAB 7.12 environment. The two algorithms are tested on 15 randomly generated instances.

4.1 Test problems

Test problems are generated randomly using the instance generation scheme proposed by Kumar et al. (2018). These test problems are of different size and complexity. The complexity of the problems is varied by reducing project durations, resource availability and enhancing the amount of interdependencies. A total of 15 test problems are generated.

4.2 Parameter settings

The population size (M) for the proposed GA is kept 20. The crossover probability (pc) and mutation probability (pm) are taken as 0.8 and 0.01 respectively. The size of the tabu list in TS algorithm is taken as 40 % of the problem size (number of projects). Termination criterion is set to 100 iterations for both the algorithms.

4.3 Comparison method

The performance of the proposed GA is compared with TS using percentage deviation method. The deviation of an algorithm’s output is taken from the best value obtained by any of the two algorithms. The formula for the percentage deviation is given by equation (8).

\[ PD = \left( \frac{B-X}{B} \right) \times 100 \]  

For a test problem, the value of objective function obtained by an algorithm is denoted by X. The best value found for the same instance by any of the two algorithms is denoted by B.

4.4 Results

Each test problem is solved 10 times by each algorithm. Average results are reported for comparison. The results for percentage deviation are presented in Table 5. From the results, it is clear that the proposed GA outperforms the TS algorithm. It is important to notice that for small-sized problems, there is little difference between the performances of the two algorithms. It can be seen that for problems LCP_01, LCP_02 and LCP_05 there is no difference in the performance of the two algorithms whereas the performance of proposed GA is slightly better than TS for LCP_03 and LCP_04. For the problems with medium complexity, the proposed GA performs better than TS for MCP_02, MCP_04 and MCP_05. As a major contribution of this work it can be seen that the proposed GA outperforms the TS for the problems with high complexity which are more prominent in real life scenario. To summarize, it is evident that the proposed GA is an effective solution technique for PPSSP.

<table>
<thead>
<tr>
<th>Problem Type</th>
<th>Test Problem</th>
<th>Percentage Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Proposed GA</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Outputs of the proposed GA and TS algorithm
5. Conclusion
This paper considers the integrated problem of selection and scheduling of the interdependent projects. Two types of interdependencies among projects-mutual exclusiveness and complementariness are considered. The most beneficial projects are selected in the portfolio and are scheduled simultaneously. The total expected benefit is considered as the objective function to be optimized. The benefits from the projects are considered to be time-dependent. The problem is formulated in the form of a 0-1 integer program. A modified genetic algorithm (GA) is developed, and performance is compared with the Tabu Search (TS) algorithm existing in the PPSSP literature. Both the algorithms are evaluated on a set of 15 randomly generated test problems with varying complexity. The results show that proposed GA outperforms the TS algorithm in 10 test problems out of the total 15 problems generated in this study.
As a major contribution of this work, the developed algorithm promises a potential solution approach to tackle the large and real life project selection problems faced by various organizations. For future research, the problem can be extended to consider the multiple objective and dynamic selection and scheduling of projects.

References


**Biographies**

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