Metaheuristic algorithms are sensitive to the value of parameters. In this regard, Taguchi method can be applied to tune the level of parameter to get near optimal solution. Both the work of Mousavi (2014) and Tavana (2016) used Taguchi L_9 approach to improve the quality of solution.

Inspired from previous researches a weekly periodic multi-item inventory control model is proposed considering single supplier single buyer relation with shortage and constant demand for different items. In order to apply the model in close to reality problem a limited budget, order capacity, truck space and warehouse space constraint is also included. To create a kind of green model equivalent cost of greenhouse gas (GHG) emissions and limitation on total emissions of all items are considered in the model. However, a Pareto-based multi-objective metaheuristic algorithms MOPSO is employed to find near optimum solution for different items so that total inventory cost and warehouse space is minimized. The proposed multi-objective inventory model can be used in situations in which purchasing managers desire to purchase multiple product that requires an extended storage space to locate their purchased items with limited budget. For efficient control on inventory, they must consider shortage, truck capacity and other realistic limitations.

In short, the highlights of this study are developing raw material inventory control model for multiple item under shortage with limited budget, developing a mathematical formulation for obtaining economic ordering quantity (EOQ), considering greenhouse gas (GHG) emissions cost in total inventory cost and limitation on total emissions of all items to make a kind of green supply chain and analyzing the results obtained for optimum level of parameters from Taguchi L_9 design.

The remainder of this paper is organized as follows: In Section 3, the problem is stated along with the notations and assumptions. In Section 4, the problem is formulated in a nonlinear programming model. The solution algorithm MOPSO is introduced in Section 5 to solve the problem. Section 6 provides experimental results along with analysis. Conclusion and recommendations for future works are given in Section 7.

3. Problem Definition, Assumptions and Notations

Considering a periodic inventory control model for single provider having constant and distinct demand of items. Raw materials are supplied from a single supplier using third party logistic (3PL) service to the buyer in a single period. The costs associated with the inventory control system are holding, ordering and transportation costs. Several items are considered here with real life constraints like warehouse space, order capacity and budget constraints. To prioritize environmental pollution as an integral part of proposed inventory model. Tax cost of greenhouse gas (GHG) emissions and limitation on total emissions of all items are included. Furthermore, the lead-time is assumed zero, and the decision variables are integer digits. The assumptions of this study are inspired from previous researches (Mousavi et al. 2014; Roozbeh Nia et al. 2015). The goal is to identify the inventory levels of the items and required warehouse space, such that the total inventory cost is minimized.

3.1 Assumptions

- Independent demand rate of items
- Demand rate is constant in each period.
- Same cartons or pallet boxes are used for different items. Thus order quantities must be a multiple of a fixed-sized batch.
- All truck has same capacity.
- No volume discount.
- Holding, Ordering and Shortage costs are considered.

3.2 Notations

The following parameters are decision variables used for items $i = 1, 2, \ldots\ldots\ldots\ldots\ldots, n.$
n: number of items to be purchased
Q_i: order quantity of the ith item (decision variable)
D_i: annual demand of the ith item
S_i: ordering cost per ordering an item
H_i: unit inventory holding cost for item i

I_i: shortage level of the *i*th item
E: Green House Gas (GHG) emission level
C_t: fixed emission tax cost
P_i : Number of carton or pallet boxes for an order of item *i*
C_p: truck capacity
A_i: required storage space per unit of the *i*th item
F: total available warehouse space
T_i: shipping cost per unit of demand
U_e: upper bound on total GHG emission
L_i: annual per-unit cost of shortages of the *i*th item
B_i: purchasing cost per unit of item
M: total budget

Based on the above assumptions and notations, the mathematical model of the problem is derived in the next section.

4. Mathematical Model Formulation

4.1 Objective Functions

The first objective function of the problem, the total inventory cost, is obtained as

$$Z_1 = \text{Total Inventory Cost} \tag{1}$$

$$= \text{Total Ordering Cost} + \text{Total Holding Cost} + \text{Total Shortage Cost}$$

$$+ \text{Total Carbon Emission Cost} + \text{Total Transportation Cost}$$

where each part is derived as follows.

$$\text{Total Ordering Cost, OC} = \sum_{i=1}^{n} \frac{D_i}{Q_i} S_i \tag{2}$$

$$\text{Total Holding Cost, HC} = \sum_{i=1}^{n} \frac{H_i}{2Q_i} (Q_i - I_i)^2 \tag{3}$$

$$\text{Total Shoratge Cost, SC} = \sum_{i=1}^{n} \frac{L_i}{2Q_i} {I_i}^2 \tag{4}$$

$$\text{Total Carbon Emission Cost, TE} = \sum_{i=1}^{n} E D_i C_t \tag{5}$$

$$\text{Total Transportation Cost, TC} = \sum_{i=1}^{n} D_i T_i \tag{6}$$

So the 1st objective of this problem is as follows.

$$Z_1 = \sum_{i=1}^{n} \frac{D_i}{Q_i} S_i + \frac{H_i}{2Q_i} (Q_i - I_i)^2 + \frac{L_i}{2Q_i} {I_i}^2 + E D_i C_t + D_i T_i \tag{7}$$

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2nd objective of this problem is to minimize the warehouse space required. That is:

$$Z_2 = \sum_{i=1}^{n} (Q_i - I_i)\, A_i \tag{8}$$

4.2 The Constraints

There are four non-equality constraints and some non-negativity constraints.
Carbon emission has an upper limit above which tax cost becomes higher (Roozbeh Nia et al., 2015):

$$\sum_{i=1}^{n} E \leq U_e \tag{9}$$

Since the total available budget is M and purchasing cost per unit is B_i, budget constraint is given below (Mousavi et al., 2014):

$$\sum_{i=1}^{n} B_i Q_i \leq M \tag{10}$$

Order capacity have some limitations:

$$\frac{D_i}{Q_i} \leq C_p \tag{11}$$

Warehouse have some space constraints:

$$\sum_{i=1}^{n} A_i Q_i \leq F \tag{12}$$

Non-negativity constraints are:

$$Q_{i,} P_{i,} I_{i,} > 0 \tag{13}$$

Where, *i = 1, 2, ……………, n;* where n is the number of items.

4.3 Final Model

Final mathematical model of the total inventory control is to

$$\text{Minimize,} \quad TOF = \omega Z_1 + (1-\omega) Z_2 \tag{14}$$

Where,

$$Z_1 = \sum_{i=1}^{n} \frac{D_i}{Q_i} S_i + \frac{H_i}{2Q_i}(Q_i - I_i)^2 + \frac{L_i}{2Q_i} {I_i}^2 + E D_i C_t + D_i T_i$$

$$Z_2 = \sum_{i=1}^{n} (Q_i - I_i)\, A_i$$

Subject to,

$$\sum_{i=1}^{n} E \leq U_e$$

$$\sum_{i=1}^{n} B_i Q_i \leq M$$

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$$\frac{D_i}{Q_i} \leq C_p$$

$$\sum_{i=1}^{n} A_i Q_i \leq F$$

$$Q_{i,} P_{i,} I_{i,} > 0$$

Where, *i = 1, 2, ……………, n;* where n is the number of items.

5. The Proposed Algorithm

In this research a modified version of PSO algorithm named multi-objective particle swarm optimization (MOPSO) is used. The purpose of using this algorithm is its simplicity. It is easy to implement and has the ability to deal with multiple conflicting objectives.

5.1 Multi Objective Particle Swarm Optimization (MOPSO)

In order to solve multi objective optimization problem, PSO needs some modifications. The first one is not to find one "global best" solution, but a set of solutions comprising the Pareto Front. After that an archive of non-dominated solutions is to kept, where all non-dominated solutions found at each iteration are stored. Inspired by the work of Coello Coello and Lechuga (2002), the detailed formulation is as follows.

5.2 Swarm Initialization

For particle *i*, position vector is x_i, which is a member of search space X where, $x_i(t) \in X$. Here, *t* is the time index to distinguish between discrete time steps and it shows the iteration number of the algorithm. Every particles in the swarm have velocity denoted by $v_i(t)$ which is a vector and belongs to the same space.

By interacting and learning from each other, every particle find their personal best denoted by $p_i(t)$ called the local best solution. There is a common best experience among the members of the swarm denoted by *g(t)* called the global best solution.

5.3 Mathematical model of motion

Initial position of the particle i is $x_i(t)$ and velocity is $v_i(t)$. Particles move toward the personal best and then to the global best and gain an updated position denoted by $x_i(t+1)$ and the addition of these beginning and end vector has an velocity of $v_i(t+1)$. So the equation for the position is-

$$x_i(t+1) = x_i(t) + v_i(t+1) \quad (15)$$

where,

$$v_i(t+1) = wv_i(t) + C_1\big(p_i(t) - x_i(t)\big) + C_2\big(g(t) - x_i(t)\big) \quad (16)$$

A simplified approach is used to standardize the PSO equation and that is-

$$v_i(t+1) = wv_i(t) + C_1 r_1 \left(x_{pbesti} - x_i(t)\right) + C_2 r_2 \left(x_{gbest} - x_i(t)\right) \quad (17)$$

where,

$w =$ inertia coefficient
$C_1, C_2 =$ accleration coefficients
$r_1, r_2 \in (0,1)$

Pseudocode of MOPSO (Mousavi et al., 2014) algorithm is as follows.

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```
for i = 1 to Pop
    initialize position (i)
    initialize velocity (i)
    if position (i) and velocity (i) be a feasible candidate solution
        penalty = 0
    else penalty = a positive number
    end if
end for
w = [0.4, 0.9]
do while Iter <= Gen
    for j = 1 to Pop
        Calculate new velocity of the particle
        Calculate new position of the particle
        pbest (iter) = min (pbest(i))
    end for
    gbest (iter) = min (gbest)
    w = w_max – ((w_max - w_min)/iter_max) × iter
    modifying the velocity and position of the particle
end while
```

Pseudocode 1: Pseudocode of MOPSO algorithm

6. Results and Analysis

To find near-optimum solution MOPSO algorithm is coded in MATLAB 15a. The obtained Pareto front is presented in Figure 1. The parameter values are presented in Table 1. The outcomes of this solution process are- Pareto front of all local optimum solutions, optimum solution for both objectives and related parameter values, total elapsed time to reach solution and mathematical formulation of EOQ for raw material inventory control.

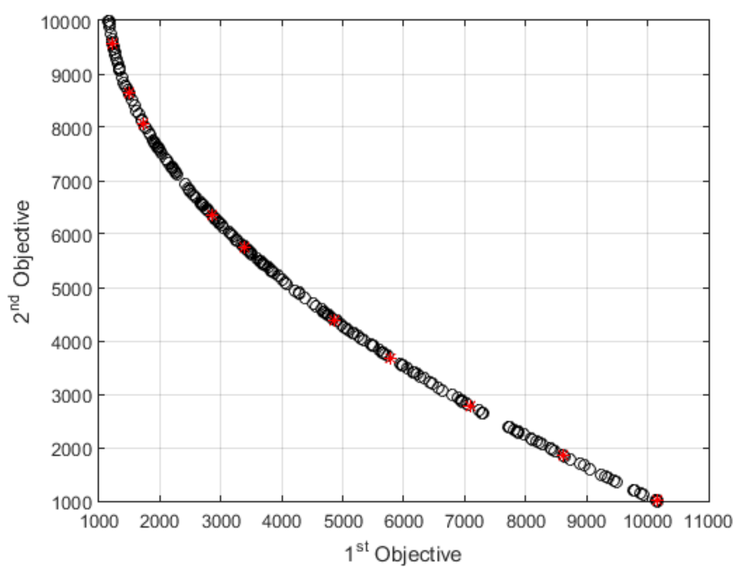

Figure 1: Pareto front of MOPSO

Table 1: Parameter values of MOPSO Pareto front

Iteration No.	C_1	C_2	Pop	Rep	1st Objective	2nd Objective	Elapsed time
200	2.5	1.5	300	10	10160	1000	44.625 s

From equation (7),

$$TC = \frac{D_i}{Q_i}S_i + \frac{H_i}{2Q_i}(Q_i - I_i)^2 + \frac{L_i}{2Q_i}I_i{}^2 + ED_iC_t + D_iT_i$$

Differentiating by Q, we get the equation of economic quantity to order for keeping raw material inventory. Thus,

$$EOQ = \sqrt{\frac{2DS}{H} + \frac{LI^2}{H} + I^2} \tag{18}$$

After choosing four factors for the algorithm, three level of value is selected for each factor based on parameter values of the algorithm from Table 1 in order to implement Taguchi L_9 design. These factors and their levels are shown in Table 2. As a result, nine different combinations of parameter value shown in Table 3 and S/N ratio for parameter levels are obtained using Minitab 18. At the end, from the mean S/N ratio plot shown in Figure 2 the optimal level of parameters' value is chosen along with their optimal values of the algorithm which are shown in Table 4.

Table 2: Parameters of MOPSO algorithm and their levels

Algorithms	Factors	Levels [1 2 3]
MOPSO	C_1 C_2 Pop Rep	[1.5 2 2.5] [1.5 2 2.5] [100 200 300] [10 30 50]

Table 3: Taguchi L_9 design along with their objective values

Run No.	A	B	C	D	MOPSO
1	1	1	1	1	10273
2	1	2	2	2	10155
3	1	3	3	3	10253
4	2	1	2	3	10156
5	2	2	3	1	10225
6	2	3	1	2	10163
7	3	1	3	2	10356
8	3	2	1	3	10250
9	3	3	2	1	10192

Table 4: The optimal levels of the algorithms' parameters

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Algorithms	Factors	Optimal Levels	Optimal Values
MOPSO	C_1 C_2 Pop Rep	2.5 1.5 300 10	10160

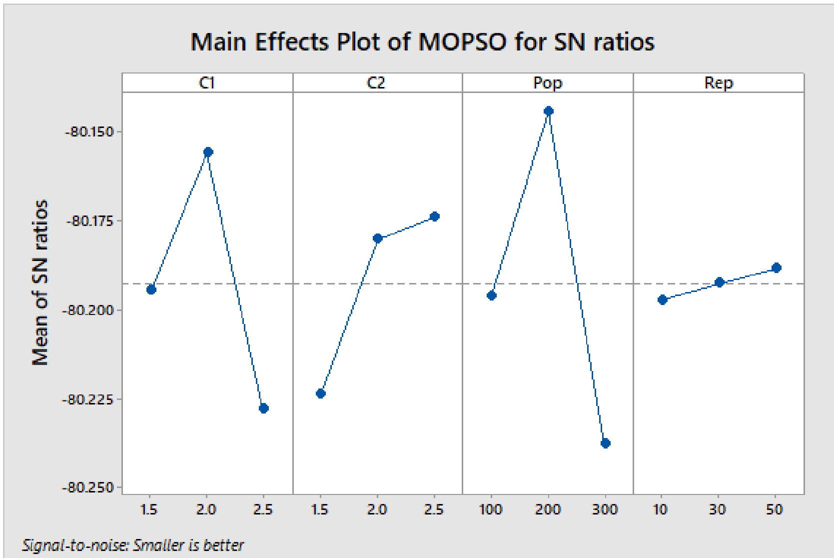

Figure 2: The mean S/N ratio plot for parameter levels of MOPSO

The results obtained with the optimal level of parameters and the one with Pareto front of MATLAB formulation show no difference. It means that MOPSO is capable of finding best result for the proposed inventory control model.

7. Conclusion and recommendation for future work

In this paper a multi-item inventory control problem with limited budget was investigated with the goals of minimizing both the total inventory cost and total required storage space. Independent demand rates of items with shortage considering no volume discount. The aim was to determine optimal order quantity such that objective function is minimized and constraints hold. The developed nonlinear programming model was solved by Pareto based multi-objective particle swarm optimization algorithm. Taguchi L_9 design was applied to calibrate the parameters of the algorithm and the combination that best suited to the objective was chosen.

Some recommendations for future work are to develop a probabilistic model using fuzzy and stochastic demand, to consider volume discount, lead time uncertainty, defective items, inflation and time value of money and other performance metrics and to apply recently developed meta-heuristic nature inspired algorithms to solve the problem.

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