Proceedings of the International Conference on Industrial Engineering and Operations Management
Bangkok, Thailand, March 5-7, 2019

Price-quantity-setting mixed duopoly models: market opening

Fernanda A. Ferreira
Polytechnic Institute of Porto, School of Hospitality and Tourism,
Applied Management Research Unit (UNIAG)
Rua D. Sancho I, 981, 4480-876 Vila do Conde, Portugal
faf@esht.ipp.pt

Flávio Ferreira
Polytechnic Institute of Porto, School of Hospitality and Tourism,
Applied Management Research Unit (UNIAG)
Rua D. Sancho I, 981, 4480-876 Vila do Conde, Portugal
fff@esht.ipp.pt

Abstract

Usually, market models analyse competition between firms with either quantity or price as decision's variables. This paper considers mixed duopoly competitions in which a state-owned public firm and a private firm produce complementary goods. We analyse, separately, the model in which the state-owned public firm sets the price and the private firm chooses the output production; and the model in which the state-owned public firm sets the output production and the private firm chooses the price. By considering domestic and international models, we analyse the effects of opening the market to a foreign firm. We also compare the results obtained with different types of decision variables. This paper contributes to the understanding of the implications of firms' decisions on social welfare. As a result, the paper shows that, in the domestic competition, social welfare is higher when the state-owned public firm sets price and the domestic private firm sets production outputs than in other competitions. Furthermore, when the market is opened to foreign firms, social welfare is higher when both firms set prices than in other competitions.

Keywords
Operations research; Game Theory; social welfare; market opening

1. Introduction

The analysis of mixed oligopoly models has received significant attention in recent years. We can find such mixed markets in industries such as telecommunications, electricity, natural gas, airlines industries, as well as services including hospitals, banking and education. The usual models consider either competitions on quantities or competitions on prices (see, for instance, DeFraja and Delbono (1989), Hashimzade et al. (2007)).
The internationalization markets is also a issue of interested. For instance, Fjel and Pal (1996) consider a mixed oligopoly model in which a public firm competes with both domestic and foreign private firms.
Also, usually, the literature focus attention on models with firms producing imperfect substitutable, or differentiated, goods, and pay less attention to models with firms producing complementary goods (see Ohnishi (2010a) and Ohnishi (2010b)).
Ohnishi (2010a) considers mixed duopoly models with firms producing imperfect substitutable goods, and compares the equilibrium outcomes of the results obtained in domestic and international competitions. Ohnishi (2010b) study the same issue, but by considering that the firms produce complementary goods.
Ferreira and Ferreira (2018) extend the work of Ohnishi (2010a) to a triopoly market and with an imported tariff fixed by the home government.
In this paper, we analyse, separately, the model in which the state-owned public firm sets the price and the private firm chooses the output production; and the model in which the state-owned public firm sets the output production and the private firm chooses the price (for models with different decision variables, see, for instance, Ohori (2014)).

We also study, separately, domestic markets and international markets. We solve and compare the four models. In addition, our results are compared with the findings of price-price competition model studied by Ohnishi (2010). This paper contributes to the understanding of the implications of firms' decisions on social welfare.
The remainder of this paper is organized as follows. In Section 2 and 3, we study, respectively, the domestic and international price-quantity-setting models. In Section 4 and 5, we study, respectively, the domestic and international quantity-price-setting models. Sections 6 recall the results for domestic and international price-price-setting models. In Section 7, we compare the results of the different market structures.

2. Domestic mixed duopoly: price-quantity competition

We consider a market competition with one state-owned public firm F_S and one domestic private firm F_D, that produce complementary goods. We assume that the representative consumer maximizes

$$U(q_S, q_D) - p_S q_S - p_D q_D,$$

where q_i is the quantity of the good produced by firm F_i and p_i is the its price, with $i = S, D$ [1]. The function U is assumed to be quadratic, strictly concave and symmetric in q_S and q_D:

$$U(q_S, q_D) = a(q_S + q_D) - \frac{1}{2}\left(q_S^2 + 2bq_S q_D + q_D^2\right),$$

where $a > 0$ and $b \in (-1, 0)$ is a measure of the degree of complementary among goods produced by both firms.
For simplicity, we assume $a = 1$ $b = -0.5$. So, the direct demand is characterized by

$$q_i = \frac{2}{3}\left(3 - 2p_i - p_j\right),$$

and, therefore, the inverse demand function is given by

$$p_i = 1 - q_i + \frac{1}{2}q_j,$$

where $i, j = S, D$ with $i \neq j$.
The profit function π_i of firm F_i is given by

$$\pi_i = (p_i - c)q_i,$$

where $c > 0$ is the marginal production cost of each firm. We assume $1/3 < c < 1$ to assure that the production outputs and prices are positive.
Domestic social welfare *W*, which is defined as the sum of consumer surplus *CS* and production surplus, is given by

$$W = CS + \pi_S + \pi_D,$$

where

$$\begin{aligned} CS &= \frac{2}{3}\left(p_S^2 + p_S p_D + p_D^2 + 3\left(1 - p_S - p_D\right)\right) \\ &= \frac{1}{2}\left(q_S^2 - q_S q_D + q_D^2\right). \end{aligned}$$

The state-owned public firm F_S aims to maximize domestic social welfare *W*, while domestic private firm F_D aims to maximize its own profit π_D.
In this section, we suppose that the state-owned public firm chooses price p_S and the domestic private firm sets the production output q_D.
The state-owned public firm F_S solves the optimization problem $\max_{p_S} W$, and the private firm solves the optimization problem $\max_{q_D} \pi_D$. By solving the system

[1] Throughtout the paper, we use the notation subscript *S* to refer the state-owned public firm and *D* to refer to the domestic private firm.

$$\begin{cases} \dfrac{\partial W}{\partial p_S} = -\dfrac{2(2p_S + p_D - 3c)}{3} = 0 \\ \dfrac{\partial \pi_D}{\partial q_D} = \dfrac{q_S - 2(2q_D + c - 1)}{2} = 0 \end{cases}$$

we find the optimal quantities[2]

$$q_S^{H,p-q} = \frac{3c-1}{2} \text{ and } q_D^{H,p-q} = 1,$$

and the prices of each good

$$p_S^{H,p-q} = 2(1-c) \text{ and } p_D^{H,p-q} = 1-c.$$

Furthermore, firms' profits, consumer surplus and social welfare are as follows:

$$\pi_S^{H,p-q} = -(1-c)^2,\ \pi_D^{H,p-q} = (1-c)^2,$$

$$CS^{H,p-q} = \frac{3(1-c)^2}{2},\ W^{H,p-q} = \frac{3(1-c)^2}{2}.$$

We note that the domestic private firm gets higher profits than the state-owned public firm.

3. International mixed duopoly: price-quantity competition

In this section, we consider a competition between a domestic state-owned public firm F_S, that chooses price p_S, and a foreign private firm F_F, that sets the production output q_F. Both firms produce complementary goods. Domestic social welfare is now given by

$$W = CS + \pi_S.$$

As in the previous section, we solve the system

$$\begin{cases} \dfrac{\partial W}{\partial p_S} = \dfrac{4(c - p_S)}{3} = 0 \\ \dfrac{\partial \pi_F}{\partial q_F} = \dfrac{q_F - 2(2q_D + c - 1)}{2} = 0 \end{cases}.$$

Thus, we get the optimal quantities

$$q_S^{I,p-q} = \frac{10(1-c)}{7} \text{ and } q_F^{I,p-q} = \frac{6(1-c)}{7},$$

and the prices of each good

$$p_S^{I,p-q} = c \text{ and } p_F^{I,p-q} = \frac{6(1-c)}{7}.$$

Furthermore, firms' profits, consumer surplus and social welfare are as follows:

$$\pi_S^{I,p-q} = 0,\ \pi_F^{I,p-q} = \frac{36(1-c)^2}{49},$$

$$CS^{I,p-q} = \frac{38(1-c)^2}{49},\ W^{I,p-q} = \frac{38(1-c)^2}{49}.$$

We note that the foreign private firm gets higher profits than the state-owned public firm.

[2] Throughtout the paper, we use the notation superscript *H* and *I*, to refer the domestic and international competition, respectively; and we use the notation superscript *p-q*, *q-p* and *p-p* to refer the price-quantity-setting, quantity-price-setting and price-price-setting competitions, respectively.

4. Domestic mixed duopoly: quantity-price competition

Now, we consider a similar domestic market competition as in Section 2, with the following differences: the domestic state-owned public firm F_S chooses production output q_S, and the domestic private firm F_D sets price p_D. Domestic social welfare is given by

$$W = CS + \pi_S + \pi_D .$$

The state-owned public firm F_S solves the optimization problem $\max_{q_S} W$, and the private firm solves the optimization problem $\max_{p_D} \pi_D$. By solving the system

$$\begin{cases} \dfrac{\partial W}{\partial q_S} = -\dfrac{2(2q_S - q_D + 2c - 2)}{3} = 0 \\ \dfrac{\partial \pi_D}{\partial p_D} = -\dfrac{2(p_S + 4p_D - 2c - 3)}{3} = 0 \end{cases}$$

we find the optimal quantities

$$q_S^{H,q-p} = \frac{3(1-c)}{2} \text{ and } q_D^{H,q-p} = 1-c ,$$

and the prices of each good

$$p_S^{H,q-p} = c \text{ and } p_D^{H,q-p} = \frac{3+c}{4} .$$

Furthermore, firms' profits, consumer surplus and social welfare are as follows:

$$\pi_S^{H,q-p} = 0 , \ \pi_D^{H,q-p} = \frac{3(1-c)^2}{4} ,$$

$$CS^{H,q-p} = \frac{7(1-c)^2}{8} , \ W^{H,q-p} = \frac{7(1-c)^2}{8} .$$

We note that the domestic private firm gets higher profits than the state-owned public firm.

5. International mixed duopoly: quantity-price competition

In this section, we consider a competition between a domestic state-owned public firm F_S, that chooses production output q_S, and a foreign private firm F_F, that sets the price p_F. Both firms produce complementary goods. Domestic social welfare is given by

$$W = CS + \pi_S .$$

As in the previous section, we solve the system

$$\begin{cases} \dfrac{\partial W}{\partial q_S} = -q_S - c + 1 = 0 \\ \dfrac{\partial \pi_F}{\partial p_F} = -\dfrac{2(p_S + 4p_F - 2c - 3)}{2} = 0 \end{cases} .$$

Thus, we get the optimal quantities

$$q_S^{I,q-p} = 1-c \text{ and } q_F^{I,q-p} = \frac{6(1-c)}{7} ,$$

and the prices of each good

$$p_S^{I,q-p} = \frac{3+4c}{7} \text{ and } p_F^{I,q-p} = \frac{9+5c}{14} .$$

Furthermore, firms' profits, consumer surplus and social welfare are as follows:

$$\pi_S^{I,q-p} = \frac{3(1-c)^2}{7} , \ \pi_F^{I,q-p} = \frac{27(1-c)^2}{49} ,$$

$$CS^{I,q-p} = \frac{43(1-c)^2}{98},\ W^{I,q-p} = \frac{85(1-c)^2}{98}.$$

We note that the foreign private firm gets higher profits than the state-owned public firm.

6. Price-price competition

Ohnishi (2010) studied domestic (resp. international) Bertrand mixed duopoly competition, in which a state-owned public firm and a domestic (resp. foreign) private firm produce complementary goods. From his paper we get

$$\pi_S^{H,p-p} = \frac{6(1-c)(c-1)}{7},\ \pi_D^{H,p-p} = \frac{48(1-c)^2}{49},$$

$$CS^{H,p-p} = \frac{37(1-c)^2}{49},\ W^{H,p-p} = \frac{43(1-c)^2}{49},$$

$$\pi_S^{I,p-p} = 0,\ \pi_F^{I,p-p} = \frac{3(1-c)^2}{4},$$

$$CS^{I,p-p} = \frac{7(1-c)^2}{8},\ W^{I,p-p} = \frac{7(1-c)^2}{8}.$$

7. Comparisons

In this section, we compare some of the results presented in the previous sections. Next proposition follows directly from the results above.

Proposition 1. At equilibrium, we have the following:

$$\pi_S^{H,p-q} < \pi_S^{H,p-p} < \pi_S^{H,q-p},\ \pi_S^{I,p-q} = \pi_S^{I,p-p} < \pi_S^{I,q-p},$$
$$\pi_D^{H,q-p} < \pi_D^{H,p-p} < \pi_D^{H,p-q},\ \pi_F^{I,q-p} < \pi_F^{I,p-q} < \pi_F^{I,p-p},$$
$$CS^{H,p-p} < CS^{H,q-p} < CS^{H,p-q},\ CS^{I,q-p} < CS^{I,p-q} < CS^{I,p-p},$$
$$W^{H,q-p} < W^{H,p-p} < W^{H,p-q},\ W^{I,p-q} < W^{I,q-p} < W^{I,p-p}.$$

Proposition 1 states that (i) in the domestic competition, social welfare is higher when the state-owned public firm sets price and the domestic private firm sets production outputs than in other competitions; (ii) in the international competition, that is when the market is opened, social welfare is higher when both firms set prices than in other competitions.

8. Numerical example

In this section, we present a numerical example for different values of the parameter c that represents the unit production costs. This numerical example illustrates the previous results.

$c = 0.5$		$c = 0.9$	
Private firm	State-pwned public firm	Private firm	State-pwned public firm
$\pi_D^{H,p-q} = 0.25$	$\pi_S^{H,p-q} = -0.25$	$\pi_D^{H,p-q} = 0.01$	$\pi_S^{H,p-q} = -0.01$
$\pi_D^{H,q-p} = 0.1875$	$\pi_S^{H,q-p} = 0$	$\pi_D^{H,q-p} = 0.0075$	$\pi_S^{H,q-p} = 0$
$\pi_D^{H,p-p} = 0.2449$	$\pi_S^{H,p-p} = -0.2143$	$\pi_D^{H,p-p} = 0.0098$	$\pi_S^{H,p-p} = -0.0086$
$\pi_F^{I,p-q} = 0.1837$	$\pi_S^{I,p-q} = 0$	$\pi_F^{I,p-q} = 0.0073$	$\pi_S^{I,p-q} = 0$
$\pi_F^{I,q-p} = 0.1378$	$\pi_S^{I,q-p} = 0.1071$	$\pi_F^{I,q-p} = 0.0055$	$\pi_S^{I,q-p} = 0.0043$

$\pi_F^{I,p-p} = 0.1875$	$\pi_S^{I,p-p} = 0$	$\pi_F^{I,p-p} = 0.0075$	$\pi_S^{I,p-p} = 0$

Acknowledgements
UNIAG, R&D unit funded by the FCT – Portuguese Foundation for the Development of Science and Technology, Ministry of Science, Technology and Higher Education, under the Project UID/GES/04752/2019.

References

DeFraja, G., and Delbono, F., Alternative strategies of a public enterprise in oligopoly, *Oxford Economic Papers*, vol. 41, pp. 302-311, 1989.

Ferreira, F. A., and Ferreira, F., Privatization and subsidization in a quantity-price-setting mixed duopoly, *Proceedings of the International Conference on Mathematical Methods and Computational Techniques in Science and Engineering*, Cambridge, UK, February, 2018. American Institute of Physics, vol. 1982, pp. 020024-1–020024-6, 2018a.

Ferreira, F. A., and Ferreira, F., Domestic and international price-setting mixed triopolies, *Journal of Applied Nonlinear Dynamics*, vol. 7, pp. 259-267, 2018b.

Fjell, K., and Pal, D., A mixed oligopoly in the presence of foreign private firms, Canadian Journal of Economics, vol. 29, pp. 737-743, 1996.

Ohnishi, K., Domestic and international mixed models with price competition, *International Review of Economics*, vol. 57, pp. 1-7, 2010.

Ohnishi, K., Price-setting mixed duopoly models with complementary goods, *Modern Economy*, vol. 1, pp. 43-46, 2010.

Ohori, S., Price and Quantity competition in a mixed duopoly with emission tax, *Theoretical Economics Letters*, vol. 4, pp. 133-138, 2014.

White, M., Mixed oligopoly, privatization and subsidization, *Economics Letters*, vol. 53, pp.189-195, 1996.

Biographies

Fernanda A. Ferreira is a fulltime associate professor in the Department of Mathematics at the School of Hospitality and Tourism of Polytechnic Institute of Porto, Portugal. She holds a BS in Mathematics and a PhD in Applied Mathematics from the University of Porto. She obtained also a Diploma of Advanced Studies in Statistics and Operations Research from Vigo University. A member and Coordinator of the Applied Management Research Unit (UNIAG), her publications, mostly journal and conference papers, cover the research interest areas of industrial organization, game theory and tourism (ORCID ID: orcid.org/0000-0002-1335-7821). Co-author of two books published in the Mathematics area. Supervised many dissertations in the areas of Game Theory and Management and participated in projects/researches. She also organizes Symposium on "Operational Research and Applications" in several International Conferences and has collaborated as reviewer with several journals.

Flávio Ferreira is Full Professor and the Dean of the School of Hospitality and Tourism of the Polytechnic Institute of Porto, Portugal. He earned PhD in Applied Mathematics from University of Porto. Flávio Ferreira is member of the Applied Management Research Unit (UNIAG). His research interests include industrial organization and game theory. He is co-author of a book published by Springer-Verlag, and author or co-author of more than 40 journal papers and conference proceedings.