New Valid Inequalities for Capacitated Plant Location Problem

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Abstract

With the contribution of Sharma and Muralidhar (2009) to problem Simple Plant Location Problem (SPLP), we now have two weak formulations and a strong formulation SPLP. It is noted that the total number of constraints in the weak formulation is linear, whereas the number of constraints in the strong formulation go quadratic. Though relaxed weak formulations give the inferior bound but their computational time is much less compared to the relaxed strong formulation of SPLP. Many researchers observed that this computational advantage of the weak formulation is significant for smaller sized problems however for large sized problems bound provided by it is poor. Hence hybrid formulations are proposed by many. We propose the similar model for capacitated plant location problem (CPLP). We modify the Sharma and Muralidhar (2009) weak formulations and add to them only the most promising strong constraints to have advantages of both. Here in this paper, we also introduce an additional constraint that the number of plants located is less than some number 'n' or greater than 'n+1'and hope that this will lead to significantly better LP relaxation bounds (we hope that this improvement will be dependent on the value of 'n'). Two criteria for computing 'n (n_1 and n_2)' are also explored. We further modify the formulation by adding one more constraint based on the assumption of opening up all plants to have bound based on simple transportation problem. This is also done with the hope of getting better bounds without causing significant computational complexity addition.

Key Word: CPLP, Weak Formulation, Strong Formulation

1. Introduction

Plant/facility location is a vital decision in determining a geographical site for the operation of any manufacturing or service industry. It is a complex process including tangible and intangible factors. The intangible factors vary from one site to another and therefore hard to measure whereas the tangible factors like construction cost, labor cost, availability of raw materials, proximity to market and suppliers, market demand, etc. can be found. In literature, the tangible cost can be divided into two categories. First, the location-specific fixed cost and the second being the transportation cost. This problem when further constrained by the production capacity of the plant is defined as capacitated plant location problem (CPLP). Like in an automobile manufacturing unit, the product of cycle time and some active assembly lines determines the capacity of that unit. The CPLP is a well-studied optimization problem belonging to the class of the NP-Hard problems for which a variety of heuristic, meta-heuristic approaches are available in the literature. We provide different formulations to compare the efficacy of the constraints that give superior LP relaxation.

2. Problem Formulation

Indices used:

i : index for plants; where i = 1,2,3,…., I;
I: set of possible number of plants
k : index for markets; where k = 1,2,3,…., K;
K: set of possible number of markets

Variable Definition:

x_{ik}: Quantity received by the market 'k' from the plant 'i' as a fraction of the total market demand
Therefore, $x_{ik} = X_{ik}/\sum_k D_k$
X_{ik}: Quantity received by the market 'k' from the plant 'I'
y_i: Location variable (=1 If the plant is located at point 'I' and, 0 otherwise)

Constants:

D_k : The demand at market 'k'.
d_k: The demand at market 'k' as a fraction of the total market demand.
Therefore, $d_k = D_k/\sum_k D_k$
f_i: Fixed cost of locating a plant at the potential location 'i'.
C_{ik}: Cost of transporting all the markets' demands ($\sum_k D_k$) from plant 'i' to market 'k'.
CAP_i : Max quantity that can be delivered by the plant 'i' as a fraction of the total market demand
Therefore, $CAP_i = (Total\ Plant\ Capacity\ at\ 'i')/\sum_k D_k$

Objective Function:

$$\text{Minimize } Z = \sum_i \sum_k (C_{ik} * x_{ik}) + \sum_i (f_i * y_i) \quad \ldots\ldots\ldots (1)$$

Subject to

$$\sum_i \sum_k x_{ik} = 1 \quad \ldots\ldots\ldots (2)$$

$$-(\sum_i x_{ik}) \geq -d_k \quad \forall k \quad \ldots\ldots\ldots (3)$$

$$y_i \geq \sum_k x_{ik} \quad \forall i \quad \ldots\ldots\ldots (4)$$

$$d_k * y_i - x_{ik} \geq 0 \quad \forall i,k \quad \ldots\ldots\ldots (5)$$

$$\sum_k x_{ik} \leq CAP_i \quad \forall i \quad \ldots\ldots\ldots (6)$$

$$\sum_i (CAP_i * y_i) \geq \sum_k d_k \quad \ldots\ldots\ldots (7)$$

$$x_{ik} \geq 0 \quad \forall i,k \quad \ldots\ldots\ldots (8)$$

$$y_i \in \{0,1\} \quad \forall i \quad \ldots\ldots\ldots (9)$$

Here, constraint (4) is the weak constraint and constraint (5) is the strong constraint.

Weak / Strong Formulation

Four formulations of CPLP are proposed here which comprises different constraints:

Formulation 1: Weak Formulation (or Standard CPLP Model)

Minimize (1); subject to (2) to (4), and (6) to (9).

Formulation 2: Most Promising Strong Formulation (Adding a few Strong Constraints)

Minimize (1); subject to (2) to (4), (5a), and (6) to (9).

Here we have modified the strong constraint (5) (proposed by Sharma and Muralidhar, 2009) as constraint (5a) and added to Formulation 1 such as,

$d_{k*} * y_i - x_{ik*} \geq 0 \qquad \forall\, i, k*$ ……… (5a)

where $d_{k*} = minset\ (d_k)$.

For $minset\ or\ k*$ values we may add first five or smallest 5% d_k values as constraint (5a) instead of all d_k .

Formulation 3: Adding a few Strong Constraints based on location and transportation cost intensity.

Sharma and Jha (2018) introduced two strong constraints that does not affect significantly the computational complexity by creating a capacity set of plants(*SET-CAP*) based on maximum possible plant locations under the given constraint scenario. In the case where transportation cost is a much smaller number compared to other costs such as location cost, the deciding factor will be the location cost. In such situations, the number of plants will be smaller and vice versa if the transportation costs are very high. Let 'n_1' be the estimated maximum number of plants that must be opened for problem CPLP (determined by their heuristic that compares the ratio of transportation and fixed costs). We get an estimate of n_1 by keeping on adding plants as per ascending capacity till the total capacity is just greater than total demand. This is achieved by the following constraints (10) and (11) by adding one more binary variable 'z_1'.

$\sum_i y_i - M * z_1 \leq n_1$ ………. (10)

$\sum_i y_i + M * (1 - z_1) \geq (n_1 + 1)$ ………. (11)

Where z_1 is the binary variable.

Another way to generate the set *SET-CAP* is we can keep on adding plants as per smallest fixed/location cost until total capacity is just greater than total demand. The cardinality of *SET-CAP* can be a good estimate of 'n' (say 'n_2'). The idea being that number of plants estimate 'n_1 or n_2' chosen should be as close to optimal 'n' so that we can get the maximum computational advantage (at least in implicit enumeration scheme due to Erlenkotter). Note the number of plants whether less than or greater than n_2 will be subject to the circumstances (same as n_1) based on transportation and location cost comparisons.

$\sum_i y_i - M * z_2 \leq n_2$ ………. (12)

$\sum_i y_i + M * (1 - z_2) \geq (n_2 + 1)$ ………. (13)

Where z_2 is again a binary variable.

Then we can have another strong formulation as,

Minimize (1); subject to (2) to (13).

We note that these constraints become more effective when we have tailor-made implicit enumeration procedure (due to Erlenkotter); and also note standard solvers do not give users that kind of flexibility.

To this one can also add 5(a) for a beneficial result.

Formulation 4: Determination of lowest transportation cost (min_TrC) and adding related constraint

Let min_TrC = Minimum transportation cost, when plants are opened at all possible locations.

Then a constraint can be added to the formulation such as,

$\sum_{i,k} (C_{ik} * x_{ik}) > \min\_TrC$ ………. (14)

In above, compute $\sum_i \mathrm{f_i} = FIX1$; wherever there is positive outflow from an 'i'.

Then we add the following $\sum_i \mathrm{f_i} * y_i \leq FIX1$ ………….. (15)

Then another strong formulation can be,

Minimize (1); subject to (2) to (15)

Again to this one can also add 5(a) for further beneficial result.

Finally we do the following addition to strong constraints:

Take 'i$_s$', such that f_i/CAP_i is smallest, and add to a set say PO (plant open). Keep doing it until for 'i' belonging to *PO,* it has sum of included CAPi just greater than or equal to 1 . Compute $FIX2 = \sum_{i\in PO} f_i$, and add the following constraint

$\sum_i \mathrm{f_i} * y_i \geq FIX2$ ……….. (16)

Let the associated TrC (with (16)) be min_TrC2; then along the lines of (15) we give the following constraint:

$\sum_{i,k} \ (C_{ik} \ * \ x_{ik}) <= \min\_TrC2$ ………….. (17)

We can use Singh and Sharma algorithm (2008) to get a good solution to this problem (say SPS*) and put an additional constraint (18) to get a good solution.

obj fn value $Z \ <= \ SPS^*$ ... ... (18)

3. Discussion

It is noted that various inequalities for the problem CPLP (as developed in this paper) are linear; And are expected to boost the LP relaxation bounds in a Branch and bound solution procedure. In particular, we plan to solve three formulations for each problem set, CPLP-weak, CPLP-strong, and CPLP- weak plus most promising strong plus various other inequalities given in this paper

We are undertaking an empirical investigation that plans to determine the relative efficacy of different constraints that give superior LP relaxations.

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Biographies

Dr R.R.K. Sharma has had 30 years of career to date. Started as graduate engineer trainee with TELCO (Pune) (now Tata Motors India) during 1980-82, and later went on to do PhD. in management at I.I.M., Ahmadabad, India. After Ph. D. in management, he worked with TVS Suzuki (for 9 months) as executive assistant to GM (marketing). Now he has 26 years of teaching and research experience at the department of Industrial and Management Engineering, I.I.T., Kanpur, 208 016 India. He has taught over 22 different courses in management at IIT Kanpur India (to B. Tech., M. Tech. and M.B.A. students) and is well versed with all the facets of management and has unique ability to integrate different areas of the subject. To date he has written over 507 (grand total) publications (223 Full Length Papers and 287 Extended Abstracts Outlining Theoretical Framework) in international/national journals and six research monographs. He has developed over 8 software products. Till date he has guided 58 M TECH and 15 PhD theses at IIT Kanpur. He has guided 129 Special Studies Projects for MBA IInd year students of IME, IIT Kanpur. He has been Sanjay Mittal Chair Professor at IIT KANPUR (15.09.2015 to 14.09.2018).

Ajay Jha is currently a fulltime research scholar at Indian Institute of Technology Kanpur. Mr. Jha holds a Bachelor of Technology degree in Mechanical Engineering from Harcourt Butler Technological Institute, Kanpur and a Master of Technology in Industrial and Management Engineering from Indian Institute of Technology, Kanpur. He has rich experience of production and marketing domains of over 10 years and also of teaching Mechanical Engineering and Operations Management courses of 10 years. His research areas include Supply Chain Management and Strategy.

Himanshu Rahangdale is a full-time student of Master of Technology in Industrial and Management Engineering program at Indian Institute of Technology. He did Bachelor of Engineering degree in mechanical engineering from Bhilai Institute of Technology, Durg, India. He has also worked for Jindal Steel and Power Limited, Raipur, India as Assistant Engineer (Projects) in the duration of November 2014 to June 2016. His interest areas include Operations Management and Advance statistics.