New Valid Inequalities for Simple Plant Location Problem

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Abstract

Plant location is a crucial decision in meeting the customer demand efficiently and effectively. The researchers have given different formulations for solving this optimization problem and such problems are NP Hard due to various integer variables and non linear formulation. Sharma and Muralidhar (2009) approach to model relaxed Simple Plant Location Problem (SPLP) resulted in two weak formulations and a strong formulation. It is found that weak formulations give an inferior bound compared to strong formulation but their computational time is comparatively very less. The reason being the constraints in strong formulations go quadratic in number. It has been observed by many that for smaller sized problems, the weak formulation of SPLP is superior to Strong formulation of SPLP in terms of computational time. Here hybrid formulations are proposed to accommodate benefits of both the formulations. We take full weak formulation (either weak one or weak two) and add to it only the most promising strong constraints. It is claimed that this approach (hybrid) is better than weak and strong formulation of SPLP in terms of upper bound and computational time. We also provide additional strong constraints based on various transportation and location costs assumptions that can improve the relaxed solutions of the SPLP.

Key Word: SPLP, Weak Formulation, Strong Formulation, Integer Relaxation

1. Introduction

In supply chain network planning a key decision is to determine the set of potential sites for the location of plants/facilities. The objective of the facility location problem is to determine sufficient number of facilities (such as plants, warehouse, fire station etc.) in order to minimize the total cost. This problem deals with the locating the plants with unlimited capacities to serve certain markets demand (also known as simple plant location problem (SPLP)). The costs involved in meeting the customer demands include the fixed costs of setting up plants, and the transportation cost of supplying customers from these plants to the demand destinations. SPLP is well studied in literature and is considered to be the NP Hard problem. Literature discusses various exact and heuristic techniques for solving. Here we provide different formulations that compare the efficiency of various modified constraints to find the formulation that gives the best non-integer/relaxed solution.

2. Problem Formulation

Indices used:

i : index for plants; where i = 1,2,3,…., I;
I :set of possible number of plants
k : index for markets; where k = 1,2,3,…., K;
K : set of possible number of markets

Variable Definition:

x_{ik}: Quantity received by the market "k" from the plant "i" as a fraction of the total market demand
Therefore, $x_{ik} = X_{ik} / \sum_k D_k$
X_{ik}: Quantity received by the market "k" from the plant "i"

y_i: Location variable (=1 If the plant is located at point "i" and, 0 otherwise)

Constants:

D_k : The demand at market "k"
d_k: The demand at market "k" as a fraction of total market demand.
Therefore, $d_k = D_k / \sum_k D_k$
f_i: Fixed cost of locating a plant at potential location "i".
Plants are assumed to have unlimited capacities.
C_{ik}: Cost of transporting all the markets' demands ($\sum_k D_k$) from plant "i" to market "k".

Objective Function:

Minimize $Z = \sum_i \sum_k (C_{ik} * x_{ik}) + \sum_i (f_i * y_i)$ .... (1)

Subject to

$\sum_i \sum_k x_{ik} = 1$ .... (2)

$-(\sum_i x_{ik}) \geq -d_k \quad \forall k$ .... (3)

$y_i \geq \sum_k x_{ik} \quad \forall i$ .... (4)

$d_k * y_i - x_{ik} \geq 0 \quad \forall i, k$ .... (5)

$y_i - \frac{\sum_k (x_{ik}/d_k)}{K} \geq 0 \quad \forall i$ ....(6)

$x_{ik} \geq 0 \quad \forall i, k$ .... (7)

$y_i \in \{0,1\} \quad \forall i$ .... (8)

Here, constraint (4) and (6) are the weak constraints and constraint (5) is the strong constraint

Weak / Strong Formulations

Four formulations of SPLP are proposed here which comprises of different constraints:

Formulation 1: Weak Formulation (or Standard SPLP)

Minimize (1); subject to constraints (2) to (4), and (6) to (8).

Formulation 2: Most Promising Strong Formulation (Adding a few Strong Constraints)

Minimize (1); subject to constraints (2) to (4), (5a), and (6) to (8).

Here we have modified the strong constraint (5) (proposed by Sharma and Muralidhar, 2009)) as constraint (5a) and added to Formulation 1 such as,

$d_{k*} * y_i - x_{ik*} \geq 0 \quad \forall i, k*$ .... (5a)

where $d_{k*} = minset(d_k)$.

Note for $minset$ or $k*$ values we may add first five or less smallest dk values as constraint (5a) instead of all d_k .

Formulation 3: Adding a few Strong Constraints based on location and transportation cost intensity.

Let 'n' be the estimated maximum number of plants that must be opened for problem SPLP (determined by heuristic given by Sharma and Jha (2018) that compares the ratio of transportation and fixed costs). In the case when transportation cost is a much smaller number (compared to other costs such as location cost), then 'n' will be smaller (and vice-versa)). This is achieved by following constraints by adding one more binary variable 'z'.

$$\sum_i y_i - M * \ z \leq n \qquad \forall i \qquad \text{..........} \ (9)$$

$$\sum_i y_i + M * (1 - z) \geq (n + 1) \qquad \forall i \qquad \text{..........} \ (10)$$

Where z is the binary variable.

Then, we have another strong formulation as

Minimize (1); subject to constraints (2) to (4), and (6) to (10)

We note that constraints will be more effective when we have tailor made implicit enumeration procedure (due to Erlenkotter); and also note standard solvers do not give users that kind of flexibility.

To this one can also add 5(a) for beneficial result.

Formulation 4: Determination of lowest transportation cost (min_TrC) and adding related constraint

Let min_TrC = Minimum transportation cost, when plants are opened at all possible locations

Then a constraint can be added to the formulation such as,

$$\sum_{i,k}(C_{ik} \ * \ x_{ik}) \ > \min\_TrC \qquad \text{.................} \ (11)$$

In above, compute $\sum_i \mathrm{f_i} = FIX1$, wherever there is positive outflow from an 'i'.

Then we add following $\sum_i \mathrm{f_i} * y_i \leq FIX1$ ………….. (12)

Hence we have another strong formulation as

Minimize (1); subject to constraints (2)-(4), (6) to (8), (11) and (12).

We note that to this constraint (5a) can be added, again for fruitful results.

It is advised to use Singh and Sharma algorithm (2008) to get a good solution to this problem (say *SPS**) and put an additional constraint (13) as bound on objective function value Z, to get a good solution.

$$Z \ <= \ SPS^* \qquad \text{………………} \ (13)$$

3. Conclusion

Here in this paper we introduce an additional constraint that number of plants to be located is less than some number 'n' or greater than 'n+1' based on contribution of transportation or fixed location cost in total objective function. We hope that this will lead to significantly better LP relaxation bounds. We also provide an additional constraint based on minimization of transportation cost (min_TrC) under the assumption all plant locations are utilized. This accommodated in formulation of SPLP will speed up computational performance.

Here we wish to compare weak formulation of SPLP (constraint (2) to (4), & (6) to (8); excluding constraint (5)) with various strong formulations of SPLP (constraint (2) to (4), (5a) & (6) to (13)). It is to be noted that all constraints are linear in nature and therefore we expect significantly better LP relaxation bounds without significant increment in the complexity.

We are undertaking an empirical investigation that plans to determine relative efficacy of different constraints in terms of better LP relaxations.

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Biographies

Dr R.R.K. Sharma has had 30 years of career to date. Started as graduate engineer trainee with TELCO (PUNE) (Now Tata Motors India) During 1980-82, And Later Went On To Do Phd. In Management At I.I.M., Ahmadabad, India. After Ph. D. in management, he worked with TVS Suzuki (for 9 months) as executive assistant to GM (marketing). Now he has 26 years of teaching and research experience at the department of Industrial and Management Engineering, I.I.T., Kanpur, 208 016 India. He has taught over 22 different courses in management at IIT Kanpur India (to B. Tech., M. Tech. and M.B.A. students) and is well versed with all the facets of management and has unique ability to integrate different areas of the subject. To date he has written over 507 (grand total) publications (223 Full Length Papers and 287 Extended Abstracts Outlining Theoretical Framework) in international/national journals and six research monographs. He has developed over 8 software products. Till date he has guided 58 M TECH and 15 PhD theses at IIT Kanpur. He has guided 129 Special Studies Projects for MBA IInd year students of IME, IIT Kanpur. He has been Sanjay Mittal Chair Professor at IIT KANPUR (15.09.2015 to 14.09.2018).

Ajay Jha is currently a fulltime research scholar at Indian Institute of Technology Kanpur. Mr. Jha holds a Bachelor of Technology degree in Mechanical Engineering from Harcourt Butler Technological Institute, Kanpur and a Master of Technology in Industrial and Management Engineering from Indian Institute of Technology, Kanpur. He has rich experience of production and marketing domains of over 10 years and also of teaching Mechanical Engineering and Operations Management courses of 10 years. His research areas include Supply Chain Management and Strategy.

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