

Optimal Control Applied to an Economic Model for Reducing Unemployment Problems in Bangladesh

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Abstract

In this paper, a nonlinear mathematical model of unemployment is described and analyzed for reduction of unemployed persons using two controls ‘creation of new vacancies’ and ‘skilled manpower’. For maximization of the number of employed people at the end of implemented policy, we find the control profile of two controls using Pontryagin’s Maximum Principle. This study concludes two effective controls so that the number of unemployed persons at the end of implemented policies of the government is minimized while minimizing the systematic cost of policy making. Our numerical results as illustrated in the figures strongly agree with the analytical findings.

Keywords: Unemployment, Socio-economic damage, Mathematical model, Optimal control, Numerical simulation.

1. Introduction

Unemployment problems have become the most immense concerns all over the world. This issue is significantly an alarming concern in Bangladesh as well. Unemployment can be defined as a state of workless for a person, who is fit, capable and willing to work. It is well known that the main reason for socio-economic damage, deletion of morality and social values is unemployment. With the increasing tendency of unemployed population in a country significantly affects the other factors, such that the income per person, health costs, quality of healthcare and poverty and seriously hamper the growth of national economy. In this paper, we propose a nonlinear mathematical model to assess the unemployment problems in Bangladesh. Optimal control techniques have been applied by introducing two controls, viz. ‘*creating new vacancies*’ and ‘*providing skilled manpower*’, in the proposed model to reduce the unemployed persons. The model has been analyzed both analytically and numerically. The analytical findings have been validated with the numerical simulations. According to World Employment Social Outlook of ILO in 2017, Global unemployment levels and rates are predicted to remain high in the short term, as the global labour force continues to grow. In particular, the global unemployment rate is predicted to rise modestly in 2017 to 5.8 per cent (from 5.7 per cent in 2016) representing 3.4 million more unemployed people globally (bringing total unemployment to just over 201 million in 2017). So unemployment is one of the most serious issues for every country.

Several authors have contributed to different mathematical models to analyze and design optimal control strategies for unemployment problem. According to most of them, the population of developing countries has increased enormously but new opportunities for employment have not increased in the same proportion. So creating new opportunities is a priority for any vibrant economy. In 2011, assuming all entrants to category of the unemployed are fully qualified and competent to do any job, Misra and Singh (2013; 2011) discussed and analyzed a system of nonlinear ordinary differential equations for the control of unemployment. It is also considered that the number of unemployed persons increases continuously at a constant rate. Having motivated by (Misra and Singh 2011), Pathan and Bhatahwal (2015) proposed a model for reduction of unemployment problem with self-employment. In that model, they considered no time delay by government and private sector in creating new vacancies. In 2016, Munoli and Gani (2016) proposed an unemployment model and discussed the optimality conditions for optimal solutions of

that nonlinear mathematical model. We refer readers to (Munoli *et al.* 2017; Nikolopoulos and Tzanetis 2003; Pathan and Bhatahwala 2017; Sirghi and Neamtu 2014) for more details and some related works on unemployment problems.

For modeling process, the number of unemployed persons, employed persons and vacancies are denoted by $U_s(t)$, $E_s(t)$ and $V_s(t)$ respectively at any time t . We know that the change of unemployed persons depend on new faces in job market, recruited persons against vacancies, migrated persons etc. Similarly, employed persons are increased for recruitment and vacancies are changed for recruited persons against vacancies, creation of new vacancies. So, we introduce a new concept to reduce unemployed persons for providing skilled manpower (Mallick and Biswas 2017). From this conception, we formulate the following system of nonlinear differential equations:

$$\dot{U}_s(t) = A - (kV_s(t) + \alpha_1 + \alpha_3)U_s(t) + \gamma E_s(t), \quad U_s(0) = U_0 \quad (1)$$

$$\dot{E}_s(t) = (kV_s(t) + \alpha_3)U_s(t) - (\alpha_2 + \gamma)E_s(t), \quad E_s(0) = E_0 \quad (2)$$

$$\dot{V}_s(t) = (\alpha_2 + \gamma)E_s(t) - \delta V_s(t) - kV_s(t)U_s(t) + \phi U_s(t), \quad V_s(0) = V_0 \quad (3)$$

where, all parameters are described in Table 1.

Table 1: Values and explanation of parameters

Parameters	Value	Explanation
A	5000	The constant rate of new faces in Jobs market
k	0.00009	Employed rate for vacancies
α_1	0.04	Rate of migration and death of unemployed
α_2	0.05	Rate of retirement and death of employed
α_3	0.05	Rate of skilled person from training
γ	0.001	Rate of persons who fired from their jobs
ϕ	0.007	Rate of creating new vacancies
δ	0.05	Diminution rate of vacancies for lack of funds

At present, the demand of skilled manpower has increased. So demands of time, policy of government provides skilled manpower with nurturing talent pool within the educational systems. So, we choose the control $u_1(t)$ for the implemented policies of government to provide skilled manpower from unemployed population with nurturing talent pool within the educational systems or training or motivating program. Hence, $u_2(t)$ is the control of creation of new vacancies. So, we assume that the following system of nonlinear differential equations for unemployment problem:

$$\dot{U}_s(t) = A - (kV_s(t) + \alpha_1 + \alpha_3 u_1(t))U_s(t) + \gamma E_s(t), \quad U_s(0) = U_0 \quad (4)$$

$$\dot{E}_s(t) = (kV_s(t) + \alpha_3 u_1(t))U_s(t) - (\alpha_2 + \gamma)E_s(t), \quad E_s(0) = E_0 \quad (5)$$

$$\dot{V}_s(t) = (\alpha_2 + \gamma)E_s(t) - \delta V_s(t) - kV_s(t)U_s(t) + \phi u_2(t)U_s(t), \quad V_s(0) = V_0 \quad (6)$$

We want to find the control strategies so that the number of employed at end of implemented policy of government is maximized while minimizing the cost of policy making. Hence, we are maximizing the difference. Thus, the objective functional is chosen to be

$$\text{Maximize } J(u_1, u_2) = \int_{t_s}^{t_f} (A_1 E_s(t) - B_1 u_1^2(t) - B_2 u_2^2(t)) dt. \quad (7)$$

Here the cost function is a nonlinear function of $u_1(t)$ and $u_2(t)$; we choose them quadratic cost function for concavity. Since the right hand sides of the state equations are linearly bounded with respect to $u_1(t)$ and $u_2(t)$, these bounds ensure the compactness needed for the existence of the optimal control (Biswas 2014; Biswas *et al.*, 2014; Fleming and Rishel 1975; Neilan and Lenhart 2010). So, we can apply the Pontryagin's Maximum Principle (Pontryagin *et al.*, 1962) in our proposed model for optimal solution. Also, the parameters $A_1, B_1, B_2 \geq 0$ represents the desired 'weights' on the achievement and systematic cost. Our aim is to find the control profile $u_1^*(t)$ and $u_2^*(t)$ of satisfying

$$\max \left\{ J(u_1(t), u_2(t)) \mid 0 \leq u_1(t), u_2(t) \leq 1 \right\} = J(u_1^*(t), u_2^*(t)) .$$

2. Analysis of the Mathematical Model without Control Variables

We analyze the model (1)-(3) without control variables using stability theory of differential equations. For this reason, we find the region of attraction (Munoli and Gani 2016) for the model in the form of the following Lemma 1.

Lemma 1: The feasible set

$$\Omega = \left\{ (U_s(t), E_s(t), V_s(t)) : 0 \leq U_s(t) + E_s(t) \leq \frac{A}{\alpha}; 0 \leq V_s(t) \leq \frac{A(\psi + \alpha)}{\alpha\delta} + \frac{\phi}{\delta} \right\},$$

where, $\alpha = \min(\alpha_1, \alpha_2)$ and $\psi = \max(-\alpha_1, \gamma)$ is a region of attraction for the mathematical model and attracts all solutions initiating in the interior of positive octant.

Proof: Adding the first and second equation of the model (1)-(3), we get

$$\begin{aligned} \dot{U}_s(t) + \dot{E}_s(t) &= A - \alpha_1 U_s(t) - \alpha_2 E_s(t) \\ \Rightarrow \dot{U}_s(t) + \dot{E}_s(t) &= A - \alpha \{U_s(t) + E_s(t)\} \text{ where } \alpha = \min(\alpha_1, \alpha_2) \end{aligned}$$

Taking the limit supremum, we get

$$\lim_{t \rightarrow \infty} \sup \{U_s(t) + E_s(t)\} \leq \frac{A}{\alpha}$$

Similarly, from the third equation of the model, we get

$$\lim_{t \rightarrow \infty} \sup \{V_s(t)\} \leq \frac{A(\psi + \alpha)}{\alpha\delta} + \frac{\phi}{\delta}.$$

3. Optimal Control Model for Unemployment Problem

To reformulate the optimal controlled model (4)-(6) with objective functional (7), we write as follows:

$$\text{Maximize } \int_{t_s}^{t_f} L(t, x(t), u(t))$$

subject to

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t), \forall t \in [t_s, t_f]$$

$$u(t) \in U(t), \forall t \in [t_s, t_f]$$

$$x(0) = x_0$$

where, $x(t) = (U_s(t), E_s(t), V_s(t))$, $L(t, x(t), u(t)) = A_1 E_s(t) - B_1 u_1^2(t) - B_2 u_2^2(t)$,

$$f(x) = \begin{pmatrix} A - kU_s(t)V_s(t) - \alpha_1 U_s(t) + \gamma E_s(t) \\ kU_s(t)V_s(t) - \alpha_2 E_s(t) - \gamma E_s(t) \\ \alpha_2 E_s(t) + \gamma E_s(t) - \delta V_s(t) - kU_s(t)V_s(t) \end{pmatrix}, g(x) = \begin{pmatrix} -\alpha_3 U_s(t) & 0 \\ \alpha_3 U_s(t) & 0 \\ 0 & \phi U_s(t) \end{pmatrix}, u(t) = \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}$$

We represent the definitions of parameters and their value [5] in Table 1.

4. Evaluation of the Maximum Principle

For evaluation of the necessary optimality condition in the form of Maximum Principle (Pontryagin *et al.*, 1962; Vinter 2000) for the above optimal control problem, we have a standard Hamiltonian function defined by

$$H[x, p, u] = \lambda L(x, u) + \langle p(t), f(x) + g(x)u(t) \rangle, \quad \lambda \in \mathbb{R}$$

for the maximization of $J(u_1, u_2)$, where $p = (p_U, p_E, p_V)$ denotes the adjoint variables.

Suppose, $(x^*(t), u^*(t))$ be an optimal solution. From the maximum principle, there exists a scalar $\lambda \geq 0$, an absolutely continuous function $p(t)$ such that the following conditions hold:

- i. Nontriviality condition: $\max \{ |p(t)| : t \in [t_s, t_f] \} + \lambda > 0$
- ii. Adjoint equation: $\dot{p}(t) = -H_x[x] = -\lambda L_x[t] - \langle p(t), f_x[t] + g_x[t]u^*(t) \rangle$
- iii. Transversality condition: $p(t_f) = (0, 0, 0)$
- iv. Weierstrass condition: $H(x^*(t), p(t), u^*(t)) = \max_u \{ H(x^*(t), p(t), u(t)) \mid 0 \leq u \leq 1 \}$

Here the time argument $[t]$ denotes the evaluation along the optimal solution.

Now the adjoint equations (ii) with adjoint variables $p = (p_U, p_E, p_V)$ in normal form (*i.e.* $\lambda = 1$) are explicitly given by

$$\dot{p}_U(t) = (p_U - p_E + p_V)kV_s(t) + \alpha_1 p_U + u_1(t)\alpha_3(p_U - p_E) - \phi u_2(t)p_V \quad (14)$$

$$\dot{p}_E(t) = -\gamma p_U + (\gamma + \alpha_2)(p_E - p_V) - A_1 \quad (15)$$

$$\dot{p}_V(t) = (p_U - p_E + p_V)kU_s(t) + \delta p_V \quad (16)$$

From (iv), we get an explicit characterization of optimal control pair in terms of the multipliers $p = (p_U, p_E, p_V)$.

$$\langle p \cdot f(x^*(t)) + g(x^*(t))u^*(t) \rangle + L(x^*(t), u^*(t)) \geq \langle p \cdot f(x^*(t)) + g(x^*(t))u(t) \rangle + L(x^*(t), u(t)) \quad (17)$$

We get the optimal controls after simplifying the above inequalities (17) as

$$u_1^*(t) = \max \left\{ \min \left\{ \frac{(p_E - p_U)\alpha_3 U_s(t)}{2B_1}, 1 \right\}, 0 \right\} \text{ and } u_2^*(t) = \max \left\{ \min \left\{ \frac{\phi p_V U_s(t)}{2B_2}, 1 \right\}, 0 \right\}.$$

5. Numerical Analysis

Using forward-backward sweep method (Lenhart and Workman, 2007; Neilan and Lenhart, 2010), we solve the optimal controlled problem numerically. The numerical optimal solution of the state equations and adjoint equations with objective function have been found in MATLAB (R2014a) using the value of parameters from the Table 1 with weight parameters $A_1 = 20$, $B_1 = 45000$, $B_2 = 2500$ and initial conditions $U_s(t_s) = 10^4$, $E_s(t_s) = 10^3$, $V_s(t_s) = 10^2$ are considered same as in (Munoli and Gani, 2016). For iterative process, we consider 1152 time-grid of time 150 units and get increment of time $\Delta t = 0.13$. Since our optimal control problem is solved by indirect method, we accept convergence tolerance of cost function at 10^{-8} .

Now, firstly we discuss the solution of the system of nonlinear ordinary differential equations when both controls are set to zero that means no policy is taken. We see that the continuously increasing unemployed persons shows the asymptotical behaviour and the number of unemployed persons is 1.2383×10^5 at final time (see Fig. 1). In Fig. 2, we observe that the employed persons are decreasing and the number of employed persons is 699 at final time when both controls are set to zero that means taking no policy. We observe that the value of objective functional is 2.4589×10^6 . We also see that vacancies are increasing for some times at first but decreasing most of the time. We see the asymptotical behaviour of vacancies in Fig. 3 and it is 30.69 at final time. Since the number of unemployed persons are very high and vacancies are very low. So, it is a critical situation for any nation.

For minimization of unemployed persons, we have to need maximum production of trained person (i.e. $u_1 = 1$) and maximum investment for creation of new vacancies (i.e. $u_2 = 1$). So, in model (1)-(3), we choose two controls $u_1 = 1$ and $u_2 = 1$ for minimization of unemployed persons but not maximization of the value of objective functional (see Table 2). Since we choose the constant controls as $u_1 = u_2 = 1$, so, it is clear that no controls of both policies are used in the model. So, “no control of both policies” means constant maximum effort for reduction of unemployed persons. We simulate our model using this technique and see that the asymptotic behaviour of states in the Fig. 4. The unemployed persons are increasing up to the time 19 units and decreasing most of the time with asymptotical behaviour and the number of unemployed persons is 1.9363×10^4 at the final time. We also see that the employed persons are increasing with asymptotical behaviour and the number of employed persons is 8.4552×10^4 at final time. We also see that vacancies are increasing with asymptotic behaviour and the number of vacancies is 1.9203×10^4 at final time.

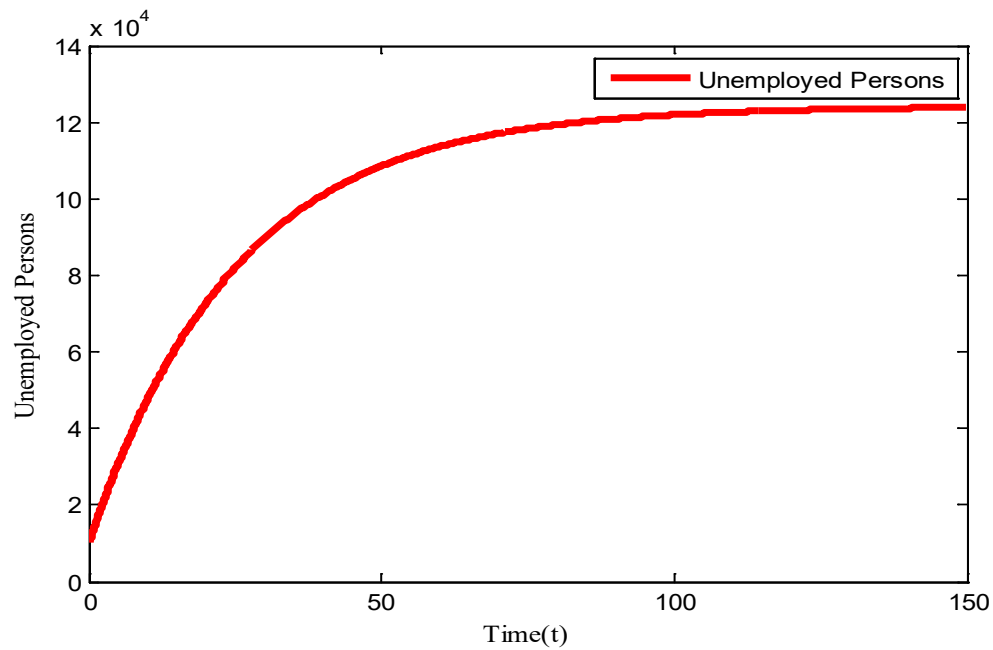


Fig. 1. Asymptotic behaviour of unemployed persons when taking no policy.

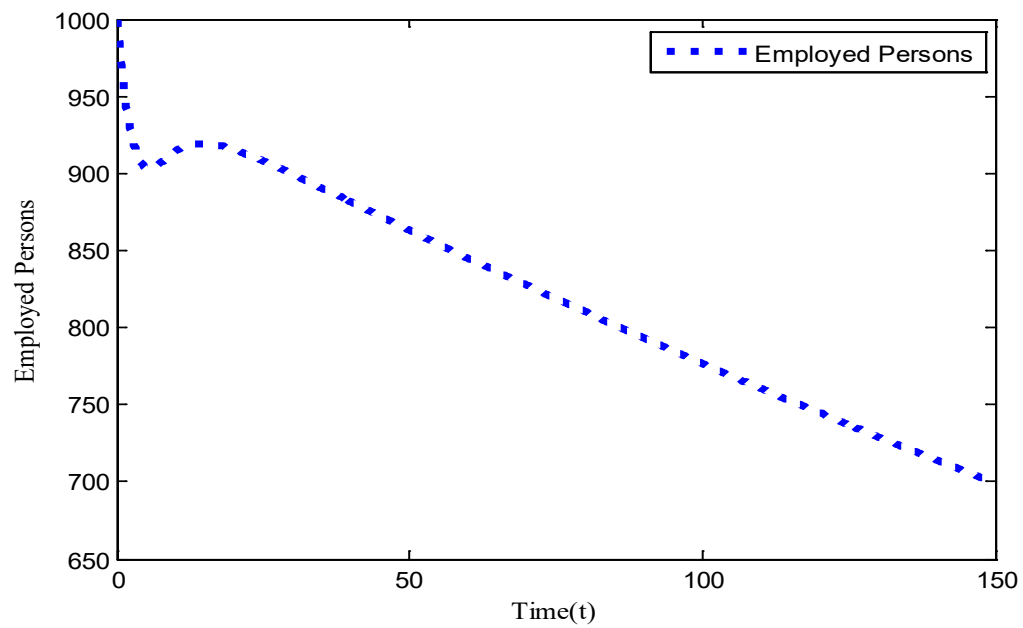


Fig. 2. Employed persons are decreasing when taking no policy.

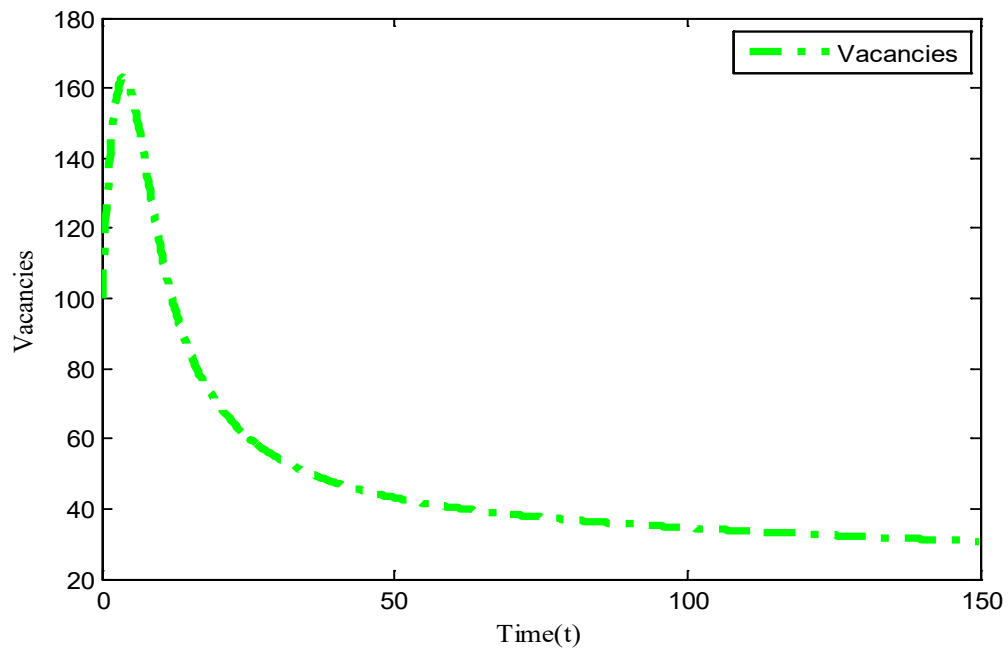


Fig. 3. Asymtotic behaviour of vacancies when taking no policy.

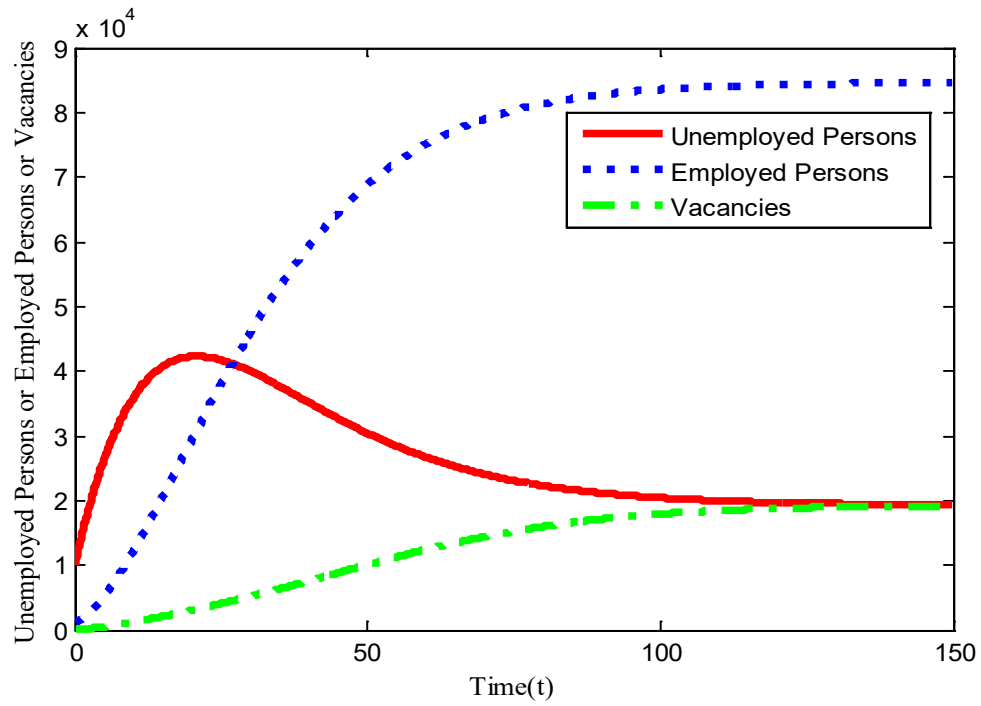


Fig. 4. State trajectories when no controls of both policies are applied.

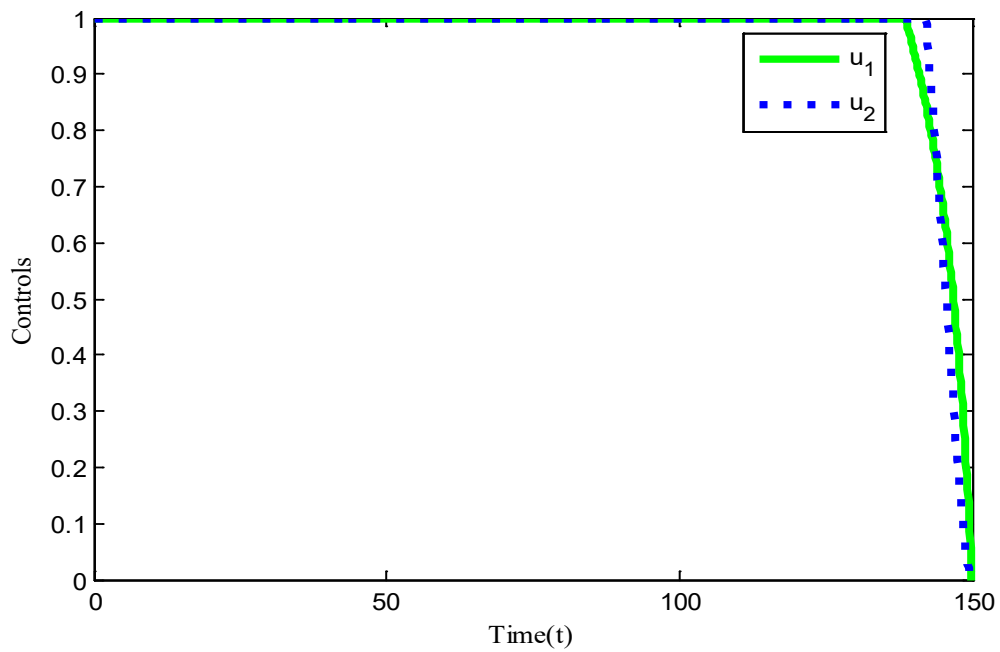


Fig. 5. Optimal control profiles for minimization of cost of policy making

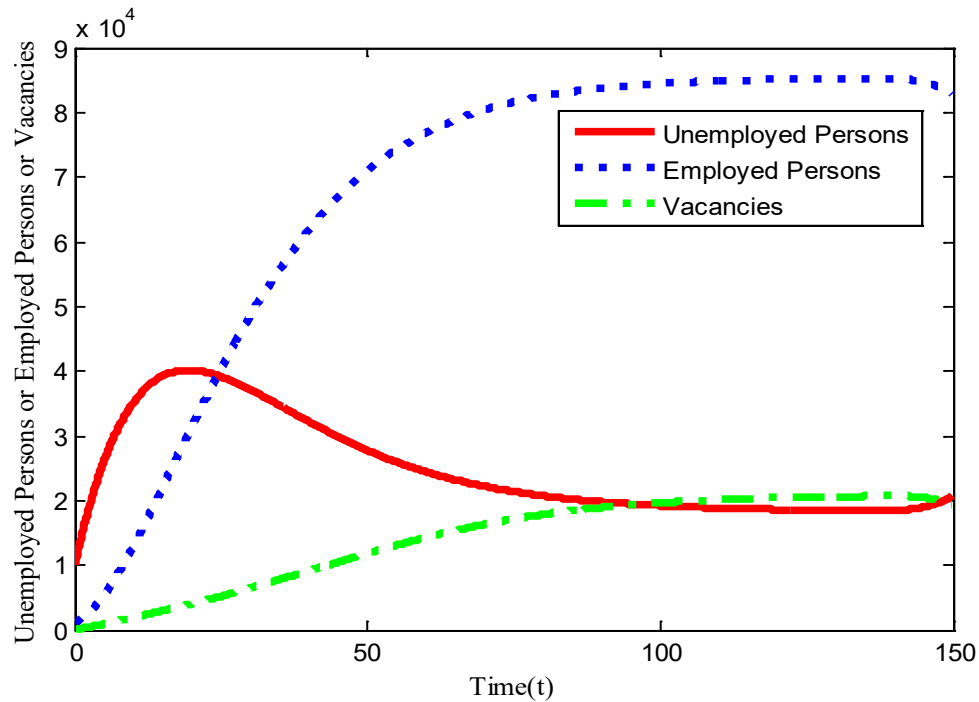


Fig. 6. Optimal state trajectories for minimization of cost of policy making.

Finally, we find the optimal solution for the policies of government as shown in Figs. 5-7 and compare to the solution without policies and no control of the policies of government. When control strategies are in use, the optimal value of objective function is 1.9655×10^8 units which is greater than the objective functional (1.9283×10^8 units) with no control of both policies.

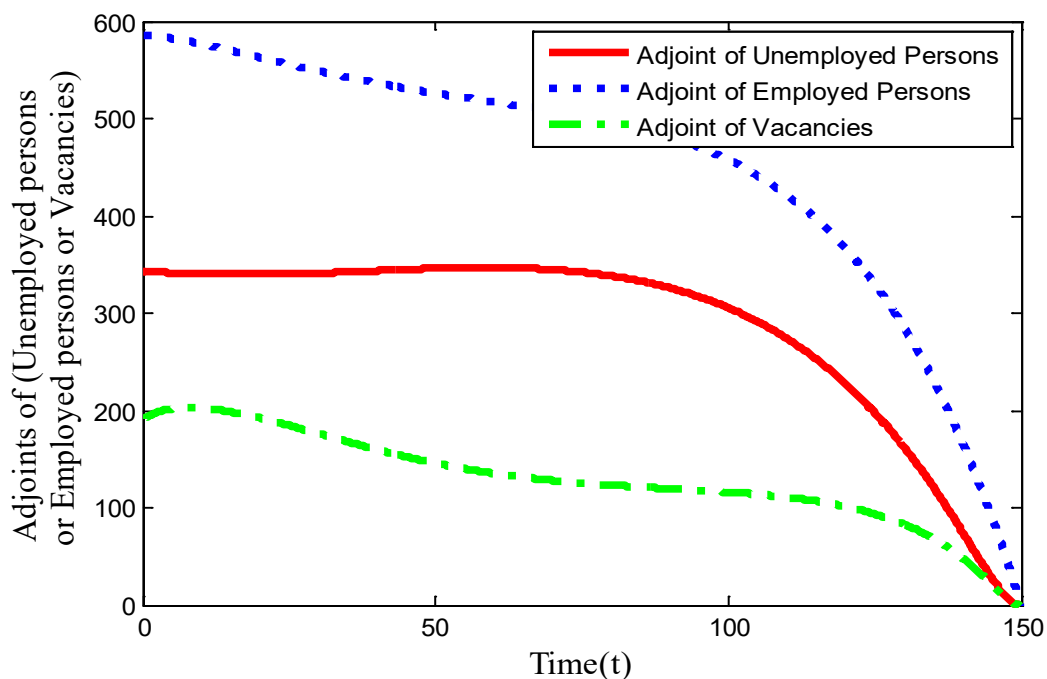


Fig. 7. Adjoint trajectories for control of both policies

Now, in Fig. 6, the optimal unemployed persons are increasing at the time 19 units and decreasing with asymptotical behaviour and the number of unemployed persons is 2.0805×10^4 at the final time. We also see that the employed persons are increasing with asymptotical behaviour and the number of employed persons is 8.2964×10^4 at final time. We also see that vacancies are increasing with asymptotical behaviour and the number of vacancies is 1.9328×10^4 at final time. Since the number of unemployed persons are near to the vacancies but greater. So, it is a desirable situation for any nation. In Fig. 7, we see that the adjoint of states satisfies the transversality condition of the optimal control problem. Because adjoint of states are zero at final time. So, the Fig. 7 ensures the completeness of optimal solution of the problem with the control profiles (see Fig. 5).

In Table 2, the objective functional is maximized for controlling of both policies of government to reduce the unemployed persons. So we decide that the control of both policies is better than no control of that policies. From this point, we believe that these control strategies can effectively reduce the unemployed persons in developing country after implementation of both policies of government. Hence we believe that the numerically analyzed control strategies of our proposed model can effectively reduce the unemployed population in the society.

Table 2: Summary of objective functional and state variables at final time

Status of Controls	Objective functional	Unemployed Persons	Employed Persons	Vacancies
$u_1 = 0, u_2 = 0$	2.4589×10^6	1.2383×10^5	6.9948×10^2	3.0692×10^1
$u_1 = 1, u_2 = 1$	1.9283×10^8	1.9363×10^4	8.4552×10^4	1.9203×10^4
$0 \leq u_1 \leq 1, 0 \leq u_2 \leq 1$	1.9655×10^8	2.0805×10^4	8.2964×10^4	1.9328×10^4

6. Conclusions

We have presented a mathematical model on unemployment introducing new vacancies and trained persons to reduce unemployed persons with maximization of objective functional using Pontryagin's maximum principle. After successful implementation of the policy of government, it is clear that the value of objective functional is maximized with the minimization of unemployed persons. We decide that the control of policies of government is more effective for maximization of objective functional with the minimization of unemployed persons than the no control of both policies. Hence, we believe that the numerically analyzed control profile of our proposed model can effectively reduce the unemployed persons in a country.

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