Incorporating Dynamic Potential market in Innovation Diffusion Model Using Stochastic Differential Equation

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Abstract

In this paper, we incorporate the idea of uncertainty in adoption process in innovation diffusion model in dynamic market. The diffusion process is sum of a deterministic component that is a function of cumulative adopters and a random fluctuation with intensity proportional to the cumulative adopters. The state of the system is modeled as a stochastic process which is governed by stochastic differential equation. The applicability and accuracy of model is illustrated by means of new product sales data. Predictive validity and mean square error have been used to check the validity of the proposed model.

Keywords:
Innovation and diffusion model, Stochastic Differential equations, random fluctuation

1. Introduction

The increased technical advancements have resulted in an outburst of production and flow of more goods and efficient services in the market. Globalization has played a major role in spreading consumer awareness, thereby making it essential for business firms to come up with new and better business ideas. With increased customer demands and expectations, there is a constant need of having new and more effective products. This process of creating something new and advanced is termed as innovation.

Once the new idea or product is formalized, the next step is to make it acceptable to the population of the respective social system. This process is termed as diffusion of Innovation. Diffusion theory analyses the causes and the pace at which the new commodity or service is being adopted by the customers in a society. These customers constitute the potential adopter population. Prediction of the adoption rate of a new product by the potential customers of the social system has been a topic of research among researchers belonging to varied fields since 1960. The most basic and popular innovation diffusion model was given by Bass (1969). This model has been further enhanced to incorporate various cases. The diffusion process gets influenced by various factors. The four important variables being the innovation itself, the means of communication via which the information is conveyed to the potential customers, the rate at which the innovation is adopted by the population and the social system comprising of individuals and organisations.

During the last few decades, the innovation diffusion models have been extensively studied and a considerable amount of research has been done by people from different fields like marketing, social sciences and economics (Easingwood, Mahajan and Muller, 1983; Mahajan and Peterson, 1985; Rogers, 1983). Numerous mathematical models have been given to study the process of diffusion of innovations which are also used for the purpose of prediction. Initially, the members of the potential adopter population are influenced by external factors to adopt the product. They are referred as innovators. Later through their word of mouth, they influence more individuals to adopt the product. These members who are influenced by internal factors are known as imitators.

The initial steps to study the diffusion process were taken by Easingwood, Mahajan and Muller (1981, 1983), Floyd (1968), Lilien, Kotler and Moorthy(1998), Mahajan , Muller and Bass (1990), Mahajan and Peterson (1978) and Sharif and Ramanathan(1981). The models undertaken were broadly categorized on the basis of three types of adopters: those which studied population consisting of only innovators, those which consist of only imitators and those which consider individuals under mixed influence. The Bass model (Bass, 1969) focuses on the timing of the first purchase of new goods, thereby examining the process of diffusion and predicting the sales growth phenomenon. This model was further modified by researchers to incorporate various fluctuations in the adoption behaviour of specific products. Lilien, Kotler and Moorthy, 1998; Mahajan and Peterson, 1978, Easingwood, Mahajan and Muller, 1981; Floyd, 1968; Fisher and Pry(1971). Kapur et al (2004) proposed an alternative form of the Bass model by using a logistic time dependent rate function.

The models considered earlier assumed that the size of the potential adopter population remains constant throughout of the product life cycle as determined in the beginning of the adoption process. But in certain cases the variability in the adoption process in innovation diffusion model may incorporate dynamic potential market. The diffusion process considered in this paper is a combination of a deterministic component that is a function of cumulative adopters and a random fluctuation with intensity proportional to the cumulative adopters. The state of the system is modelled as a stochastic process which is governed by stochastic differential equation. The applicability and accuracy of model is illustrated by means of new product sales data. Predictive validity and mean square error have been used to validate the proposed model. Noise in the adoption rate incorporating Repeat Purchasing has been studied by Chaudhary et al (2018).

The present paper is organized as follows. In section 2, we develop the framework of modeling of proposed model and solution of the proposed problem is obtained in closed form. Section 3 contains the parameter estimation results. Section 4 concludes the paper.

2. Model Development

With the advancement in technology, new products are being brought in the market regularly to cater the demands of the consumers. Prior models assumed that the size of the potential adopter population remains deterministic throughout of the product life cycle as determined in the beginning of the adoption process. The model considered here deals with a more realistic state of dynamic potential market. Here, we incorporate the effect of uncertainty in adoption system due to the size of the noise in system and the adoption process defined by $\{N(t), t \geq 0\}$ behaves as a stochastic process with continuous state space. Then adoption rate of the product with noise in system is defined by

$$\frac{dN}{dt} = \frac{b}{1+\beta_1 e^{-bt}}\left(\bar{N} - N(t)\right) + \text{"noise"}\, N(t) \tag{1}$$

Where the multiplicative "noise" term is denoted by "$\beta\,\gamma(t)$" being standard Gaussian white noise and$\beta > 0$ is the measure of the irregular fluctuations of the size of the noise of the system. Now, the uncertainty in the adoption process is reflected by the following Stochastic Differential Equation:

$$dN(t) = \frac{b}{1+\beta_1 e^{-bt}}\left(\bar{N} - N(t)\right)dt + \beta N(t) dW(t) \tag{2}$$

Where $\bar{N}$ is the expected potential adopter population, and $W(t)$ is a one-dimensional Wiener process which is formally defined as an integration of the white noise with respect to time t. Using $It\hat{o}$ formula (Oksendal,2005), solution to equation (2) with initial condition $N(0) = 0$, is given by:

$$N(t) = \bar{N} F_t^{-1} \int_0^t r(s) F_s ds \tag{3}$$

Where $F_t = \mathrm{e}^{\left(\int_0^t r(s)ds + \frac{\beta^2}{2}t - \beta W(t)\right)}$

The solution to the above equation from (3) can now be written as:

$$N(t) = \bar{N}\frac{b}{e^{bt}+\beta_1}\,\mathrm{e}^{-\frac{\beta^2}{2}t+\beta W(t)} \int_0^{\mathrm{t}} \mathrm{e}^{\frac{\beta^2}{2}s - \beta W(s) + bs}\,\mathrm{ds} \tag{4}$$

We consider the mean number adopters of product up to time *t*. As we know that the Brownian motion or Weiner Process follows normal distribution. Thus, the mean number of adopters is given as

$$E[N(t)] = \frac{\bar{N}\,b}{\left(b+\beta^2\right)} \frac{\left(1-e^{-\left(b+\beta^2\right)t}\right)}{(1+\beta_1 e^{-bt})} \tag{5}$$

3. Data Analysis and Model Validation

The proposed model has been tested and its effectiveness has been justified by estimating the parameters on sales data of three commodities namely IBM Systems-in-use Generation-I (USA), air conditioners and telephone answering machine cited in Kapur et al. (2012). Table1given below provides the sales data for these products. The marketing strategy and efforts made to promote the product are observed throughout the product's life cycle. SPSS software has been used to estimate the parameters of the given model which uses the least square technique. It is a software used for statistical analysis of data. The usefulness and relevance of the model is established by testing how well it fits the data. To test the goodness of fit of the proposed model, multiple determination (R^2) and mean square error (MSE) has been used.

3.1 Data description and estimation results

Table 1 gives the description of the datasets. The values of estimated parameter of the proposed model given by equation (5) for the all the data sets are given in Table2.

Table1: Data Description

DS-I		DS-II		DS-III	
1955	190	1	96	1	147
1956	750	2	291	2	585
1957	1750	3	529	3	1332
1958	3430	4	909	4	2795
1959	5972	5	1954	5	5441
1960	8612	6	3184	6	10559
1961	10962	7	4451	7	16336
1962	12782	8	6279	8	22318
1963	13952	9	7865	9	28280
1964	14702	10	9538	10	32911
1965	15157	11	11338		
1966	15460	12	12918		
1967	15663	13	14418		
1968	15833				
1969	15882				
1970	15911				
1971	15925				
1972	15931				
1973	15935				
1974	15939				
1975	15942				

Table2: Parameter Estimates for the Proposed Model

Parameter estimation	Data Set		
	DS-I	DS-II	DS-III
$\bar{N}$	15861.293	17173.219	38464.122
b	0.649	0.434	.688
β	.00002543	.00002094	.00002184
β_1	41.589	57.395	170.200
R^2	.999473	.999682	0.999506
MSE	16615.66	31622.17	12115.22

3.2. Goodness of Fit Curves

The following figures show the goodness of fit of the proposed model graphically.

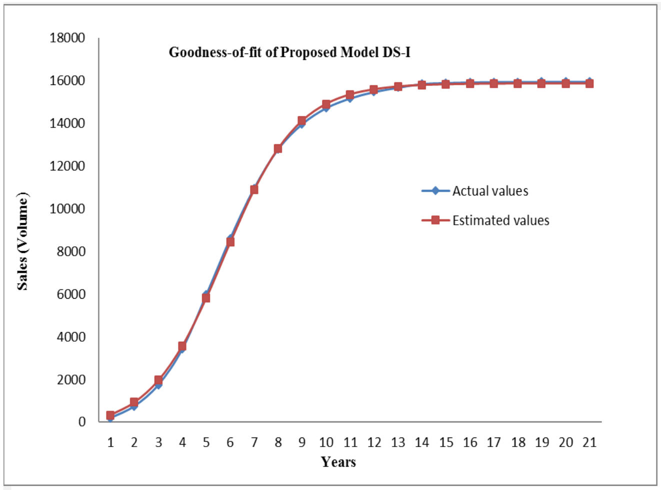


Fig 1: For DS-1

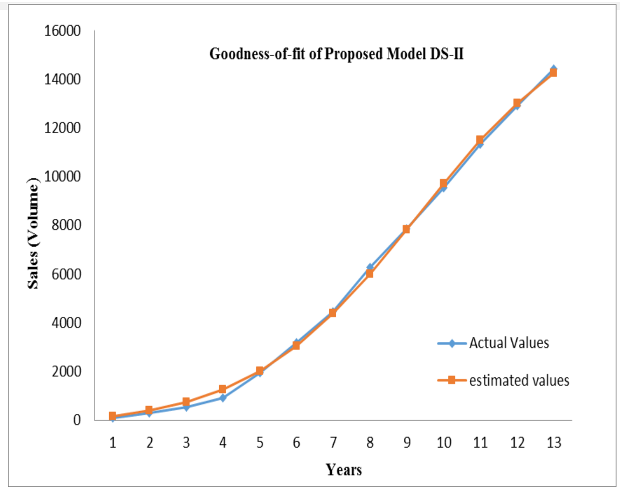
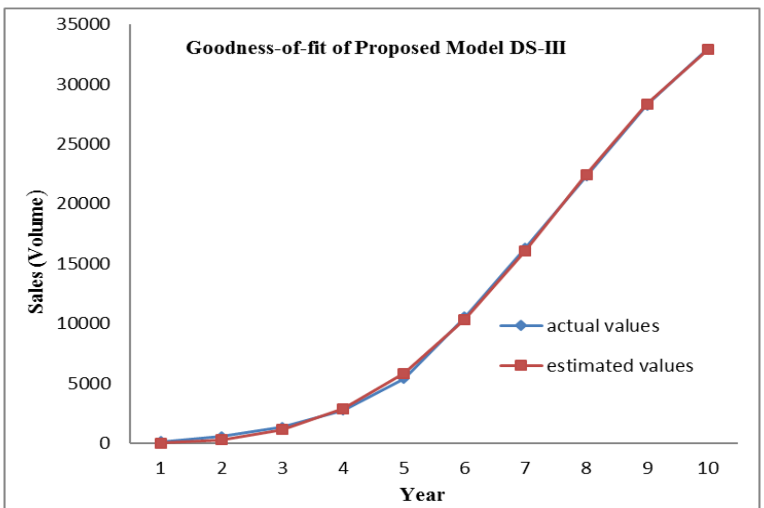


Fig 2: For DS-II



4. Discussion and Conclusion

In this paper, we proposed an innovation-diffusion model to describe the effect of multiplicative noise in system that influences on adoption process. Most of the existing models in the literature assume absence of noise in adoption process while developing the model. Here, adoption rate process is sum of a deterministic component that is a function of cumulative adopters and a random fluctuation with intensity proportional to the cumulative adopters. The state of the system is modelled as a stochastic process which is governed by stochastic differential equation. Its applicability and accuracy is illustrated by means of new product sales data. Predictive validity and mean square error have been used to check the validity of the proposed model. We summarize the estimated values of parameters of the proposed model in Table 2. It can be seen that the proposed model describes the adoption growth and fits in all the three data sets. From the Table2, we have seen that proposed model fits best in data set DS-I and DS-III with lower value of MSE and R^2 in comparison to data set DS-II.

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