Reinsurance Loss Model at Individual Risk for Short Term in Life Insurace

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Abstract

An insurance insurer has a risk of loss. The risk of loss occurs if the insurer must bear claims beyond his ability. However, the insurer can reduce this risk by utilizing reinsurance. This paper intends to examine a reinsurance loss model developed by Panjer and Willmot in 1992, to process a group of data so as to produce information needed by actuaries in making decisions. Here is described the steps that must be implemented, and the calculation process is assisted by a computer application program to make it easier. This model is used to meet the need for information on the distribution of total claims, premium stop loss and variance for various retention levels. The calculation results show this model is the most effective for the number of policies and small benefits. Because the greater the number of policies and benefits, the more number of arrays and elements needed in each iteration and computer application program. So that it can quickly produce information needed by actuaries to support decision making.

Keywords
Risk of loss, reinsurance, premium stop loss, variance, retention level.

1. Introduction

Risk is the substance of insurance. In fact, an insurer has a risk of loss. This risk of loss occurs if the insurer has to bear a claim that exceeds that ability. The risk of an insurance guarantor can be reduced using reinsurance, where another insurance guarantor (reinsurer) acts as a guarantor of insurance from an insurance company (the giver company) whose origin is at risk, this is called reinsurance (De Pril, 1986; Pacáková and Zapletal, 2013; Goffard and Laub, 2017).

Reinsurance has many forms, if a reinsurer pays an individual's loss equal to the difference in a limit; this is called reinsurance excess of loss (XL). Whereas in the case of stop-loss reinsurance, the reinsurer pays the difference in the sum of all claims combined above a limit (Bowers *et al.*, 1997; Chan and Tse, 2017).

However, it is believed that individual risk models need special attention as facts that are widely used in certain applications, especially in life and health insurance, and this is more difficult to treat mathematically, even though simple models. For this reason, further studies are needed by adding new parameters and variables to the existing models, and producing more accurate mathematical models (Burrow and Lang, 1997; Chen, 2013; Turner, et al., 2017).

This paper aims to conduct a study of adding new parameters and variables into the model, and the results of the study were applied to the analysis of case claims data on life insurance. The full discussion is described in the following sections.

2. Matematical Model

2.1 Distribution of Panjer Model Total Claims

The model is developed by Panjer (De Pril, 1986), and we will use it to produce a total distribution of claims. Suppose a portfolio consists n risk with a claim probability q_j ; $j = 1,2,...,n$ and total coverage b_j ; $j = 1,2,...,n$, the probability function of the total claim is given by,

$$f_S(x) = fx_1 * fx_2 * ... * fx_n(x) \tag{1}$$

for $x = 0,1,2,..., \sum_{j=1}^{n} b_j$.

where,

$$f_{X_j}(x) = \begin{cases} p_j = 1 - q_j, x = 0; \\ qj, x = b_j \end{cases} \tag{2}$$

the equation can be calculated recursively using partial addition $S_j = S_{j-1} + X_j$ for $j = 1,2,...,n$ start with $S_1 = X_1$. In this case the formula (1) becomes

$$f_{S_j}(x) = \begin{cases} f_{S_{j-1}}(x)f_{X_j}(0), x < b_j \\ f_{S_{j-1}}(x)f_{X_j}(0) + f_{S_{j-1}}(x - b_j)f_{X_j}(b_j), x \ge b_j \end{cases}$$
$$= \begin{cases} p_j f_{S_{j-1}}(x), x < b_j \\ p_j f_{S_{j-1}} + q_j f_{S_{j-1}}(x - b_j), x \ge b_j \end{cases} \tag{3}$$

In addition to the Panjer model, the De Pril model (De Pril, 1986) will be used for the distribution of total claims as explained in the following sections.

2.2 De Pril Model for $E[S_0]$

First, we divide the portfolio into sub-portfolios based on policy size and probability of claims (Burrow and Lang, 1997). Suppose n_{ij} symbolizes the size of the policy i (with $i = 1,2,...,r$) and probability of claims q_j (with $j = 1,2,...,m$).
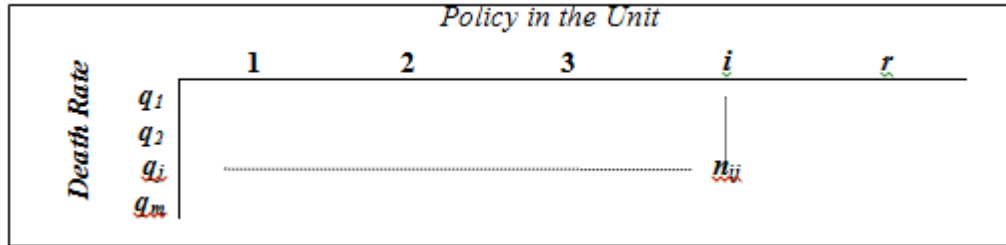

Figure 1. Model De Pril Portfolio Classification

Then, *pgf* of total claim can be written as

$$P_s(z) = \prod_{i=1}^{r} \prod_{j=1}^{m} (1 - q_j + q_j z^i)^{n_{ij}}$$

The logarithm of *pgf* is

$$\log P_S(z) = \sum_{i=1}^{r} \sum_{j=1}^{m} n_{ij} \log(1 - q_j + q_j z^i) \tag{4}$$

Now, we integrate equation (4) to produce

$$P_S'(z) = \left\{ \sum_{i=1}^{r} \sum_{j=1}^{m} iq_j n_{ij} z^{i-1} (1 - q_j + q_j z^i)^{-1} \right\} P_S(z) \tag{5}$$

If $z = 1$, on (5) the average of total claim distribution becomes

$$E(S) = P_S'(1) = \sum_{i=1}^{r} \sum_{j=1}^{m} iq_j n_{ij} \tag{6}$$

2.3 Reinsunrance Loss Model
In this case, the maximum cost incurred by the insurer for the risk of individual loss is d. Therefore, the distribution of insurer claims can be described as follows

$$f_X^1(x) = \begin{cases} f_X(x), x < d \\ 1 - F_X(d - 0), x = d \\ 0, x > d \end{cases} \tag{7}$$

where $f_X(x)$ is probability density of individual losses. Reinsurer will only suffer losses if the difference in individual losses exceeds d and reinsurer losses are as much as the excess against d. Therefore, it is likely that the reinsurer will not suffer losses for individual risks

$$p_j^* = p_j + q_j \Pr\{X \le d\} = p_j + q_j F_X(d). \tag{8}$$

Then, distribution of reinsurer losses to risk - j is

$$f_X^R(x) = \begin{cases} p_j^*, x = 0 \\ f_X(x+d), x > 0. \end{cases} \tag{9}$$

In the case of a collection or stop-loss reinsurance, the reinsurer pays the difference from the sum of all claims combined for the risk to exceed a deductible or limit, called stop-loss or attachment value (De Pril, 1986). If the distribution of losses is given by $f_S(x), x \ge 0$, then distribution of insurer losses, $S_d^I = \min(S, d)$, is as follows

$$fs_d^I(x) = \begin{cases} fs(x), x < d \\ 1 - Fs(x-0), x = d \\ 0, x > d \end{cases} \tag{10}$$

and distribution of reinsurer losses, $S_d^R = \max\{(S-d), 0\} = (S-d)_+$, is

$$fs_d^R(x) = \begin{cases} F_s(d), x = 0 \\ f_s(x+d), x > d \end{cases} \tag{11}$$

The expected value of the insurer loss is also called the stop-loss premium and evaluated as

$$
\begin{aligned}
E[S_d^R] &= E[(S-d)_+] \\
&= \int_0^\infty (x-d)dFs(x) - \int_0^d (x-d)dFs(x) \\
&= E[S] - d + \int_0^d (x-d)d(1-Fs(x)) \\
&= E[S] - \int_0^d 1 - Fs(x)dx
\end{aligned}
\tag{12}
$$

Because the expectations of reinsurer losses and expectations of insurer losses must be in the total equation of total loss expectations, so we have

$$E[S_d^I] = E[S] - E[S_d^R] = \int_0^d 1 - Fs(x)dx$$

When the stop-loss limit d increases indefinitely, the insurer costs then become

$$E[S] = \lim_{d \to \infty} E[S_d^I] = \int_0^\infty 1 - Fs(x)dx \tag{13}$$

so that the expectations of reinsurer losses (13) can be rewritten as

$$E[S_d^R] = \int_d^\infty 1 - F_S(x)dx. \tag{14}$$

Stop loss reinsurance will be chosen as the type of insurance that will be discussed in the arithmetic loss distribution section in the next section.

2.4 Arithmetic Loss Distribution

For example something is set at a positive integer value where the measurement unit is very easy, for example 1000 dollars. If $f_S(x)$ defined only on non-negative integers, then (9), (10) and (11) become

$$E[S_d^R] = \sum_{x=d+1}^\infty (x-d)f_s(x) \tag{15}$$

$$= E[S] - \sum_{x=0}^{d-1}\{1 - F_s(x)\} \tag{16}$$

$$= \sum_{x=d}^\infty \{1 - F_s(x)\} \tag{17}$$

For (17), it can be seen that stop-loss premium profits can be calculated recursively as

$$E[S_{d+1}^R] = E[S_d^R] - \{1 - F_s(d)\}, \tag{18}$$

start with $E[S_0^R] = E[S]$. This recursive formula is very easy because the initial value can be calculated directly (Burrow and Lang, 1997).

In the same way, it can be shown that the second raw moment can be calculated recursively as

$$E[(S_{d+1}^R)^2] = E[(S_d^R)^2] - 2E[S_d^R] + \{1 - F_s(d)\} \tag{19}$$

Reducing squares (17) from (18) will produce

$$Var(S_{d+1}^R) = Var(S_d^R) - F_s(d)\{2E[S_d^R] - 1 + F_s(d)\} \tag{20}$$

Recursion (20) starts with those produced separately.

3. Case Analysis

The following data groups are secondary data taken from the literature written by Panjer & Willmot (De Pril, 1986) on pages 135 and 136 entitled "Insurance Risk Models" as follows: A small-scale manufacturing company has a contract for life insurance groups for 14 permanent employees. Actuaries from insurance companies have selected the death tables of the Canadian Institute of Actuaries in 1969-1975 to represent the death rates of each employee in the group. Each employee is insured for a number of coverage from their salary rounded up to 1000 dollars. The data group of the employees is as shown in Table 1 below.

Table 1. Life Insurance Group Data

j employee	Ages	Gender	Benefit (*b_j*)	Death Rate (*q_j*)
1	20	L	15,000	0.00149
2	23	L	16,000	0.00142
3	27	L	20,000	0.00128
4	30	L	28,000	0.00122
5	31	L	31,000	0.00123
6	46	L	18,000	0.00353
7	47	L	26,000	0.00394
8	49	L	24,000	0.00484
9	64	P	60,000	0.02182
10	17	P	14,000	0.00050
11	22	P	17,000	0.00050
12	26	P	19,000	0.00054
13	37	P	30,000	0.00103
14	55	P	55,000	0.00479
	Total		**373,000**	

3.1 Determine Distribution of Total Claims

This total claim distribution model was developed by (Panjer and Willmot, 1992; Pavel, 2011; Kunreuther, 2015; Vukovic, 2015; Lefèvre, et al., 2018; Sukono, et al., 2018); suppose there is n risk with probability of claims (q_j) and the sum of coverage (b_j) for $j = 1,2,...,n$, then the function of total probability of claims that given by (2) will be very useful to determine $f_{X_j}(d)$.

In the simulation of calculating the total distribution of claims, it is determined for value $d = 0,1,2,...,20$, this value is considered sufficient to describe the steps of the formulation in the previous section (Panjer and Willmot, 1992; Tiller and Fagerberg, 1990). For value of $f_{X_1}(d); d = 0,1,2,...,20$ generated from equation (2) then it will be used to determine $f_{S_1}(d); d = 0,1,2,...,20$ by using equation (3). To determine $f_{X_1}(d); d = 0,1,2,...,20$ and $f_{S_1}(d); d = 0,1,2,...,20$ used *MS-Excel 2000*. In Figure 2, the following are shown in steps to get a value $f_{S_{14}}(d)$.

$$
\begin{array}{cccc}
f_{S_1}(0), & f_{S_1}(1), & \cdots & f_{S_1}(20) \\
f_{S_2}(0), & f_{S_2}(1), & \cdots & f_{S_2}(20) \\
\vdots & \vdots & \vdots & \vdots \\
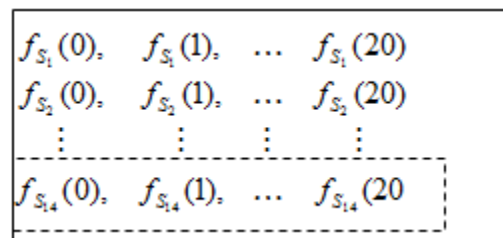f_{S_{14}}(0), & f_{S_{14}}(1), & \cdots & f_{S_{14}}(20)
\end{array}
$$

Figure 2. Distribution Chart of Panjer Model Total Claims

From the data group in Table 1, the value $d = 1,2,...,373000$ is needed, that number is a large number so to do a complete calculation, and a computer application program has been built using Delphi 6.0.

Now we will enter the calculation process, starting with determining $f_{X_1}(d)$. On the Table 2 are shown the value of $f_{X_1}(d)$ for $d = 0,1,2,...,20$, generated from equation (2) as follow:

Table 2. The value of $f_{X_1}(d)$, for $d = 0,1,2,...,20$

		$f_{X_1}(d)$			
b_1	p_1	$d = 0$	$d = 1,2,3,...,14$	$d = 15$	$d = 16,17,...,20$
15	0.99851	0.99851	0	0.00149	0

After getting the values of $f_{X_1}(d)$, for $d = 0,1,2,...,20$, the following will be shown the process of calculating the total claim distribution or value of $f_{S_j}(d)$ starting with the value $d = 0$:

a) Determining the value of $f_{S_i}(d)$ for $d = 0$

Table 3. The value of $f_{S_j}(d)$ for $d = 0$

j	b_j	Formula	$f_{S_j}(0)$	j	b_j	Formula	$f_{S_j}(0)$
1	15	$f_{S_1}(0) = f_{X_1}(0)$	0.998510000	8	24	$P_8 f_{S_7}(0)$	0.981198770
2	16	$P_2 f_{S_1}(0)$	0.997092116	9	60	$P_9 f_{S_8}(0)$	0.959789013
3	20	$P_3 f_{S_2}(0)$	0.995815838	10	14	$P_{10} f_{S_9}(0)$	0.959309118
4	28	$P_4 f_{S_3}(0)$	0.994600943	11	17	$P_{11} f_{S_{10}}(0)$	0.958829464
5	31	$P_5 f_{S_4}(0)$	0.993377583	12	19	$P_{12} f_{S_{11}}(0)$	0.958311696
6	18	$P_6 f_{S_5}(0)$	0.989870961	13	30	$P_{13} f_{S_{12}}(0)$	0.957324635
7	26	$P_7 f_{S_6}(0)$	0.985970869	14	55	$P_{14} f_{S_{13}}(0)$	**0.952739050**

From table 3 generated the value of $f_{S_i}(d)$ for $d = 0$ is **0.95273905**.

b) Determining the value of $f_{S_j}(d)$ for $d = 1$

Table 4. The value of $f_{S_j}(d)$ for $d = 1$

j	b_j	Formula	$f_{S_j}(1)$	j	b_j	Formula	$f_{S_j}(1)$
1	15	$f_{S_1}(1) = f_{X_1}(1)$	0	8	24	$P_8 f_{S_7}(1)$	0
2	16	$P_2 f_{S_1}(1)$	0	9	60	$P_9 f_{S_8}(1)$	0
3	20	$P_3 f_{S_2}(1)$	0	10	14	$P_{10} f_{S_9}(1)$	0
4	28	$P_4 f_{S_3}(1)$	0	11	17	$P_{11} f_{S_{10}}(1)$	0
5	31	$P_5 f_{S_4}(1)$	0	12	19	$P_{12} f_{S_{11}}(1)$	0
6	18	$P_6 f_{S_5}(1)$	0	13	30	$P_{13} f_{S_{12}}(1)$	0
7	26	$P_7 f_{S_6}(1)$	0	14	55	$P_{14} f_{S_{13}}(1)$	**0**

From the above calculation by fulfilling the condition equation (2) a value is generated $f_{S_i}(d)$ for $d = 1$ is **0**. Referring to the condition of the total claim probability function developed by Panjer and the value of $f_{X_1}(d)$ that is shown by table 2 where the value of $d < b_j$ is 0, the intended value b_j is the smallest in the data group above, i.e. 14. This result in the value of $f_{S_j}(d) = 0$ that applies for $d = 1, 2, ..., 13$ is $f_{S_j}(d) = 0$. After determining the value of $f_{S_j}(d)$ for $d = 1, 2, ..., 20$, we will also get the value of distribution function $F_S(d)$.

3.2 Determinining Net Stop Loss Premiums

a) Recursive Calculation Method

To apply the recursive formula using the value q_j and matrix line n_{ij} not zero, the data group is shown in Table 5. as follows:

Table 5. Illustration of value q_j and Matrix Line Non-zero n_{ij}

j	$1000q_j$	$i = 14$	15	16	17	18	19	20	24	26	28	30	31	55	60
														n_{ij}	
1	0.50	1	0	0	1	0	0	0	0	0	0	0	0	0	0
2	0.54	0	0	0	0	0	1	0	0	0	0	0	0	0	0
3	1.03	0	0	0	0	0	0	0	0	0	0	1	0	0	0
4	1.22	0	0	0	0	0	0	0	0	0	1	0	0	0	0
5	1.23	0	0	0	0	0	0	0	0	0	0	0	1	0	0
6	1.28	0	0	0	0	0	0	1	0	0	0	0	0	0	0
7	1.42	0	0	1	0	0	0	0	0	0	0	0	0	0	0
8	1.49	0	1	0	0	0	0	0	0	0	0	0	0	0	0
9	3.53	0	0	0	0	1	0	0	0	0	0	0	0	0	0
10	3.94	0	0	0	0	0	0	0	0	1	0	0	0	0	0
11	4.79	0	0	0	0	0	0	0	0	0	0	0	0	1	0
12	4.84	0	0	0	0	0	0	0	1	0	0	0	0	0	0
13	21.82	0	0	0	0	0	0	0	0	0	0	0	0	0	1

After all the columns are filled correctly and accurately, then we can do the next step which is determining the value $E[S_0^R] = E[S]$. Using equation (6) we determine the value $E[S]$ that is shown in the following operations and to abbreviate the calculation process, we can ignore multiplication operations with a value so that it becomes as follows:

$$E[S] = 14(0.50)(1)+15(1.49)(1)+16(1.42)(1)+17(0.50)(1)+18(3.53)(1)+19(0.54)(1)+20(1.28)(1)+24(4.84)(1)$$
$$+26(3.94)(1)+28(1.22)(1)+30(1.03)(1)+31(1.23)(1)+55(4.79)(1)+60(21.82)(1)$$
$$= 7 + 22.35 + 22.72 + 8.50 + 63.54 + 10.26 + 25.60 + 116.16 + 102.44 + 34.16 + 30.90 + 38.13$$
$$= 2054.41$$
$$= \mathbf{2.05441} \text{ (1000 unit)}$$

b) Raw Moment

$$\mu_1' = \sum_{j=1}^{14} q_j b_j = 2.05441$$

c) Stop Loss and Variance Net Premiums

Through equation (18) we can determine the stop loss and variance net premium as shown in Table 6. below.

Table 6. Stop Loss Premiums and Variances for Various Stop Loss Levels

D	$F_S(d)$	$E\left[S_d^R\right]$	$Var\left[S_d^R\right]$	D	$F_S(d)$	$E\left[S_d^R\right]$	$Var\left[S_d^R\right]$
0	0.95273905	2.05441	102.5335632	11	0.95273905	1.53454	64.920907
1	0.95273905	2.00715	98.6639570	12	0.95273905	1.48728	62.041903
2	0.95273905	1.95989	94.8844060	13	0.95273905	1.44002	59.554059
3	0.95273905	1.91263	91.1949100	14	0.95321566	1.39276	56.554059
4	0.95273905	1.86537	87.5954680	15	0.95463736	1.34597	53.943459
5	0.95273905	1.81811	84.0860810	16	0.95599217	1.30061	51.416933
6	0.95273905	1.77084	80.6667480	17	0.95646878	1.25660	48.972259
7	0.95273905	1.72358	77.3374710	18	0.95984386	1.21307	46.610094
8	0.95273905	1.67632	74.0982480	19	0.96035862	1.17291	44.319921
9	0.95273905	1.62906	70.9490790	20	0.96157969	1.13327	42.105154
10	0.95273905	1.58180	67.8899660				

3.3 For Portfolios with 14 policies

a) Number of Arithmetic Operations at Each Iteration

In the Panjer algorithm there are 2 (two) types of arithmetic operations needed, namely:
- Multiplication operations : minimum $n-1$ and maximum $2(n-1)$
- Addition / subtraction operations: minimum = 0 and maximum $2(n-1)$

Based on the data group used for analyzing the model, it is known the number of policies = n = 14, so that:
- Multiplication operations: minimum $n-1=14-1=13$ and maximum $2(n-1)=2(14-1)=2\times13=26$

- Addition / subtraction operations : minimum = 0 and maximum $2(n-1)= 2(14-1) = 2\times13 = 26$

  Total arithmetic operations: minimum 13 and maximum 52.

b) Number of Elements in Array for Next Iteration
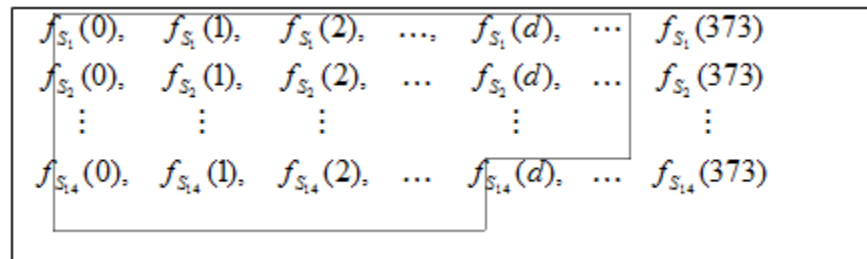
Consider Figure 3. below.

Figure 3. Number of Elements Must Be Stored in the Panjer Algorithm

It takes a number of elements contained in a dashed line that must be stored; this is because the elements needed are uncertain so that all the elements produced must be stored. The number of array elements needed is as many as $dn + (n-1)$.

So for 1 (one) iteration is needed as many as $1(14) + (14-1) = 27$ and maximum $(373)(14)+(14-1) = 5222 + 13 = 5235$ element.

The number of array elements above is for policies as many as 14 policies which are actually a small amount. Suppose the number of policies is 100 times or 1,000 times the number of arrays that must be stored in each iteration with this model is certainly very large.

3.4 For Large Size Portfolios

Suppose the data group in Table 2. we modify the number of policies and display them in the form of sub-portfolios as in Table 7. below.

Table 7. Illustration of Portfolio with Amount of Large Policy (1400 policies)

j	$1000q_j$	$i = 14$	15	16	17	18	19	20	24	26	28	30	31	55	60
								N_{ij}							
1	0.50	100	0	0	100	0	0	0	0	0	0	0	0	0	0
2	0.54	0	0	0	0	0	100	0	0	0	0	0	0	0	0
3	1.03	0	0	0	0	0	0	0	0	0	0	100	0	0	0
4	1.22	0	0	0	0	0	0	0	0	0	100	0	0	0	0
5	1.23	0	0	0	0	0	0	0	0	0	0	0	100	0	0
6	1.28	0	0	0	0	0	0	100	0	0	0	0	0	0	0
7	1.42	0	0	100	0	0	0	0	0	0	0	0	0	0	0
8	1.49	0	100	0	0	0	0	0	0	0	0	0	0	0	0
9	3.53	0	0	0	0	100	0	0	0	0	0	0	0	0	0
10	3.94	0	0	0	0	0	0	0	0	100	0	0	0	0	0
11	4.79	0	0	0	0	0	0	0	0	0	0	0	0	100	0
12	4.84	0	0	0	0	0	0	0	100	0	0	0	0	0	0
13	21.82	0	0	0	0	0	0	0	0	0	0	0	0	0	100

a) Number of Arithmetic Operations at each Iteration

Based on the data group used for analyzing the model, it is known the number of policies $= n = 1400$, so that:
- Multiplication operations: minimum $n-1 = 1400 - 1 = 1399$ and maximum $2(n-1) = 2(1400-1) = 2\times1399 = 2798$.
- Addition/subtraction operations: minimum $= 0$ and maximum $2(n-1) = 2(1400-1) = 2\times1399 = 2798$.
  Total arithmetic operations: minimum 1399 and maximum 5596.

b) Number of Elements in the Array for next Iteration

The number of array elements needed is as many as $dn + (n-1)$. So for 1 (one) iteration is needed as many a $1(1400) + (1400-1) = 2,799$, and maximum $(37300)(1400)+(1400-1) = 52,220,000+1,399 = 52,220,399$ element array.

Based on the analysis above, it can be summarized briefly in a comparison table of the number of arithmetic operations and array elements for the number of small and large policies as below.

Table 8. Comparison of the Amount of Arithmetic Operations and Array Elements to the Amount of Policy

	Data : 14 Policy M = 373	Data 140 Policy M = 3730	Data : 1400 Policy M = 37300
Number of Arithmetic Operations	13 to 52	139 to 556	1,399 to 5,596
Number of Array Elements	27 to 5,235	279 to 523,599	2,799 to 52,221,399

4. Conclusion

This Panjer and Wilmot (1992) model can be used as an option for actuaries to support important decisions in reinsurance. Information generated from this model meets the needs of total claim distribution, stop loss premium and variance for various levels of retention. The large number of arithmetic operations and array elements needed in each iteration makes this model more efficient and effective for the number of small policies. This is because arithmetic operations and the number of elements that must be stored in the array will increase as the number of policies increases and the number of benefits we need. The greater the number of policies and benefits of the b_j involved make the calculation process easier if supported by an adequate computer program and device application. This is related to the CPU time needed in the calculation process.

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