A New 3-D Chaotic System with Three Nonlinearities, its Adaptive Synchronization and Circuit Simulation

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Abstract

Chaotic systems deal with nonlinear dynamical systems which are highly sensitive to changes in the initial conditions. This paper reports the finding of a new 3-D chaotic system with 3 nonlinearities. The phase plots and dynamic analysis of the new chaotic system are described by means of MATLAB plots, Lyapunov exponents, etc. The chaotic system has a unique saddle-point equilibrium at the origin. As a control application, the adaptive synchronization of the new chaotic system with itself is obtained using Lyapunov stability theory. Finally, an electronic circuit of the new chaotic system with MultiSIM is designed and a good match between the plots of the theoretical chaotic model and the circuit model is obtained. The electronic circuit model validates the new theoretical chaotic model developed in this work.

Keywords
Chaos, chaotic systems, adaptive control, synchronization and electronic circuit.

1. Introduction

Chaos theory deals with chaotic systems which are nonlinear dynamical systems highly sensitive to small changes in the initial conditions (Azar and Vaidyanathan, 2016). Chaotic systems are also confirmed by the presence of a positive Lyapunov exponent. If the sum of the Lyapunov exponents of the chaotic system is negative, it is classified as a dissipative chaotic system (Azar and Vaidyanathan, 2016). The first dissipative chaotic system with two quadratic nonlinearities was discovered by Lorenz (1963), while he was working on a 3-D weather convection model (Lorenz, 1963).

Chaos theory has several applications in science and engineering such as memristors (Buscarino *et. al*., 2012; Sun *et. al*., 2013; Muthuswamy, 2010), weather systems (Lorenz, 1963), biological systems (Vaidyanathan 2015a, 2015b, 2015c, 2015d, Voorsluijs and Decker, 2016), neural systems (Vaidyanathan 2015e, 2015f, 2015g, 2015h), oscillators (Pakiriswamy and Vaidyanathan, 2012; Rasappan and Vaidyanathan, 2012; Vaidyanathan 2012a, 2012b, 2014, 2015i, 2015j; Vaidyanathan and Rasappan, 2011; Vaidyanathan and Sampath, 2012), chemical reactors (Vaidyanathan, 2015k), circuits (Daltzis *et al*., 2018; Mamat *et al*., 2018; Pham *et al*., 2016; Sambas *et al*., 2018a, 2018b; Vaidyanathan, 2018a, 2018b, 2018c, 2018d, 2018e), finance (Zhao *et al.,* 2011), robotics (Sambas *et al*., 2016; Vaidyanathan *et al.* 2017), steganography (Vaidyanathan *et al.,* 2018f), cryptography (Murali, 2000; Wu *et al.,* 2014), image encryption (Xue *et al.,* 2018), secure communications (Li *et al*., 2005), etc.

In this work, we describe a new 3-D chaotic system with three nonlinearities – two quadratic nonlinearities and a quartic nonlinearity. It is interesting to know that the new chaotic system has a positive Lyapunov exponent confirming its chaotic behavior. We show that the new chaotic system has rotation symmetry about the x_3-axis. We also show that the new chaotic system has three unstable equilibrium points – a saddle-point equilibrium at the origin and two saddle-focus equilibrium points on the $(x_1, x_3)-$plane.

In Section 2, we give the model of the new chaotic system and analyze the dynamic properties of the new chaotic system via phase plots, Lyapunov exponents, Kaplan-Yorke dimension, etc. In Section 3, we describe the adaptive synchronization of the new chaotic system with itself via adaptive control method. In Section 4, we present an electronic circuit of the new chaotic system via MultiSIM to verify the feasibility of the theoretical model. Our simulations show a good match between the MATLAB plots and the MultiSIM simulations of the new chaotic system. Finally, Section 5 contains the conclusions of this work.

2. A New 3-D Chaotic System with Three Nonlinearities

In this work, we report a new 3-D system given by the nonlinear dynamics

$$\begin{cases} \dot{x}_1 = x_2 + x_2 x_3 \\ \dot{x}_2 = x_1 - a x_2 - x_1 x_3 \\ \dot{x}_3 = -b x_3 + x_1^4 \end{cases} \tag{1}$$

In (1), $X = (x_1, x_2, x_3)$ is the state and a, b are constant parameters.

The system (1) has two quadratic nonlinearities and a quartic nonlinearity.

We shall show that the system (1) exhibits a chaotic attractor for the parameter values

$$a = 0.6, \quad b = 0.4 \tag{2}$$

The Lyapunov exponents of the new system (1) for $(a,b) = (0.6, 0.4)$ and $X(0) = (0.2, 0.2, 0.2)$ are calculated for $T = 10^5$ seconds using Wolf's algorithm (Wolf *et al.,* 1985) as follows:

$$LE_1 = 0.2583, \quad LE_2 = 0, \quad LE_3 = -1.2583 \tag{3}$$

From (3), we note that the new 4-D system (1) is chaotic since it has a positive Lyapunov exponent.

Also, the new system (1) is dissipative since the sum of the Lyapunov exponents is negative.

Furthermore, the Kaplan-Yorke dimension of the new system (1) is determined as follows:

$$D_{KY} = 2 + \frac{LE_1 + LE_2}{|LE_3|} = 2.2053 \tag{4}$$

We note that the new chaotic system (1) remains invariant under the change of coordinates given by

$$(x_1, x_2, x_3) \mapsto (-x_1, -x_2, x_3) \tag{5}$$

This shows that the new hyperchaotic system (1) has rotation symmetry about the x_3 − coordinate axis. Hence, all the non-trivial trajectories of the new chaotic system (1) must have twin trajectories associated with them.

We get the equilibrium points of the new chaotic system (1) by solving the following equations for the chaotic case of the parameters $(a,b) = (0.6, 0.4)$:

$$x_2(1 + x_3) = 0 \tag{6a}$$

$$x_1 - ax_2 - x_1x_3 = 0 \tag{6b}$$

$$-bx_3 + x_1^4 = 0 \tag{6c}$$

Solving the system (6), it is easy to see that the chaotic system (1) has three equilibrium points given as follows: $E_0 = (0,0,0)$, $E_1 = (0.7953, 0, 1)$ and $E_2 = (-0.7953, 0, 1)$.

The Jacobian of the new chaotic system (1) for $(a,b) = (0.6, 0.4)$ at any point $\boldsymbol{x} = (x_1, x_2, x_3)$ is given by

$$J = \begin{bmatrix} 0 & 1 + x_3 & x_2 \\ 1 - x_3 & -0.6 & -x_3 \\ 4x_1^3 & 0 & -0.4 \end{bmatrix} \tag{7}$$

The eigenvalues of $J_0 = J(E_0)$ are obtained as $\lambda_1 = -0.4$, $\lambda_2 = -1.3440$ and $\lambda_3 = 0.7440$. This shows that E_0 is a saddle point equilibrium of the new chaotic system (1), which is unstable. The eigenvalues of $J_1 = J(E_1)$ are obtained as $\lambda_1 = -1.9427$ and $\lambda_{2,3} = 0.4714 \pm 1.3599i$. The eigenvalues of $J_2 = J(E_2)$ are obtained as $\lambda_1 = 1.2776$ and $\lambda_{2,3} = -1.1388 \pm 1.3612i$. Thus, E_1 and E_2 are saddle-focus equilibrium points of the new chaotic system (1), which are unstable. Hence, the strange attractor associated with the new chaotic system (1) is a *self-excited* attractor.

The phase plots of the new hyperchaotic system (1) for $X(0) = (0.2, 0.2, 0.2)$ and $(a,b) = (0.6, 0.4)$ are shown in Figures 1-3. Also, Figure 4 shows the Lyapunov exponents of the new chaotic system (1).

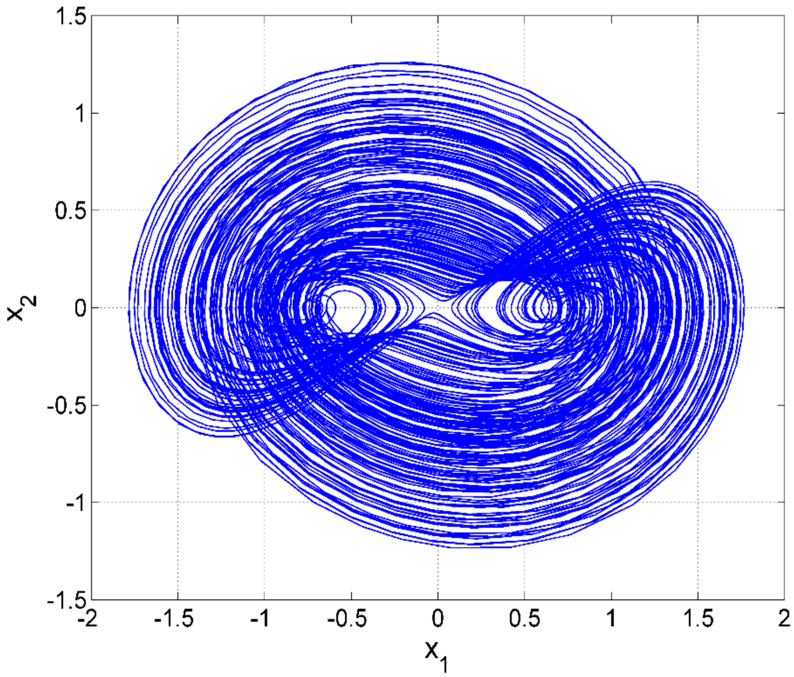


Figure 1. 2-D phase plot of the new chaotic system (1) in the (x_1, x_2) – plane for $X(0) = (0.2, 0.2, 0.2)$ and $(a, b) = (0.6, 0.4)$

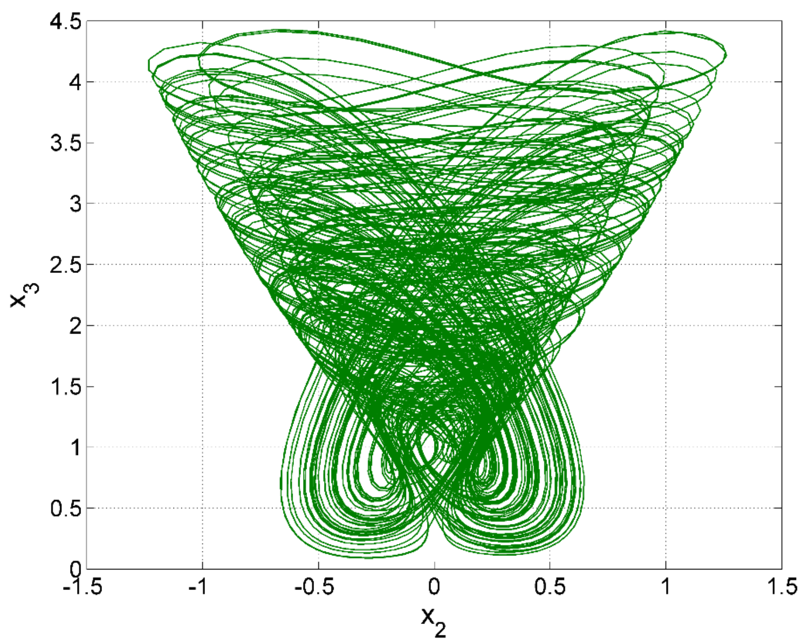


Figure 2. 2-D phase plot of the new chaotic system (1) in the (x_2, x_3) – plane for $X(0) = (0.2, 0.2, 0.2)$ and $(a, b) = (0.6, 0.4)$

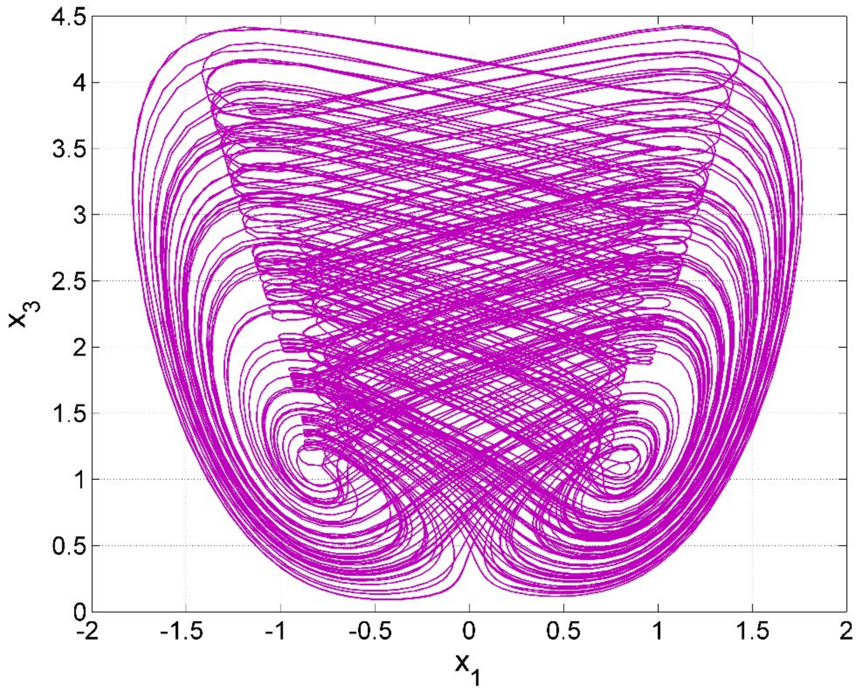


Figure 3. 2-D phase plot of the new chaotic system (1) in the $(x_1, x_3)-$plane for $X(0) = (0.2, 0.2, 0.2)$ and $(a, b) = (0.6, 0.4)$

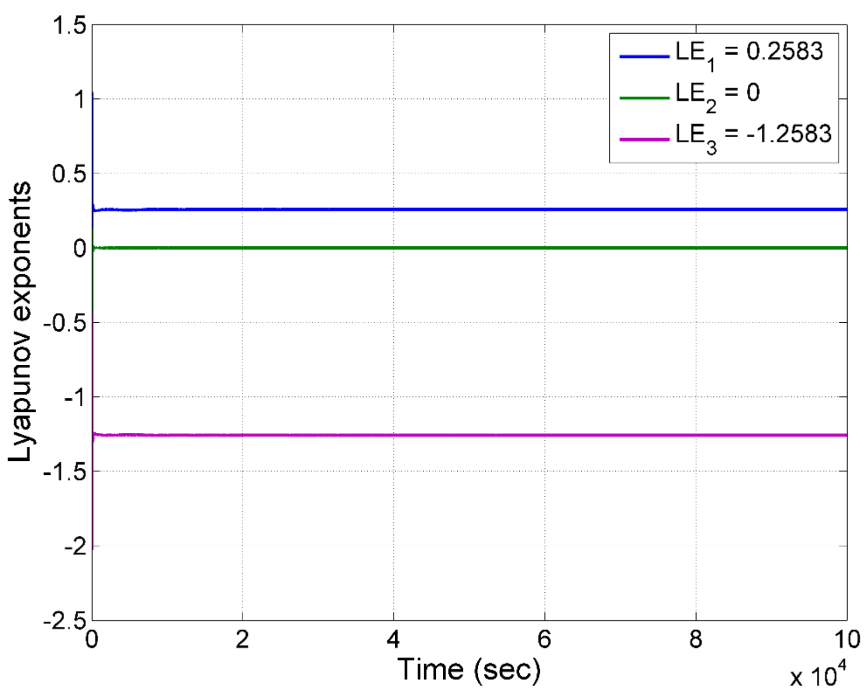
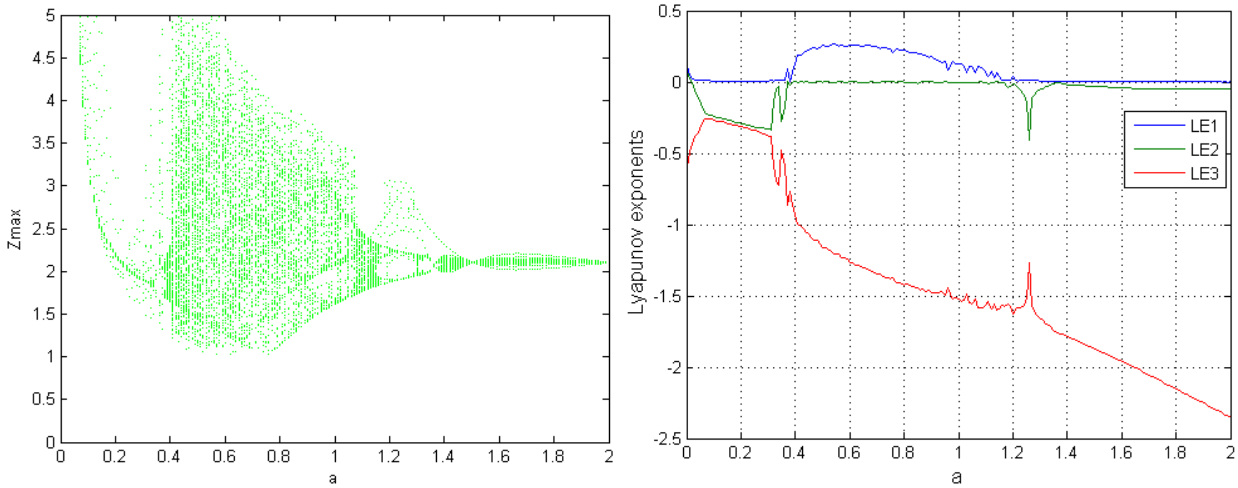
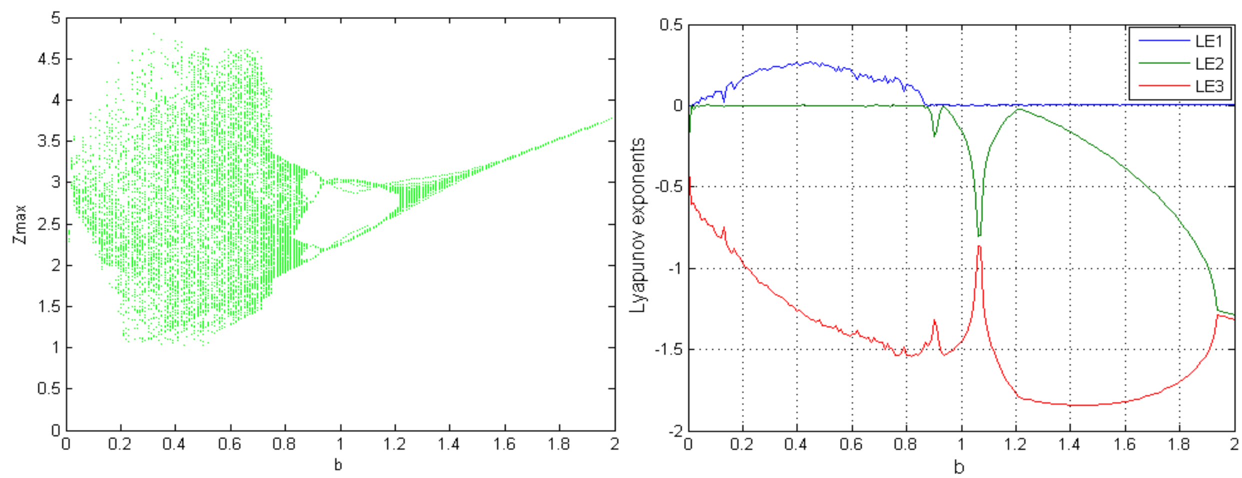


Figure 4. Lyapunov exponents of the new system (1) for $X(0) = (0.2, 0.2, 0.2)$ and $(a, b) = (0.6, 0.4)$



(a) (b)

Figure 5. (a) Bifurcation diagram of system (1) versus the parameter a for $b = 0.4$ and initial conditions $(x(0), y(0), z(0)) = (0.2, 0.2, 0.2)$; (b) Lyapunov spectrum of system (1) when varying the parameter a for for $b = 0.4$, and initial conditions $(x(0), y(0), z(0)) = (0.2, 0.2, 0.2)$.



(a) (b)

Figure 6. (a) Bifurcation diagram of system (1) versus the parameter b for $a = 0.6$ and initial conditions $(x(0), y(0), z(0)) = (0.2, 0.2, 0.2)$; (b) Lyapunov spectrum of system (1) when varying the parameter b for for $a = 0.6$, and initial conditions $(x(0), y(0), z(0)) = (0.2, 0.2, 0.2)$.

The bifurcation diagram and Lyapunov exponents are displayed when changing the value of the bifurcation parameter a as reported in Figure 5 (a) and Figure 5(b), respectively. In more detail, in the range $a < 0.4$, the system exhibits periodic behavior. For the value of the parameter $0.4 \leq a < 1.2$, a chaotic behavior is obtained. Also, for the value of $a \geq 1.2$, system (1) remains always in periodic state. Moreover, dynamical analysis of system (1) has been studied by varying the parameter b. Figures 6(a), (b) present the bifurcation diagram and the diagram of Lyapunov exponents of system (1), respectively. As shown in Figures 6(a) and Figures 6(b), the system can exhibit periodical and chaotic behaviors with different values of the parameter b.

3. Global Chaos Synchronization of the New Chaotic Systems by Adaptive Control Method

In this section, we consider the adaptive synchronization of the new chaotic system with itself using adaptive control method.

As the master system, we consider the new chaotic system given by

$$\begin{cases} \dot{x}_1 = x_2 + x_2 x_3 \\ \dot{x}_2 = x_1 - a x_2 - x_1 x_3 \\ \dot{x}_3 = -b x_3 + x_1^4 \end{cases} \tag{8}$$

In (8), $X = (x_1, x_2, x_3)$ is the state and a, b are system parameters which are not available for measurement.

As the slave system, we consider the new chaotic system with controls given by

$$\begin{cases} \dot{y}_1 = y_2 + y_2 y_3 + u_1 \\ \dot{y}_2 = y_1 - a y_2 - y_1 y_3 + u_2 \\ \dot{y}_3 = -b y_3 + y_1^4 + u_3 \end{cases} \tag{9}$$

In (9), $Y = (y_1, y_2, y_3)$ is the state and $\boldsymbol{u} = (u_1, u_2, u_3)$ is the adaptive control using the estimates $A(t), B(t)$ in lieu of the unknown system parameters $a, b,$ respectively.

The synchronization error between the new chaotic systems (8) and (9) is given by

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases} \tag{10}$$

The synchronization error dynamics is obtained as follows:

$$\begin{cases} \dot{e}_1 = e_2 + y_2 y_3 - x_2 x_3 + u_1 \\ \dot{e}_2 = e_1 - a e_2 - y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 = -b e_3 + y_1^4 - x_1^4 + u_3 \end{cases} \tag{11}$$

We consider the adaptive control given by

$$\begin{cases} u_1 = -e_2 - y_2 y_3 + x_2 x_3 - k_1 e_1 \\ u_2 = -e_1 + A(t) e_2 + y_1 y_3 - x_1 x_3 - k_2 e_2 \\ u_3 = B(t) e_3 - y_1^4 + x_1^4 - k_3 e_3 \end{cases} \tag{12}$$

In (12), k_1, k_2, k_3 are positive constants and $A(t), B(t)$ are parameter estimates.

We define the parameter estimation error as follows:

$$e_a = a - A(t), \quad e_b = b - B(t) \tag{13}$$

Differentiating (13), we obtain the following:

$$\dot{e}_a = -\dot{A}(t), \quad \dot{e}_b = -\dot{B}(t) \tag{14}$$

By substituting the adaptive control law (12) into (11), we get the closed-loop error dynamics as follows:

$$\begin{cases} \dot{e}_1 = -k_1 e_1 \\ \dot{e}_2 = -[a - A(t)]e_2 - k_2 e_2 \\ \dot{e}_3 = -[b - B(t)]e_3 - k_3 e_3 \end{cases} \tag{15}$$

Using the definition (13), it is possible to simplify the closed-loop error dynamics (15) as follows:

$$\begin{cases} \dot{e}_1 = -k_1 e_1 \\ \dot{e}_2 = -e_a e_2 - k_2 e_2 \\ \dot{e}_3 = -e_b e_3 - k_3 e_3 \end{cases} \tag{16}$$

We consider the quadratic Lyapunov function V defined as follows:

$$V(e_1, e_2, e_3, e_a, e_b) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) + \frac{1}{2}\left(e_a^2 + e_b^2\right) \tag{17}$$

Differentiating V along the trajectories of (16) and (14), we obtain the following:

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a\left[-e_2^2 - \dot{A}\right] + e_b\left[-e_3^2 - \dot{B}\right] \tag{18}$$

In view of (18), we propose the following parameter update law:

$$\begin{cases} \dot{A} = -e_2^2 \\ \dot{B} = -e_3^2 \end{cases} \tag{19}$$

Thus, we prove the following main result of this section.

The new result is established using Lyapunov stability theory (Khalil, 2002).

Theorem 1. The new chaotic systems (8) and (9) with unknown system parameters a, b are globally and exponentially synchronized for all initial conditions $X(0), Y(0) \in R^3$ with the adaptive feedback control law (12) and the parameter update law (19), where the control gains k_1, k_2, k_3 are positive constants.

Proof. First, we note that the candidate Lyapunov function V given in Eq. (17) is a quadratic and positive definite function on R^5. Next, we substitute the parameter update law (19) into Eq. (18). This results in the following:

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \tag{20}$$

This shows that $\dot{V}$ is a negative semi-definite function on R^5. Hence, by Barbalat's Lemma (Khalil, 2002), we conclude that the controlled error variables $e_1(t), e_2(t), e_3(t)$ of the closed-loop error dynamics (16) exponentially converge to zero as $t \to \infty$. This completes the proof. ■

For numerical simulations, we take the control gains as $k_i = 10,\ i = 1,2,3.$ Also, we take the parameter values as in the chaotic case (2), *i.e.* $(a,b) = (0.6, 0.4)$. As the initial states of the new chaotic systems (8) and (9), we take $X(0) = (2.5,\ 6.8,\ 3.4)$ and $Y(0) = (7.2,\ 2.9,\ 1.5)$, respectively. We also take $(A(0), B(0)) = (9,\ 14)$.

Figures 7-9 show the synchronization of the states of the new chaotic systems (8) and (9), while Figure 10 shows the time-history of the synchronization error between these two systems, when the adaptive control law (12) and the parameter update law (16) are implemented.

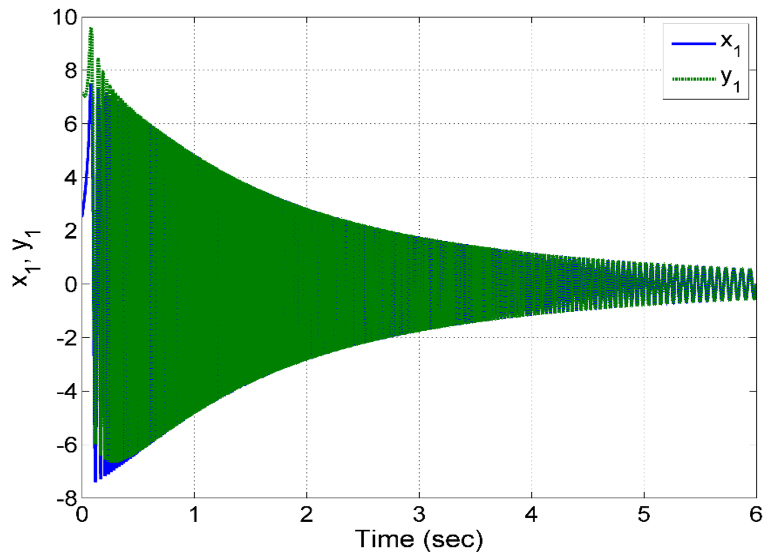


Figure 7. Synchronization of the states x_1 and y_1 of the new chaotic systems (8) and (9)

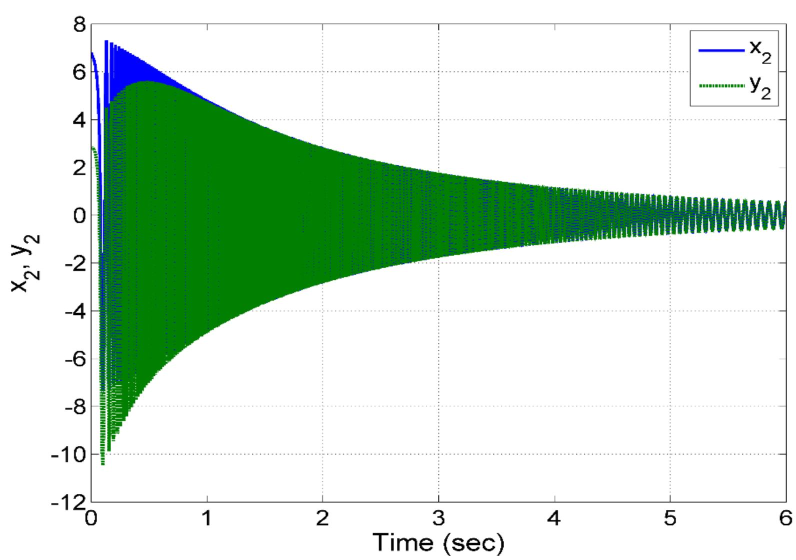


Figure 8. Synchronization of the states x_2 and y_2 of the new chaotic systems (8) and (9)

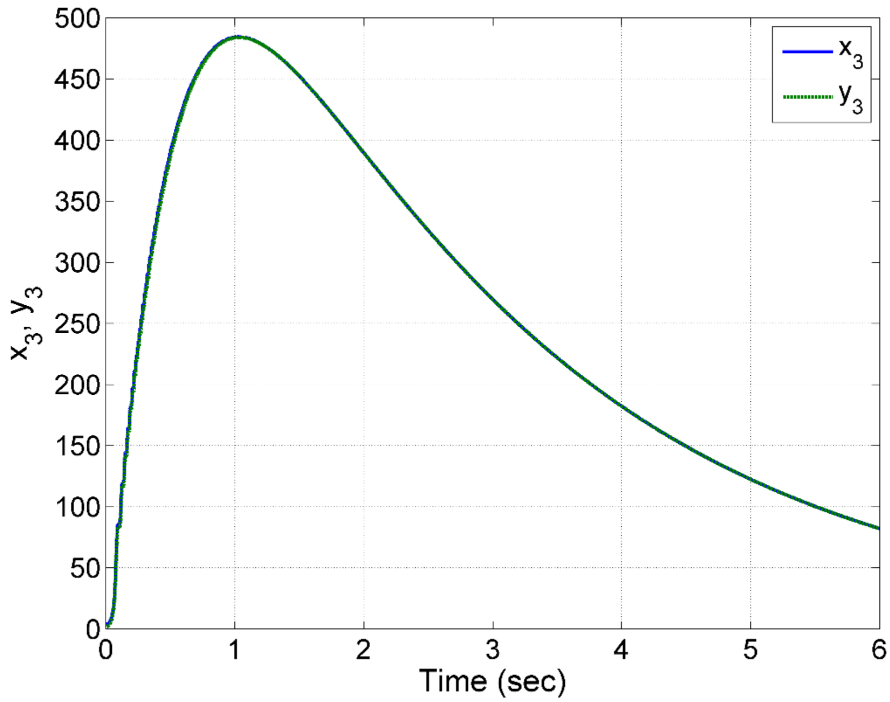


Figure 9. Synchronization of the states x_3 and y_3 of the new chaotic systems (8) and (9)

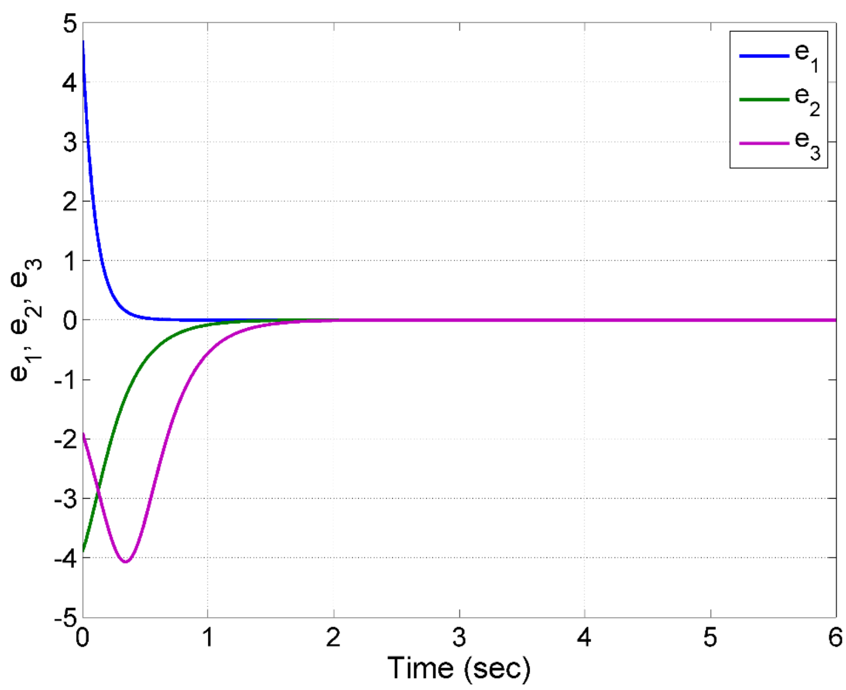


Figure 10. Time-history of the synchronization error between the new chaotic systems (8) and (9)

4. Circuit Simulation of the New 3-D Chaotic System with Three Nonlinearities

The circuit comprises of electronic components such as resistors, capacitors, and operational amplifiers. Figure 11 shows the circuit schematic for implementing the new 3-D hyperchaotic system Eq. (1). There are 3 capacitors, 11 resistors, 5 opamps and 5 multipliers in the circuit. Here the state variables in system (1) are the voltages across the capacitors. The dynamical equations of the circuit are given by

$$\begin{cases} \dot{x}_1 = \dfrac{1}{C_1R_1}x_2 + \dfrac{1}{10C_1R_2}x_2x_3 \\ \dot{x}_2 = \dfrac{1}{C_2R_3}x_1 - \dfrac{1}{C_2R_4}x_2 - \dfrac{1}{10C_2R_5}x_1x_3 \\ \dot{x}_3 = -\dfrac{1}{C_3R_7}x_3 + \dfrac{1}{1000C_3R_6}x_1{}^4 \end{cases} \tag{21}$$

In (21), x_1, x_2, x_3 are the voltages of corresponding capacitors C_1, C_2, C_3. The values of components in Figure 11 are selected to match the model (21) as follows: $R_1 = R_3 = 400$ kΩ, $R_2 = R_5 = 40$ kΩ, $R_4 = 666.67$ kΩ, $R_6 = 400$ Ω, $R_7 = 1$ MΩ, $R_8 = R_9 = R_{10} = R_{11} = 100$ kΩ, $C_1 = C_2 = C_3 = 1$ nF. The MultiSIM outputs, showing phase portraits of the new 3-D hyperchaotic system as seen in Figure 12, agree well with the MATLAB simulation of the equations (1) shown earlier.

5. Conclusions

In this work, we reported the modelling of a new 3-D chaotic system with 3 nonlinearities. The 2-D phase plots and bifurcation analysis of the new chaotic system are described by means of MATLAB plots, Lyapunov exponents, etc. It was shown that the chaotic system has a unique saddle-point equilibrium at the origin, which is unstable. As a control application, the adaptive synchronization of the new chaotic system with itself was derived using Lyapunov stability theory. Finally, an electronic circuit of the new chaotic system was built with MultiSIM and a good match between the plots of the theoretical chaotic model and the circuit model was obtained. The electronic circuit model validates the new theoretical chaotic model developed in this work confirming that the new chaotic system is very suitable for engineering and real-world applications such as encryption, secure communication, etc.

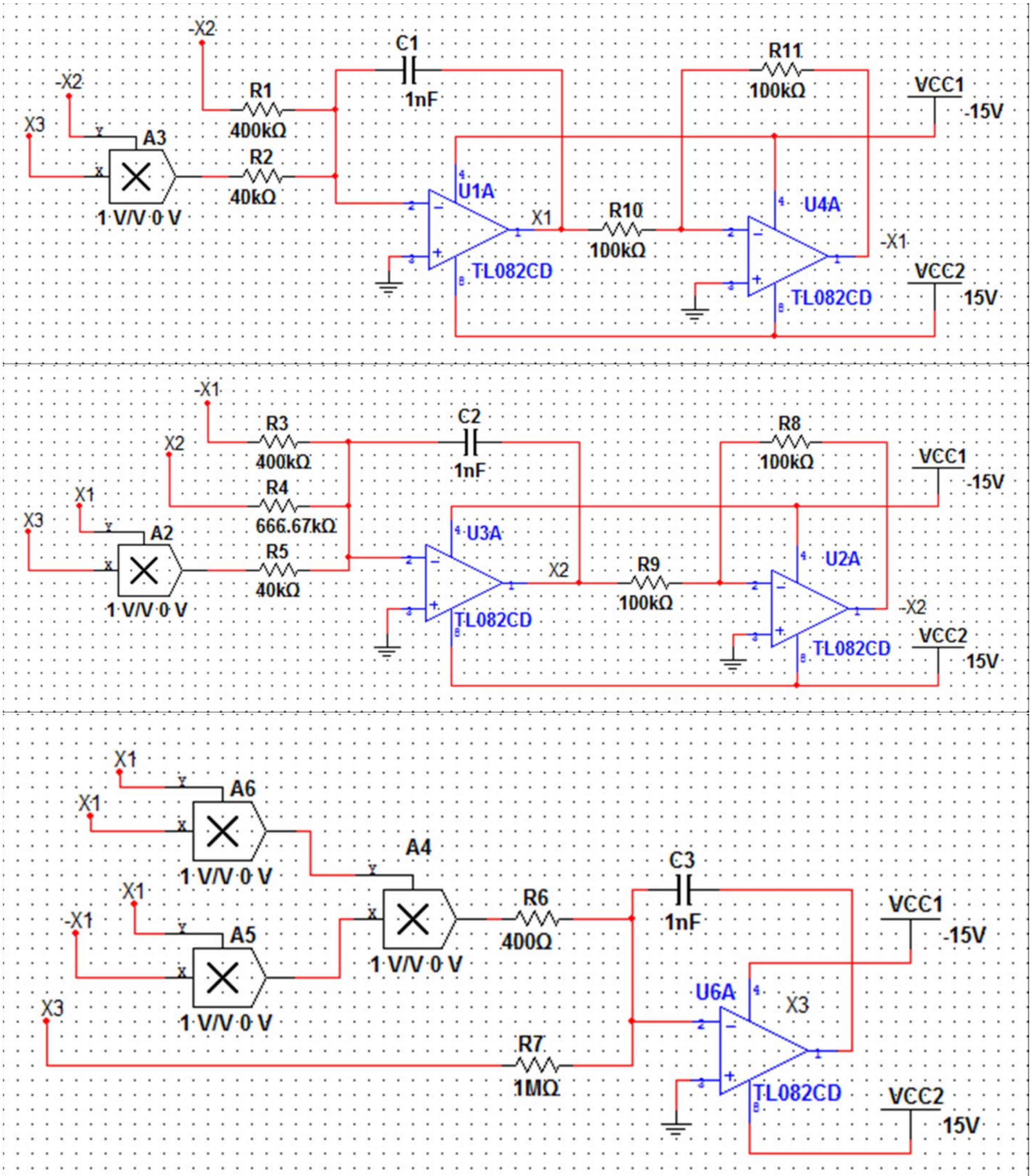
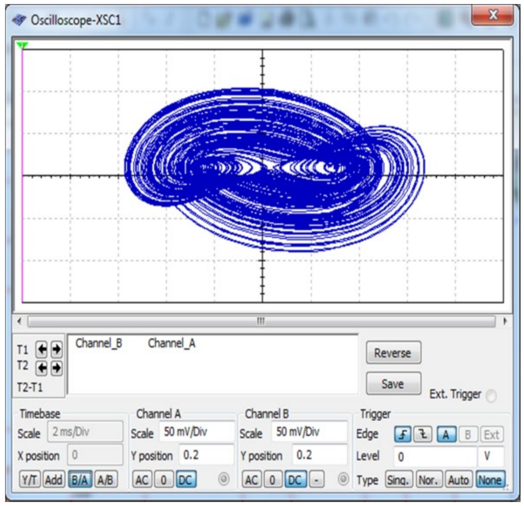
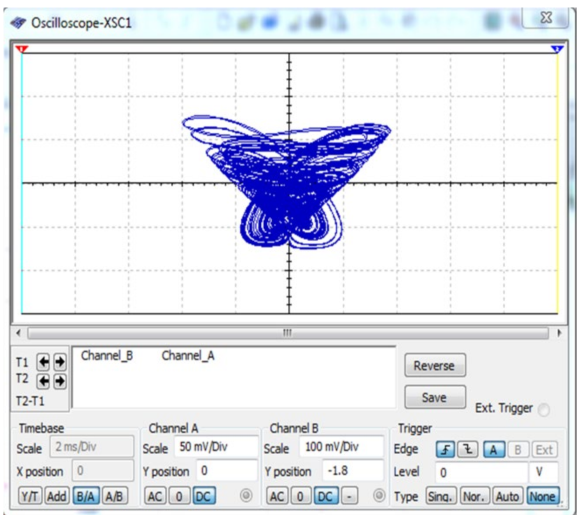


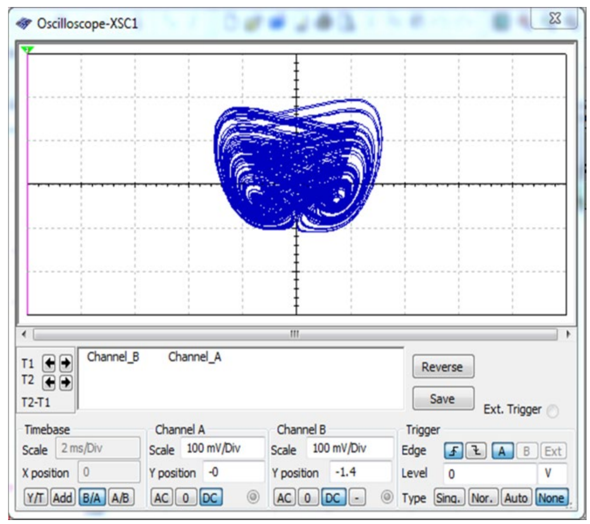
Figure 11 Circuit design for the new 3-D chaotic system with three nonlinearities by MultiSIM



(a)



(b)



(c)

Figure 12 The phase portraits of the new 3-D chaotic system with three nonlinearities observed on the oscilloscope in different planes (a) x_1-x_2, (b) x_2-$x_3$plane and (c) x_1-x_3 plane by MultiSIM

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