Numerical Simulation of Finite Difference Time Domain (FDTD) for Solving the Boundary Value Problem (BVP) in Earth Layer

Adam Sukma Putra, Idham Syah Alam
Department of Physics, Faculty of Mathematics and Natural Sciences,
Universitas Gajah Mada, Indonesia
adamsukmaputra@ugm.ac.id

Sukono
Department of Mathematics, Faculty of Mathematics and Natural Sciences,
Universitas Padjadjaran, Indonesia
sukono@unpad.ac.id, betty.subartini@unpad.ac.id, napitupuluherlina@gmail.com

Wahyu Srigutomo
Department of Physics, Faculty of Mathematics and Natural Sciences,
Bandung Institute of Technology, Indonesia
wahyu@itb.ac.id

Abdul Talib Bon
Department of Production and Operations,
University Tun Hussein Onn Malaysia, Malaysia
talibon@gmail.com

Abstract

electromagnetic problem. The FDTD is derived by discretizing the Maxwell's Equation using the finite difference (FN) method. We test the governing equation by numerically constructing the Yee Algorithm in 1-Dimensional (1-D) System to describe the distribution of the Transverse Electric (TE) and Transverse Magnetic (TM) within two type boundary condition. First, we simulate the system using Perfectly Electrically/Magnetically Conducting (PEC/PMC) boundary condition, second with Absorbing Boundary Condition (ABCs). We assume that TE and TM are propagate in homogenous and isotropic media. Thereore, the conductivity σ, permeability, μ and permittivity remains constant time by time. For a further studiy, we apply the simulation to the isotropic and homogenous 2 dimensional earth layers that have a various condition of the BVP. The result leads to the conclusion that for the ideal condition of the layered earth model, the simulation is able to give a best solution for each earth layer problem.

Keywords: Electromagnetic, FDTD, BVP, Finite Difference, numerical simulation. Earth Layer Problem

The Finite Difference Time Domain (FDTD) method is an application of the finite difference method, commonly used on solving differential equations to solve Maxwell's equations. The time-dependent Maxwell's equations (in partial differential form) are discretized using central-difference approximations to the space and time partial derivatives. (rathiv *et al, 2012*)

The resulting finite difference equations are solved in FDTD, space is divided into small portions called cells. On the surfaces of each cell, there are assigned points. Each point in the cell is required to satisfy Maxwell's equations. (Chan, A, 2006). In this way, electromagnetic waves are simulated to propagate in a numerical space.. FDTD is one of the commonly used methods to analyse electromagnetic phenomena at radio and microwave frequencies. The FDTD The technique was first proposed by K. Yee in early 70s

Theory

The theory on the basis of the FDTD method is to solve an electromagnetic Problem by discretizing the Maxwell's equations both in time and space with central difference approximations.(Yee, 1966). As we know clearly that any classical electrodinamic phenomena can be described physically and mathematically by the Maxwell's equation

Let us see again the complete four set ofMaxwell equation.

$$\nabla \cdot \mathbf{D} = \rho \qquad (1)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad (2)$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \qquad (3)$$

$$\nabla \cdot \mathbf{H} = 0 \qquad (4)$$

where $\mathbf{D} = \varepsilon\mathbf{E}$ $\mathbf{B} = \mu\mathbf{H}$

where **B** is the magnetic flux density, **E** is the electric field intensity, **D** is the electric flux density, **H** is the magnetic field intensity and ε is the electric permittivity and μ is the magnetic permeability.

In order to better understand, Let consider the maxwell's equation for the free space, therefore there is source of charge (ρ=0) and the conductivity is zero (σ=0). The maxwell's equation for the free space become

$$\nabla \cdot \mathbf{D} = 0 \qquad (5)$$

$$\nabla \times \mathbf{E} = -\mu\frac{\partial \mathbf{H}}{\partial t} \qquad (6)$$

$$\nabla \times \mathbf{H} = \epsilon\frac{\partial \mathbf{E}}{\partial t} \qquad (7)$$

$$\nabla \cdot \mathbf{H} = 0 \qquad (8)$$

This set of equation is also valid for the homogenous and isotropic media.

Yee Algorithm

The FDTD method has been developed for one-dimensional, two-dimensional and three-dimensional forms, but for simplicity here, the two-dimensional problem for the transverse magnetic case (Yee, 1966).

Let see the equation (6) and (7).

$$\nabla \times \mathbf{E} = -\mu\frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{H} = \epsilon\frac{\partial \mathbf{E}}{\partial t}$$

For 1-D case, we can rewrite the equation to be

$$-\frac{1}{\varepsilon}\frac{\partial \mathbf{H}_y}{\partial \mathrm{z}} = \frac{\partial \mathbf{E}_x}{\partial t} \qquad (9)$$

$$-\frac{1}{\mu}\frac{\partial \mathbf{E}_x}{\partial \mathrm{z}} = \frac{\partial \mathbf{H}_y}{\partial t} \qquad (10)$$

That represent a plane wave travelling in the z-direction. (Sadiku,2006).

We can see that the Transverse Electric (TE) mode and Transverse Magnetic (TM) mode are completely uncoupled from one another; in other words, they contain no common vector field components and can therefore exist independently from one another. That represent a plane wave travelling in the z-direction. (Sadiku,2006).

We now separately consider the discretization of Equations (9) and (10), respectively, for TE and TM modes using central difference

For TE mode

$$\frac{\partial \mathbf{E}_x}{\partial t} = \frac{E_x^{n+\frac{1}{2}}(k) - E_x^{n-\frac{1}{2}}(k)}{\Delta t} \qquad (11)$$

$$\frac{\partial \mathbf{H}_x}{\partial z} = \frac{H_y^n\left(k+\frac{1}{2}\right) - H_y^n\left(k-\frac{1}{2}\right)}{\Delta z} \qquad (12)$$

Therefore, the equation 9 become

$$\frac{E_x^{n+\frac{1}{2}}(k) - E_x^{n-\frac{1}{2}}(k)}{\Delta t} = -\frac{1}{\varepsilon}\frac{H_y^n\left(k+\frac{1}{2}\right) - H_y^n\left(k-\frac{1}{2}\right)}{\Delta t} (13)$$

For TM mode

$$\frac{\partial \mathbf{H}_y}{\partial t} = \frac{H_y^{n+1}(k+\frac{1}{2}) - H_y^{n-1}(k-\frac{1}{2})}{\Delta t} \qquad (14)$$

$$\frac{\partial \mathbf{E}_y}{\partial z} = \frac{E_x^{n+\frac{1}{2}}(k+1) - E_x^{n-\frac{1}{2}}(k)}{\Delta z} \qquad (15)$$

Therefore, the equation (10) become

$$\frac{H_y^{n+1}(k+\frac{1}{2}) - E_y^{n-1}(k-\frac{1}{2})}{\Delta t} = -\frac{1}{\mu}\frac{E_x^{n+\frac{1}{2}}(k+1) - E_x^{n-\frac{1}{2}}(k)}{\Delta z} (16)$$

Equations (13) and (16) show the usefulness of Yee's scheme in order to have a central difference approximation for the derivatives. In particular, the left term in equation (13) says that the derivative of the **E** field at time nΔt can be expressed as a central difference using **E** field values at times (n+1/2)Δt and (n-1/2)Δt. The right term in equations (13) approximates instead the derivative of the **H** field at point kΔx as a central difference using **H** field values at points (k+1/2)Δz and (k-1/2)Δz. This scheme is known as "leap-frog" algorithm.(Andrew, 2004) See figure. (1)

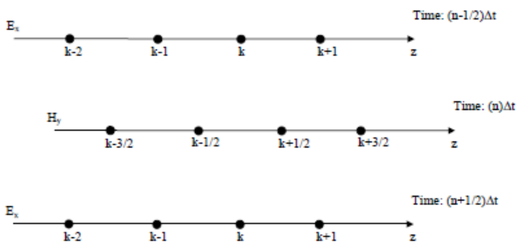

Figure 1. Yee's one-dimensional scheme for updating EM fields in space and time

The explicit FDTD equation can be derived from (13) and (16) obtaining

$$E_x^{n+\frac{1}{2}}(k) = E_x^{n-\frac{1}{2}}(k) + \frac{\Delta t}{\varepsilon \Delta z}\left(H_y^n\left(k-\frac{1}{2}\right) - H_y^n\left(k+\frac{1}{2}\right)\right) (17)$$

$$H_y^{n+1}\left(k+\frac{1}{2}\right) = H_y^n\left(k+\frac{1}{2}\right) + \frac{\Delta t}{\mu \Delta z}\left(E_x^{n-\frac{1}{2}}(k) - E_x^{n+\frac{1}{2}}(k+1)\right) (18)$$

To avoid computational problem, (taflove, 2004) in his book, taflove introduced a normalization of the E field

$$E = \sqrt{\frac{\epsilon}{\mu}}E \qquad (19)$$

Now, equation (17) and (18) become

$$E_x^{n+\frac{1}{2}}(k) = E_x^{n-\frac{1}{2}}(k) + \frac{1}{\sqrt{\epsilon\mu}}\frac{\Delta t}{\Delta z}\left(H_y^n\left(k-\frac{1}{2}\right) - H_y^n\left(k+\frac{1}{2}\right)\right) (19)$$

$$H_y^{n+1}\left(k+\frac{1}{2}\right) = H_y^{n}\left(k+\frac{1}{2}\right) + \frac{1}{\sqrt{\epsilon\mu}}\frac{\Delta t}{\Delta z}\left(E_x^{n-\frac{1}{2}}(k) - E_x^{n+\frac{1}{2}}(k+1)\right) \quad (20)$$

For the stability reason,

$$\frac{1}{\sqrt{\epsilon\mu}}\frac{\Delta t}{\Delta z} \le 1 \qquad (21)$$

So,

$$\Delta t \le \frac{\Delta z}{\sqrt{\epsilon\mu}} \qquad (22)$$

Or,

$$\Delta t \le \frac{\Delta z}{c\sqrt{d}} \qquad (23)$$

Where $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$, and $\sqrt{d} = \sqrt{\epsilon_r\mu_r}$

for numerical purpose, we define the $\sqrt{d} = 2$. (Taflove,2004).

signal source

to simplify the simulation, we consider to chose the sinusoidal signal as he source of the wave. With E_0 and as the initial pulse at t=0

$$E_x = E_{x0} + \sin(\omega t) \qquad (24)$$

$$E_x = H_{x0} + \sin(\omega t) \qquad (25)$$

Boundary Condition

To simplify the simulation, we define the boundary condition at node:

$$E_x(0,n) = 0$$

$$H_y(0,n) = 0$$

Finally, the equation (19) and (20) can be simulate in MATLAB

I. Simulation and Result

By using MATLAB, we then solve the equation (19) and (20) numerically. The result shows the sinusoidal field propagation of both TE and TM mode. (see figure. 2).

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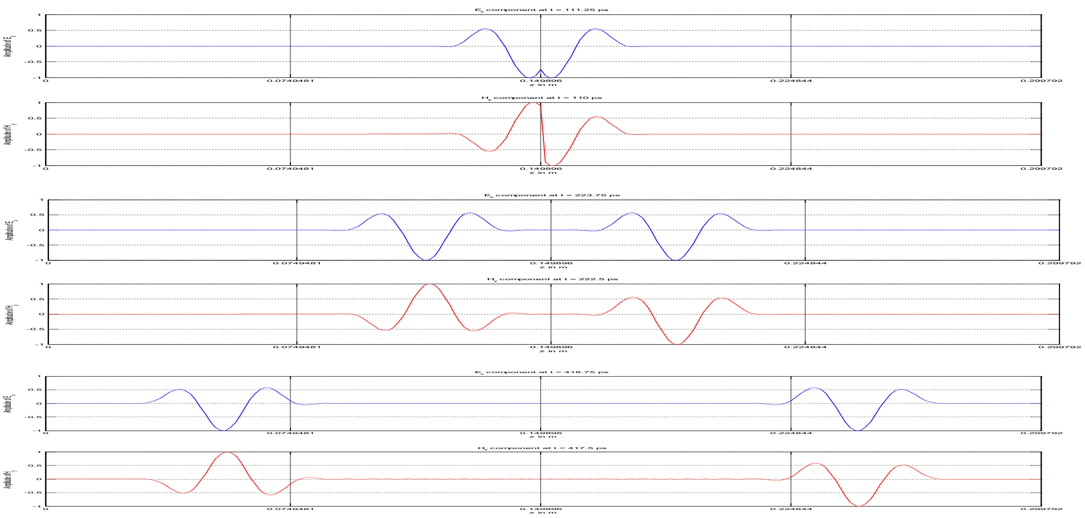


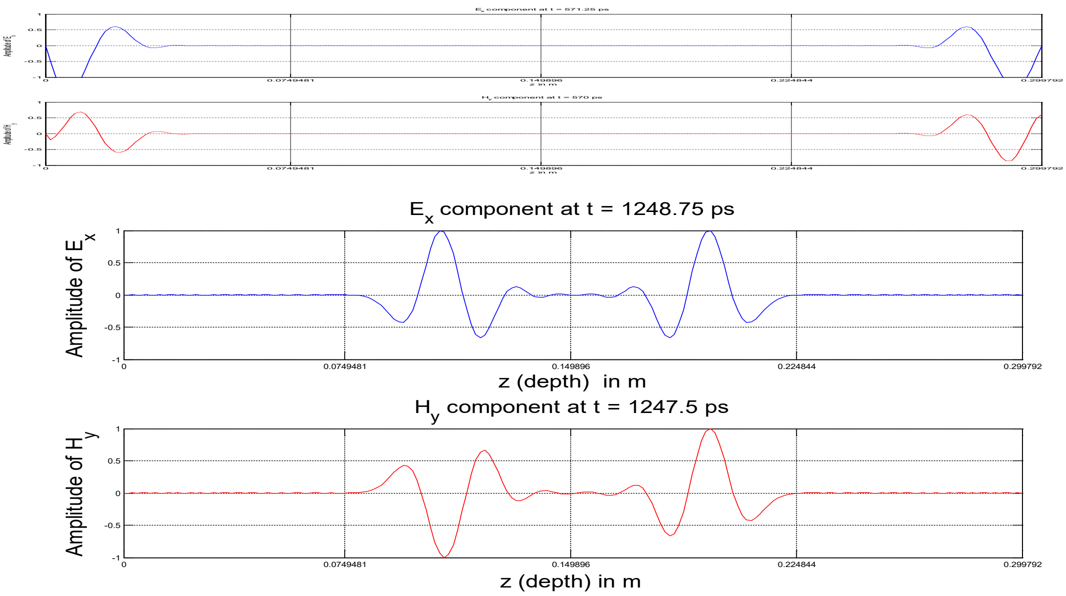


Figure 2. the E_x and H_y propagation along the z direction and reflected by the boundary. The lower image is the final state at t=1250 ps.

From the simulation we can see that E_x and H_x separated into two wave which propagate in opposite direction. Where the E_x has the same the opposite phase with H_x in the positive direction, and opposite phase in the negative direction. It is not important, because we define it as the initial condition. The most important is, the condition when the wave collide the boundary. It is reflected in the same magnitude, but with opposite pahse. It means that the boundary is acting like an operator for changing the phase but still preserve the magnitude. This boundary condition called Perfectly Electrically Conducting (PEC). Perfectly Magnetical Conducting (PMC).

PEC and PMC boundaries are specified by simply setting the boundary electric field node Ex = 0 or the boundary magnetic field node Hy = 0, respectively.

Simulation with absorbing Boundary Condition

we need to model a region that "trap" the field inside. In most of the problems, however, we need to simulate open space regions. In these cases, since our simulation region MUST be limited, we need to find a way to "simulate" the open space. These boundary conditions are called Absorbing Boundary Conditions (ABCs). (chan,2006)

Let us see again the equation (19) and (20)

The absorbing boundary condition for the 1-D case can be therefore expressed by

For z=1

$$E_x^{n+\frac{1}{2}}(1) - E_x^{n-2+\frac{1}{2}}(K) \qquad (26)$$

For the left side of the mesh

$$E_x^{n+\frac{1}{2}}(KE) - E_x^{n-2+\frac{1}{2}}(KE-1) \qquad (27)$$

for the right side of the mesh. With these conditions, in the 1D simulation described in the previous section the wave will be completely "absorbed" by the termination (boundary). (chan,2006).

The boundary condition proposed by G.Mur (2009) can be explained by considering the wave equation that the electromagnetic field obey. Considering the 1D case, we can write the following wave equation

$$\frac{\partial^2 \mathbf{E}_x}{\partial t^2} - \frac{1}{c^2}\frac{\partial^2 \mathbf{H}_x}{\partial \mathrm{t}^2} = 0 \qquad (28)$$

The equation (28) can be rewrite as

$$\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)\mathbf{E}_x=0 \qquad (29)$$

Engquist and Madja (2009) have shown that an absorbing boundary condition for the left side of the grid (z=0) can be derived applying the condition.

$$\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)\mathbf{E}_x=0 \qquad (30)$$

By substituting the equation (26) & (27) to the equation (30), we obtain

$$\frac{1}{\Delta z}\left(E_x^n(1)-E_x^n(K)\right)-\frac{1}{c\Delta t}\left(E_x^{n+\frac{1}{2}}(1)-E_x^{n-2+\frac{1}{2}}(K)\right)=0 \quad (31)$$

Therefore

$$\frac{1}{c\Delta t}\left(\frac{E_x^{n+1/2}(2)-E_x^{n-1/2}2}{2}-\frac{E_x^{n+1/2}(1)-E_x^{n-1/2}(1)}{2}\right)-\frac{1}{c\Delta t}\left(\frac{E_x^{n+\frac{1}{2}}(1)-E_x^{n-\frac{1}{2}}2}{2}-\frac{E_x^{n+\frac{1}{2}}(1)-E_x^{n-\frac{1}{2}}(2)}{2}\right)=0 \quad (31)$$

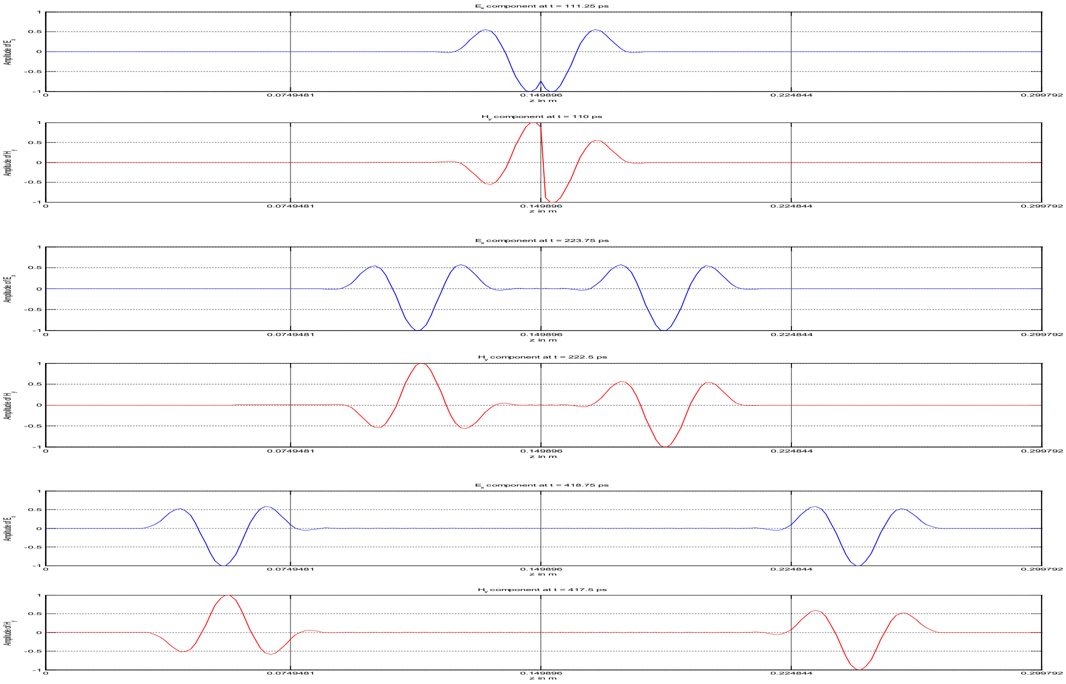
Simplification for the left boundary. (chan,2006)

$$E_x^{n+\frac{1}{2}}(1)=E_x^{n-2+\frac{1}{2}}(2)+\frac{c\Delta t-\Delta z}{c\Delta t+\Delta z}\left(E_x^{n+\frac{1}{2}}(2)-E_x^{n-\frac{1}{2}}(1)\right) \quad (32)$$

And for the right boundary

$$E_x^{n+\frac{1}{2}}(KE)=E_x^{n-2+\frac{1}{2}}(KE-1)+\frac{c\Delta t-\Delta z}{c\Delta t+\Delta z}\left(E_x^{n+\frac{1}{2}}(KE-1)-E_x^{n-\frac{1}{2}}(KE)\right) \quad (33)$$

Finally, we simulate the equation (32) and (33) using MATLAB.



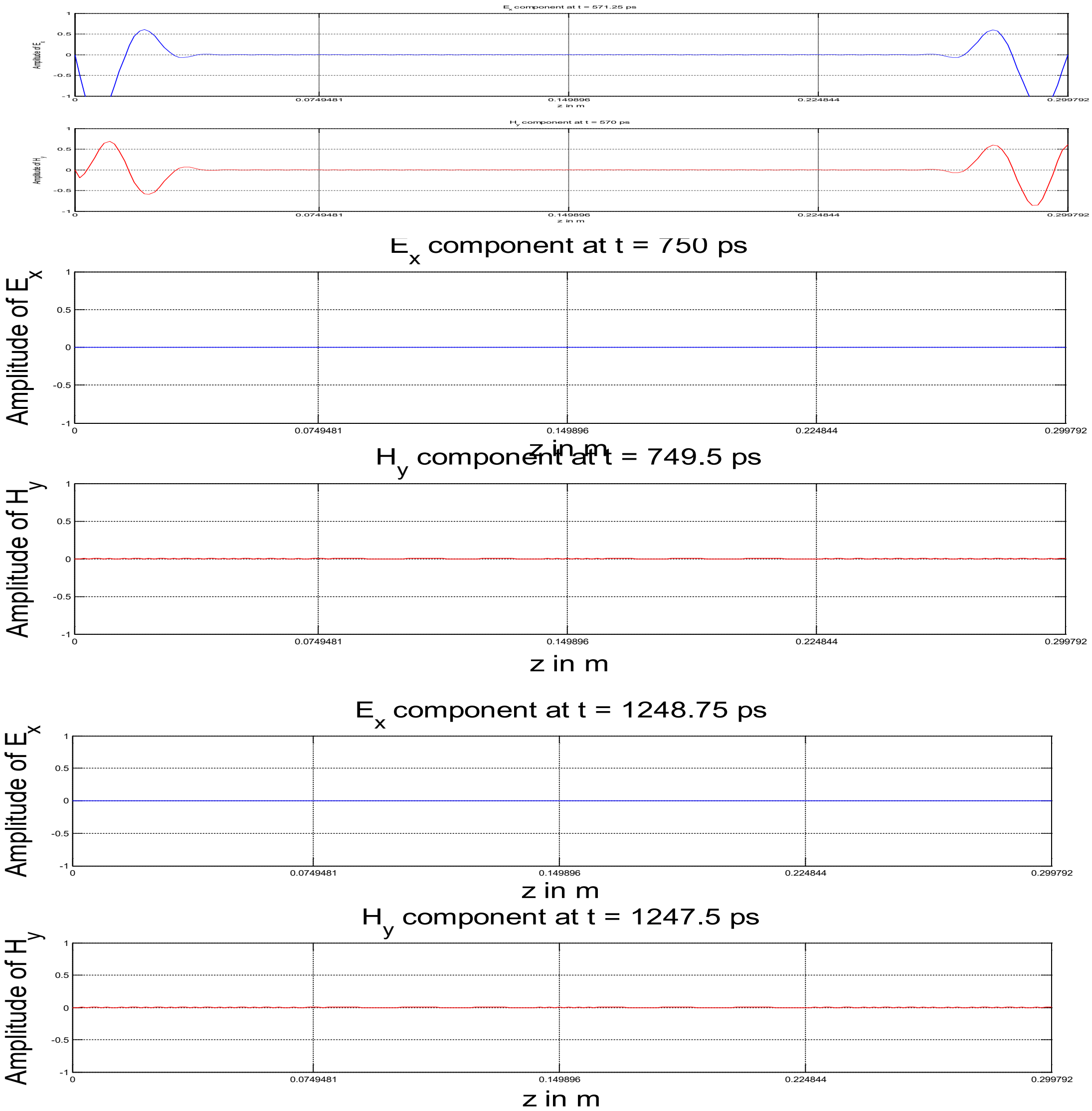


Figure 3. the E_x and H_y propagation along the z direction and reflected by the boundary ABCs

From the figure above, we can see that after the wave is collide with the boundary, there are no reflected wave both for TE and TM mode. It is called as ABCs boundary condition which can absorb the magnitude of both TE and TM.

Conclusion

The FDTD method is widely used because it is simple to solve numerically. It can solve Maxwell's time-dependent curl equations by using finite differences to discretize them. This report describes the design of 1- FDTD simulation for both TE and TM mode. This report also successfully demonstrates a working 1D-FDTD code that correctly implements PEC, PMC, and ABCs boundaries.

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Biographies

Adam Sukma Putra is a lecturer at the Department of Physics, Faculty of Mathematics and Natural Sciences, Universitas Gajah Mada, the field of Geophysisc, with a field of concentration of Seismic.

Idham Syah Alam is a lecturer at the Department of Physics, Faculty of Mathematics and Natural Sciences, Universitas Gajah Mada, the field of Geophysisc, with a field of concentration of computational particle physics.

Sukono is a lecturer in the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran. Currently serves as Head of Master's Program in Mathematics, the field of applied mathematics, with a field of concentration of financial mathematics and actuarial sciences.

Abdul Talib Bon is a professor of Production and Operations Management in the Faculty of Technology Management and Business at the Universiti Tun Hussein Onn Malaysia since 1999. He has a PhD in Computer Science, which he obtained from the Universite de La Rochelle, France in the year 2008. His doctoral thesis was on topic Process Quality Improvement on Beltline Moulding Manufacturing. He studied Business Administration in the Universiti Kebangsaan Malaysia for which he was awarded the MBA in the year 1998. He's bachelor degree and diploma in Mechanical Engineering which his obtained from the Universiti Teknologi Malaysia. He received his postgraduate certificate in Mechatronics and Robotics from Carlisle, United Kingdom in 1997. He had published more 150 International Proceedings and International Journals and 8 books. He is a member of MSORSM, IIF, IEOM, IIE, INFORMS, TAM and MIM.

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