Multi-Product Multi-Vehicle Inventory Routing Problem With Mixed Integer Linear Programming

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Abstract

Multiproduct Multivehicle Inventory Routing (MMIRP) is an extension of the Inventory Routing Problem (IRP) that integrating route and inventory decisions simultaneously to minimize total transportation and inventory costs. This paper aims to implement The MMIRP method to design the routes of LPG distribution in Indonesian LPG supply chain.The MMIRP model of this article considers two products: 5.5 kg and 12 kg LPG and 2 vehicles to dispersed sub-agent and retailer customers. The existing distribution activities the company does not pay attention a route sequence and inventory at the customers so the expenses of transportation costs are very high and when the customer visited the inventory level is still high. The problems are solved using Mixed Integer Linear Programming with LINGO software. The results obtained after applying MMIRP resulted in a total distance can savings of 1.75% or 9.77 km, reducing total fuel consumption of 13.39% or 11.84 liters and minimizing total transportation and inventory costs by 16.01%.

Keywords
Inventory Routing Problem, Multiproduct, Multivehicle, Mixed Integer Programming

1. Introduction

Transportation and inventory decisions are a very influential problem in a company's supply chain management process. Inventory Routing Problem (IRP) is a combination of transportation decisions and inventory decisions, namely route problems and inventory management problems that refer to the Vendor Managed Inventory (VMI) system. IRP is applied to various types of industries including maritime logistics, distribution of liquefied natural gas, paper industry raw materials, food distribution in supermarket supply chains, automobile components, perishable products, food and beverages, cement, fuel, blood and organic oil waste. In its development, in the IRP there appeared several variants in the form of Multiproduct and Multivehicle problems which are closer to real life problems, this problem is known as the Multiproduct Multivehicle Inventory Routing Problem (MMIRP). MMIRP is a problem where suppliers are in charge of distributing several products to various customers that are spread geographically using a number of vehicle fleets simultaneously [2]. Since IRP is the result of the development of VRP cases which are included in the NP-Hard problem, the resulting combination will become very complex. Computing time in NP-Hard cases will increase as the size of the problem increases [3]. Some MMIRP methods are solved by Heuristic methods [4],[5] and Exact methods [1],[6], [7] and [8]. The use of the Exact method has advantages in the level of effectiveness in producing optimal solutions [9]. Since transportation activities have a big impact on the environment, logistics activities taking into account environmental aspects have been developed to get sustainable policies without impacting the environment [10]. There are several studies that consider CO2 emissions and consider the fuel consumption factors of vehicles including from previous studies [7], [11], [10] and [12].

PT. Gading Mas Indah is one of the LPG agents in Susanto street No. 1D Ciptomulyo, Malang City which distributes PSO products namely LPG 3 Kg and Non-PSO LPG namely Bright Gas 5.5 Kg, 12 Kg LPG and 50 Kg LPG. In this study focused on the distribution of Bright Gas products of 5.5 Kg and 12 Kg of LPG with a total of 46 customers scattered in Malang City using a fleet of 2 car pickup vehicles. Each product can be loaded into a vehicle simultaneously in each shipment provided it does not exceed the vehicle capacity of 1300 Kg. Distribution is carried out every day from Monday to Saturday. During this time the distribution of the product by the company did not use special methods for determining the delivery route and stocking supplies. In daily life, shipments are carried out around the customer according to the driver's subjectivity. As a result, the total mileage becomes even greater, more fuel consumption and a greater total cost.

Distribution activities at PT. Gading Mas Indah is an IRP problem, especially the Multiproduct Multivehicle Inventory Routing Problem (MMIRP). Where the company distributes 2 products with 2 vehicles to 46 customers. Companies always try to meet customer demand every day and do not expect a stockout at the customer. Then it takes simultaneous decision for the driver so that the route that will be passed optimally while still meeting the customer's daily demand and avoid the occurrence of a stockout. Therefore, in this study will apply the MMIRP method with MILP with the aim of minimizing the total transportation and inventory costs. In the total transportation costs, the vehicle fuel consumption is calculated by considering the load, distance and rate of vehicle fuel consumption.

2. Research Method

2.1 The Characteristic of the System

Based on the characteristics of the company, each vehicle can travel more than once per day. Vehicles can transport 5.5 Kg Bright Gas products and 12 Kg of LPG in each trip to 46 customers spread across the Klojen, Blimbing, Kedungkandang, Lowokwaru and Sukun areas. The vehicle has a payload capacity of 1300 Kg and each product has a mass of 12.6 Kg and 27.1 Kg. Product distribution includes reverse logistic processes in the form of unit. So if there is an empty tube in the customer, it will be replaced with a tube of contents when it is visited according to the number of the empty unit.

2.2 Data Collection

The data needed in this study include product and mass data, vehicle data (vehicle capacity, fuel consumption when full and current loads without load), customer data (weekly demand, product storage capacity, product purchase price, initial inventory, the distance to the agent, the distance between the customers), the fraction of the storage cost, the price of the vehicle fuel, the initial route, the initial load and the number of products the customer receives every day.

2.3 Data Processing

2.3.1 The calculation of fuel consumption rate

Fuel consumption is calculated to find the level of consumption of a vehicle's fuel when it is free and when the load is full. Determination of vehicle fuel consumption in liters / Km is obtained by the full to full method which is approximated by equation (1) according to [13] by volume method bellows.

$$C = \frac{V(1+\alpha[T_o - T_F])}{L} \times 100 \tag{1}$$

Furthermore, the fuel consumption rate is calculated so that it is obtained in liters / Kg / Km according to the formulation of [14] in the equation bellows (2).

$$\alpha = \frac{\rho^* - \rho}{Q} \tag{2}$$

2.2.2 The mathematical model formulation

Mathematical models are used according to deterministic mathematical models of [7]. Moreover, the researcher develops a mathematical model according to the characteristics of LPG companies. This development is found in the equation (7) dan (12), so that the following mathematical models are obtained.

Index Set

$\mathcal{V}$: Point Set, $\mathcal{V} = \{0,1,\dots,N\}$, where 0 shows thesupplier

$\mathcal{V}$ ' = $\mathcal{V}$ \ 0 : Customer

$\mathcal{A}$: Path Set$\mathcal{A} = \{(i,j)|i,j \in \forall, i \neq j\}$

$\mathcal{M}$: Product Set$\mathcal{M} = \{1,\dots,M\}$

$\mathcal{K}$: Vehicle Set$\mathcal{K} = \{1,\dots,K\}$

$\mathcal{T}$: Period Set $\mathcal{T} = \{1,\dots,T\}$

Decision Variable

x_{ijk}^t : Binary numbers, value 1 if vehicle k visits point j after point *i*, beside that is 0 $(i,j \in \mathcal{V}, k \in \mathcal{K}, t \in \mathcal{T})$

q_{ijkm}^t : The total number of product *m* carried from point *i* to point *j* by vehicle *k* in time *t* $(i,j \in \mathcal{V}, m \in \mathcal{M}, k \in \mathcal{K}, t \in \mathcal{T})$

I_{im}^t : Product inventory rate*m* in point*i* in the end of the period *t* $(i \in \mathcal{V}', m \in \mathcal{M}, t \in \mathcal{T})$

R_{im}^t : The total number of products received at the point*i*from supplier in time *t* $(i \in \mathcal{V}', m \in \mathcal{M}, t \in \mathcal{T})$

Parameter

s_m^t : Total number of product *m* available to send by the supplier in time *t*$(t \in \mathcal{T})$

c_{im}^t : Product demand *m* in point *i* in time *t* $(i \in \mathcal{V}', m \in \mathcal{M}, t \in \mathcal{T})$

h_{im}	: Inventory cost per unit for products *m* in point *i* $(i \in \mathcal{V}', m \in \mathcal{M})$
I_{im}^{0}	: Inventory level initials *i* for products *m* $(i \in \mathcal{V}', m \in \mathcal{M})$
HC_{im}	: Product inventory capacity*m*in point*i*$(i \in \mathcal{V}')$
d_{ij}	: Distance from point *i* to point $j(i, j \in \mathcal{V})$
Q_k	: Vehicle capacity $k(k \in \mathcal{K})$
a_k	: Vehicle fuel consumption *k* per kilometer without load $(k \in \mathcal{K})$
b_k	: Vehicle fuel consumption *k* per kilogram unit load per kilometer $(k \in \mathcal{K})$
w_m	: Product mass per unit $m(m \in \mathcal{M})$
u	: Fuel price per liter

The Objective Function

$$\min u \sum_{(i,j)\in\mathcal{A}} d_{ij} \left(\sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} a_k x_{ijk}^t + \sum_{m\in\mathcal{M}} \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} b_k q_{ijkm}^t w_m \right) + \sum_{i\in\mathcal{V}'} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} I_{im}^t h_{im} \quad (3)$$

The Constraint Function

$$\sum_{j\in\mathcal{V}, j\neq i} \sum_{k\in\mathcal{K}} x_{ijk}^t \leq 1 \; \forall i \in \mathcal{V}', \forall t \in \mathcal{T} \quad (4)$$

$$\sum_{j\in\mathcal{V}, j\neq i} x_{ijk}^t = \sum_{j\in\mathcal{V}, j\neq i} x_{jik}^t \quad \forall i \in \mathcal{V}', \forall k \in \forall t \in \mathcal{T} \quad (5)$$

$$\sum_{k\in\mathcal{K}} x_{ijk}^t \leq 1 \qquad \forall (i,j) \in \mathcal{A}, \forall t \in \mathcal{T} \quad (6)$$

$$\sum_{j\in\mathcal{V}'} x_{0jk}^t \geq 1 \qquad \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \quad (7)$$

$$\sum_{j\in\mathcal{V}, j\neq i} \sum_{k\in\mathcal{K}} q_{jikm}^t = R_{im}^t + \sum_{j\in\mathcal{V}, j\neq i} \sum_{k\in\mathcal{K}} q_{ijkm}^t \qquad \forall i \in \mathcal{V}', \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \quad (8)$$

$$I_{im}^{t-1} + R_{im}^t = c_{im}^t + I_{im}^t \forall i \in \mathcal{V}', \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \quad (9)$$

$$\sum_{m\in\mathcal{M}} q_{ijkm}^t w_m \leq Q_k x_{ijk}^t \qquad \forall (i,j) \in \mathcal{A}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \quad (10)$$

$$\sum_{j\in\mathcal{V}'} \sum_{k\in\mathcal{K}} q_{0jkm}^t \leq s_m^t \quad \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \quad (11)$$

$$R_{im}^t + I_{im}^{t-1} \leq HC_{im} \quad \forall i \in \mathcal{V}', \forall m \in \mathcal{M} \forall t \in \mathcal{T} \quad (12)$$

$$q_{ijkm}^t \geq 0 \qquad \forall i,j \in \mathcal{V}, \forall m \in \mathcal{M}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \quad (13)$$

$$x_{ijk}^t \in \{0.1\} \qquad \forall i,j \in \mathcal{V}, \forall k \in \mathcal{K}, \forall t \in \mathcal{T} \quad (14)$$

$$I_{im}^t \geq 0 \qquad \forall i \in \mathcal{V}, \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \quad (15)$$

$$R_{im}^t \geq 0 \qquad \forall i \in \mathcal{V}', \forall m \in \mathcal{M}, \forall t \in \mathcal{T} \quad (16)$$

The objective function in equation (3) aims to minimize total costs. The total cost is obtained from the total transportation costs plus the total of inventory costs. Equation (4) ensures that each customer is visited as much as once in each time period. Equation (5) guarantees that if a vehicle arrives at point i in the time period t, it must leave that point in the same period. Equation (6), as many as one vehicle, can use lines (i, j) in each time period. In equation (7), guarantee that each vehicle travels at least once in each period t. Equation (8) ensures that the difference between the number of incoming and outgoing products m at point i must equal the number of products m received by point i in the time period t. Equation (9) gives the level of product inventory m for each point i in period t. Equation (10) is a limitation of the capacity of a vehicle in which the mass of the product carried cannot exceed the capacity of the vehicle. Equation (11) ensures that the total number of products m sent by suppliers cannot exceed the amount of product availability at the supplier. Equation (12) guarantees that the amount of product m received by customer i with the inventory level in the t-1 period must not exceed the product capacity m. Equations (13), (14), (15), (16) provide integer (integer) and non-negative limits for decision variables. To obtain optimal distribution decision results, mathematical models in equations (3) to (16) are solved using LINGO software to produce optimum feasible or global solutions.

2.2.4 The calculation of total transportation and inventory costs

The total transportation and inventory costs are obtained directly from the LINGO software output or using equation (3) and manually calculating the values of the decision variables that have been generated.

3. Results and Discussion

Mathematical models have been translated and run using LINGO software on ASUS A455LN notebooks with specifications Intel® Core™ i3-4030U CPU @ 1.90 GHz 64-bit OS Windows 8.1 Pro dan RAM 2,00 GB. Produces feasible solutions starting at 30 seconds.

3.1 Fuel consumption level

Based on the equation (1) and (2) the results of the fuel consumption level without load is 0.125 liters / Km, the full time is 0.2 liters / Km and the slope rate is 0.0000576923 liters / Kg / Km.

3.2 The determination of the quantity of shipment per trip

The determination of the quantity of delivery produced by variables Q_{0jmkt} according to LINGO output results. The 6-day delivery planning period starts from Monday to Saturday. So that the results of the decision on vehicle delivery quantity 1 and 2 are obtained in table 1 and 2.

Table 1 Decision on vehicle delivery quantity 1

Trip	Bright Gas 5,5 Kg						LPG 12 Kg					
	Mon	Tue	Wed	Thu	Fri	Sat	Mon	Tue	Wed	Thu	Fri	Sat
1	2	0	0	3	0	0	29	20	40	6	20	20
2	0	3	0	0	4	0	10	6	20	10	36	6
3	-	0	0	4	-	2	-	10	23	34	-	3
4	-	0	5	2	-	6	-	7	40	20	-	39
5	-	6	-	-	-	-	-	23	-	-	-	-

Table 2 Decision on vehicle delivery quantity 2

Trip	Bright Gas 5,5 Kg						LPG 12 Kg					
	Mon	Tue	Wed	Thu	Fri	Sat	Mon	Tue	Wed	Thu	Fri	Sat
1	0	0	0	0	2	0	18	2	20	20	3	23

2	0	0	4	4	0	2	18	11	16	12	2	5
3	-	4	0	-	0	0	-	14	12	-	25	13
4	-	-	-	-	2	-	-	-	-	-	47	-

3.3 The assignment of vehicles to each customer

Vehicle assignments are obtained from variablesX_{ijkt} the value of 1. The results of the delivery route decision for vehicles 1 and 2 respectively in table 3 and table 4.

Table 3Decision of vehicle route 1

Day	Trip	Rute	Total Km
Monday	1	GMI – EDI – KCN – FKS – SKD – SMR – AMD – YYK – DHC – ADK – GMI	24.093
	2	GMI – GLR – GMI	6.116
Tuesday	1	GMI – EDI – KCN – GMI	5.578
	2	GMI – SWH – GMI	2.840
	3	GMI – CYN – GMI	9.560
	4	GMI – SBH – GMI	8.640
	5	GMI – TDN – AMD – DIN – CMR – GMI	26.799
Wednesday	1	GMI – GLR – GMI	6.116
	2	GMI – HHD – SRI – PJK – GMI	13.853
	3	GMI – MCL – BRK – TGP – IDR – GMI	21.836
	4	GMI – ADK – DHC – YYK – CMR – DIN – GMI	24.797
Thursday	1	GMI – SWH – GMI	2.840
	2	GMI – CYN – SBH – GMI	11.730
	3	GMI – TDN – AMD – DIN – GMI	21.836
	4	GMI –BGS – IDR – GMI	23.201
Friday	1	GMI – EDI – KCN – GMI	5.578
	2	GMI – UMR – MCL – BRK – LGG – TGP – IDR – GMI	23.120
Saturday	1	GMI – EDI – KCN – GMI	5.578
	2	GMI – HHD – DVA – GMI	10.284
	3	GMI – FKS – GMI	10.664
	4	GMI – SKD – AMD – CMR – MG2 – DIN – MG1 – YYK – GMI	26.525
Total Km			291.347

Tabel 4 Decision of vehicle route 2

Day	Trip	Rute	Total Km
Monday	1	GMI – UMR – TGP – IDR – GMI	22.590
	2	GMI – BGS – DIN – GMI	21.118
Tuesday	1	GMI – DTA – GMI	12.690
	2	GMI – IDR – GMI	18.140
	3	GMI – FKS – SKD – GMI	15.487
Wednesday	1	GMI – EDI – KCN – GMI	5.578
	2	GMI – UMR – MTR – GMI	14.739
	3	GMI – TDN – GMI	12.624
Thursday	1	GMI – EDI – KCN – GMI	5.578
	2	GMI – FKS – SKD – GMI	15.497
Friday	1	GMI – MGS – GMI	5.420

	2	GMI – WPG – GMI	9.966
	3	GMI – CYN – T82 – SBH – FKS – GMI	16.140
	4	GMI – TDN – SMR – DIN – MG1 – ADK – DHC – YYK – GMI	23.464
Saturday	1	GMI – DTA – MCL – TGP – IDR – SRJ – GMI	35.689
	2	GMI – SBH – GMI	8.640
	3	GMI – TDN – BRL – GMI	13.001
Total Km			256.361

From the processing of the company's initial data and processing from the results of the output solver manually a summary is obtained in table 5.

Table 5 Summary of the results of route decisions

Comparison	Start	Proposed approach
Trip	25 trip	38 trip
Distance	557.48 Km	547.71 Km
	88.39 liter	76.55 liter

From the results in table 5, the total trip from the proposed method is 38 trips, while the company method is 25 trips. So there is a difference of 13 more trips or an increase of 52%. However, the total distance generated from the proposed method is lower by 1.75% or the difference of 9.77 Km from the total initial distance. Where the total distance by means of the company is 557.48 Km, while the proposed method is 547.71 Km. This is in line with the results of the prior research [15], where in the study there were more 5 trip differences with a more minimal distance. Furthermore, the total fuel consumption is released according to the company's method of 88.39 liters while the proposed method is 76.55 liters. Thus, there is a difference between a decrease of 11.84 liters or a decrease of 13.39%. Thus, in transportation problems that take into account factors of vehicle fuel consumption even though the number of trips produced is more, but produces a total distance and a lower total fuel consumption. This is in line with the research [16] where route problems that consider fuel consumption can minimize the level of fuel consumption needed, even with more distance and number of trips.

3.4 Number of products accepted

Each customer visited on the results of the route decision has the number of products that will be received. This amount is to meet daily demand and supply. Table 6 shows the result for the number of products received by each customer.

Table 6 Number of products received by the customer

Cust.	Bright Gas 5,5 Kg						LPG 12 Kg					
	Mon	Tues	Wed	Thu	Fri	Saturday	Mon	Tue	Wed	Thu	Fri	Wed
EDI							8	13	13	13	13	13
KCN							1	7	7	7	7	7
BRK									1		1	

MGS					2						3	
PJK									2			
GLR							10		40			
GRB												
UMR							9		16		16	
DTA								2				4
MTR			4									
DVA												1
WPG											2	
HHD									15			5
DWI												
MCL									5		3	3
TGP							4		6		3	3
RTL												
FRH												
LGG											2	
IDR				2	4		5	11	11	11	11	2
SRJ												2
SHB												
SRI									3			
SWH		2	1	2	3			6		6		5
CYN								10		5	10	
SBH								7		5	10	
FKS	1	4		4		2	2	6		4	3	3
TDN								4	12	12	12	12
BRL												1
SKD							4	8		8		4
T82											2	
SBR												
SMR					1		2				2	
ANJ												
AMD	1	4		4		2				1		4
MG1						1					2	2
MG2												3
ADK			1		1		4		6		6	
DHC							4		5		3	
YYK						1	4		10		5	5
TBR												
CMR		2	4			2						2
TDR												
BWN												
DIN							13	19	19	21	17	19
BGS							5			9		

Based on preliminary data using this method, Bright Gas's initial total inventory is 72 unit and the total number of product receipts is 55 unit and a demand of 117 unit. Then, resulting in a residual difference of 10 unit. Requests for 117 unit can be fulfilled with the initial inventory amount and the amount of product receipt. Similarly, for LPG 12 Kg products where the weekly total demand is 903 unit, the total amount of initial inventory is 230 unit with the total acceptance of 683 unit. Then, resulting in a residual difference of 10 unit. Total demand is still fulfilled by the amount of initial inventory and the number of products received. This is a proof that MMIRP can fulfill all customer daily requests without experiencing a stockout, as well as stated in the prior research [1].

3.4 The Level of Inventory at Each Product

The output results in table 7 provide the total inventory level at the end of the day from all customers for 6 days.

Table 7 Total inventory level for each product

Comparison		Start (unit)	Proposed approach (unit)
Inventory	BG 5.5 Kg	403	193
Product	LPG 12 Kg	1,071	455

Based on table 7, the difference in total inventory ending between the initial method of the company and the proposed method for 5.5 Kg Bright Gas product is 210 unit. This number decreased by 52.10% of the initial method of the company. Whereas the 12 Kg LPG product produced a difference of 616 unit. This number decreased by 57.51%. Based on these results, it is found that the results of a smaller amount of inventory prove that the existence of inventory needs to be eliminated, if not possible then the amount must be minimized by ensuring that the fulfillment of the demand of the user remains smooth. This result is as stated before [17]. Although it is able to guarantee that demand is still fulfilled, the amount of inventory that is feared is not able to meet demand if at any time the demand is high or the company cannot fulfill it for certain reasons. From here it is necessary to guarantee that the inventory level at the customer cannot be empty. For that, it is necessary to add a safety stock function to each customer so that the inventory level is not empty.

3.5 The calculation of total transportation and inventory costs

After processing and discussion, the results of the initial transportation costs were Rp. 689,452 and inventory costs were Rp. 57,256 with a total cost of Rp. 746,709. The proposed method generates transportation costs of Rp. 597,081 and inventory costs of Rp. 30,038 with a total cost of Rp. 627,119. So that the proposed method produces a total transportation cost savings of Rp. 92,371 or 13.39% and the total cost of inventory saves Rp. 27,218 or 47.53%. Thus, the proposed method results in a total cost savings of Rp. 119,590 of the total initial cost or a saving of 16.01% for six days.

4. Summary of Results

From the research that has been done, the MMIRP problem at PT. Gading Mas Indah is completed using MILP capable of minimizing the total transportation and inventory costs as in research purposes. The results showed that the total distance savings were 1.75% or 9.77 Km, reducing the total fuel consumption of 13.39% or 11.84 liters, minimizing the total inventory of Bright Gas products by 5.5 Kg 52.10% or 210 unit, while in LPG products 12 Kg by 57.51% or 616 unit and minimizing the total transportation and inventory costs by

16.01% or Rp. 119,590. For further research, it is expected to add constraints to safety stock, availability of products at the agent every day, maximize the amount of product acceptance at the customer and carry out a heuristic approach to streamline route results.

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