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A Dual Approach to the Harmonized Model between Inventory Reduction and Heijyunka (Production Leveling) Based on the Minimum Average-Energy Principle

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Abstract

A major issue in manufacturing is the low cost production and supply of products in response to variable demand while ensuring no sales opportunity is missed. In previous studies, many factors affecting production allocation, such as the number of workers and working hours, have been examined. In contrast, we have worked on the development of a new method to simply determine the production quantity "balanced (harmonized) between inventory reduction and heijyunka (production leveling)," based on the demand quantity and the inventory quantity. We have proposed a harmonized model of inventory reduction (lowering energy of inventory management) and production leveling (increasing entropy of production allocation) based on the "maximum entropy principle." In this study, we attempt a "dual approach" to exchange the objective function and the constraint to the harmonized model and propose a new model based on the "minimum average-energy principle." This enables multifaceted analysis of harmonized levels between production leveling and inventory reduction.

Keywords
Aggregate Production Planning, Maximum Entropy Principle, Smoothing Coefficient, Production Allocation Entropy, Inventory Management Energy

1. Introduction

The manufacturing industry has experienced major changes in its market environment in recent years, and manufacturing and supplying at low cost, without missing sales opportunities, has become a significant issue against a backdrop of changing demand. Therefore, production and inventory management has a difficult problem to solve by finding a balance between "agile production," allowing the supply of products to the market in a timely manner for inventory reduction, and "stable (leveled) production," which is aimed at reducing manufacturing costs. It is an even more important issue for manufacturers of seasonal products, in particular, while in production planning, especially aggregate production planning, the production planning problem must be determined for each production quantity per period through the planning period (Kuroda 1994).

Kurihara and Yamashita (2013) considered this production planning problem to be one of balancing agile production (inventory reduction) in response to changing sales and leveled production in pursuit of efficiency; they proposed a harmonized model between inventory reduction and production leveling (referred to as "the base model"), which is formulated as an entropy maximization problem under constant average energy, based on the "maximum entropy principle" (Jaynes 1957).

In this study, we try a dual approach to replace the objective function and constraints of the maximum entropy principle for such a problem, based on the "minimum average-energy principle" (Fukao 1987), and propose a new harmonized model between inventory reduction and production leveling. Also, similar to the base model (Kurihara and Yamashita 2013), we verify the validity of the proposed model with simple numerical examples based on the sales results of a seasonal product (home air conditioner).

2. Literature overview

Linear programming methods offer a typical solution to the production planning problem in the aggregate production planning (Buffa and Miller 1979). In linear programming methods, the total cost, including the manufacturing cost and the inventory cost, is minimized in consideration of various factors and data, such as hours required to produce one unit of product, regular time hours, overtime hours, backlog, and each unit cost. Therefore, when applying these methods to practical problems, the number of elements and constraints on relationships between elements become complicated, which may lead to an increase in calculation time (Buffa and Miller 1979).

On the other hand, the aggregate production planning is required to timely and effectively ensure consistency between business plan (corporate planning/finance department), sales plan (sales department), and manufacturing resource plan (production department) in the company-wide sales and operations planning (S&OP) process (Bowersox et al. 2010). Furthermore, when coordinating between plans and departments, it is necessary to use common terms for planning quantity that are simple and easy to understand, and "sales (demand) quantity, production quantity, and inventory quantity" are often used to refer to planning quantity. Therefore, by focusing on the demand quantity, the production quantity, and the inventory quantity as the basic planning elements of the aggregate production planning, we have worked on the development of a new method for determining the production quantity that is "balanced (harmonized) between inventory reduction and production leveling" as simply as possible, based on the demand quantity and the inventory quantity.

Generally, in the agile production, it is necessary to reduce costs related to inventory management by reducing inventory quantity. Meanwhile, to reduce manufacturing costs, it is necessary to effectively utilize work force and production facilities through production leveling, which minimizes fluctuations in production quantity, thereby stabilizing it. Based on Yamashita (2010), if inventory management cost is captured by its workload (energy), then cost reduction on inventory management by inventory reduction corresponds to lowering (decreasing) energy of inventory management. On the other hand, if production leveling is taken as entropy, equalization (equally distributed state) of production allocation by leveling can be regarded as increasing entropy of production allocation. Therefore, the policy-mix problem of inventory reduction and production leveling can be treated as a balancing (harmonized) problem between decreasing inventory management energy and increasing production allocation entropy. Kurihara and Yamashita (2013) applied the "maximum entropy principle" (Jaynes 1957), which is positioned as the core of the basic principles described by entropy, to just such a harmonized problem.

The maximum entropy principle (Jaynes 1957) is a general principle on "extended reasoning," which probabilistically evolved reasoning, such that conclusions cannot be obtained only with given evidence (Klir and Folger 1988). That is, in the maximum entropy principle, when attempting to estimate the probability distribution from insufficient evidence, it is a rational choice to select a distribution with maximum uncertainty (entropy) among all the probability distributions according to that evidence, in order to adequately recognize that the evidence is inadequate. In order to simply estimate production quantity only with demand quantity and inventory quantity based on the maximum entropy principle, Kurihara and Yamashita (2013) formulated the following as the maximization problem φ of the production allocation entropy H, keeping the average inventory management energy E at a constant C.

$$\varphi = H + \lambda(E - C) + \mu\left(\sum_{t=1}^{T} p_t - 1\right) \to \max \tag{1}$$

However, λ and μ are Lagrange multipliers.

$$H = -\sum_{t=1}^{T} p_t \log p_t \tag{2}$$

$$E = \alpha S = \alpha \sum_{t=1}^{T} \left\{ \frac{a_{t-1} + a_t}{2} \right\} \tag{3}$$

t: planning period (month) , t = 1, 2, ... , T
d_t: demand quantity at month t
p_t : production allocation rate at month t ($\Sigma p_t = 1$)
a_t: inventory quantity at the end of month t
S: the sum of average inventory quantity for T months
α: coefficient determined by product characteristics, such as inventory management cost, risk and influence on cash flow of holding inventory, and so on (referred to as "inventory management energy coefficient")

Then, Kurihara and Yamashita (2013) put forward the following solution. First, assuming that initial inventory is set to a_0, inventory at the end of planning horizon (T) is set to a_T, and total amount of production is set to M, production quantity at month t (p_t) is expressed as in Eq. (5).

$$M = \sum_{t=1}^{T} d_t - a_0 + a_T \tag{4}$$

$$m_t = p_t M \tag{5}$$

and with $x_t = (T - t + 1/2)M$, $G = \sum_{t=1}^{T} \{a_0 - (T - t + 1/2)d_t\}$, $Q = \exp[\lambda\alpha]$, Eq. (6) and Eq. (7) are obtained.

$$p_t = \frac{Q^{x_t}}{\sum_{k=1}^{T} Q^{x_k}} \tag{6}$$

$$\sum_{t=1}^{T} Q^{x_t} \{\alpha x_t + \alpha G - C\} = 0 \tag{7}$$

We obtain the Q satisfying Eq. (7) numerically and substitute it into Eq. (6) to obtain a solution for the production allocation rate at month t (p_t) that maximizes the entropy, and we can calculate the production quantity at month t (m_t). As a result, unlike the linear programming methods (Buffa and Miller 1979) and other existing solutions (linear decision rule (Holt et al. 1960), dynamic programming method (Bishop and Rockwell 1958), etc.), we can simply derive the solution of the most leveled production quantity based on the demand quantity, under the condition that the inventory management energy according to the inventory level is constant.

3. Study Method

3.1 Approach of this study

As Fukao (1987) and Yamashita (2010) point out, a dual problem involves exchanging the objective function and the constraint exits against the maximization problem based on the maximum entropy principle, so too can we think of a dual problem against the maximization problem proposed by Kurihara and Yamashita (2013), and the basic principle in such a dual problem is the "minimum average-energy principle." This minimum average-energy principle is the principle of estimating the probability distribution that minimizes the average energy while keeping the entropy to a constant magnitude.

In this study, which is based on the minimum average-energy principle, we reconstruct the harmonized problem between inventory reduction and production leveling, and propose a new harmonized model formulated as an (average) inventory management energy minimization problem, keeping production allocation entropy to a constant magnitude (giving a leveling state) under variable demand. At that time, as a constraint condition, it is necessary to give a certain leveling state by production allocation entropy. Therefore, we will examine the correspondence between a general indicator controlling the leveling state and the production allocation entropy.

In the production planning, the exponential smoothing method is often used as a simple method of smoothing and leveling demand fluctuations, and its control indicator is a smoothing (leveling) coefficient (Kotani 1990; Korytkowski et al. 2014). For simplicity, assuming safety inventory, initial inventory, and ending inventory are set to 0, a relationship between d_t and m_t can be expressed as Eq. (8), using γ (smoothing coefficient).

$$m_t = \gamma P + (1-\gamma)d_t \tag{8}$$

However, $0 \leq \gamma \leq 1$

Here, the base production quantity P means the production quantity in the case of perfect leveling through the planning horizon, and it is expressed as Eq. (9).

$$P = \frac{M}{T} = \frac{\sum_{t=1}^{T} m_t}{T} = \frac{\sum_{t=1}^{T} d_t}{T} \tag{9}$$

And, production allocation rate at month t (p_t) can be expressed as Eq. (10) by Eq. (8) and Eq. (9).

$$p_t = \frac{m_t}{M} = \frac{\gamma P + (1-\gamma)d_t}{M} \tag{10}$$

Therefore, the production allocation entropy H showing the leveling state corresponding to the smoothing coefficient γ can be expressed as Eq. (11) by Eq. (2) and Eq. (10).

$$H = -\sum_{t=1}^{T} p_t \log p_t = -\sum_{t=1}^{T} \left(\frac{\gamma P + (1-\gamma)d_t}{M} \right) \log \left(\frac{\gamma P + (1-\gamma)d_t}{M} \right) \tag{11}$$

As a result, the smoothing coefficient, which is a general indicator, is associated with the production allocation entropy, and it becomes possible to control the leveling state through the smoothing coefficient in the proposed model.

3.2 Prerequisites

The main prerequisites for formulating the problem are as follows.

(1) The demand is known and deterministic, and safety inventory is not taken into account.
(2) There is no out-of-stock.
(3) The smoothing coefficient (for controlling the leveling state) is known.

3.3 Problem Formulation

The constant production allocation entropy (the leveling state corresponding to the smoothing coefficient, which is calculated based on Eq. (11) in the previous section) is set as C. The harmonized problem φ in this study can be formulated as in Eq. (12), considering the minimization problem of the inventory management energy E under the production allocation entropy H of Eq. (2) kept at constant C.

$$\varphi = E - \theta(H - C) + \mu\left(\sum_{t=1}^{T} p_t - 1 \right) \rightarrow \min \tag{12}$$

However, θ and μ are Lagrange multipliers.

3.4 Derivation of solutions

The solution of the harmonized problem φ in Eq. (12) is derived by the following procedure.
First, from the previous study of Kurihara and Yamashita (2013),

with $x_t = (T - t + 1/2)M$, $G = \sum_{t=1}^{T} \left\{ a_0 - \left(T - t + 1/2 \right) d_t \right\}$, Eq. (3) is as shown in Eq. (13).

$$E = \alpha \sum_{t=1}^{T} p_t x_t + \alpha G \tag{13}$$

Accordingly, the harmonized problem φ of this study can be rewritten as Eq. (14).

$$\varphi = \left(\alpha \sum_{t=1}^{T} p_t x_t + \alpha G \right) - \theta \left(-\sum_{t=1}^{T} p_t \log p_t - C \right) + \mu \left(\sum_{t=1}^{T} p_t - 1 \right) \rightarrow \min \tag{14}$$

Since Eq. (14) is convex downward with respect to p_t, φ is partially differentiated with p_t and is set to 0, and solving Eq. (14) as $Q = \exp[\alpha/\theta]$ yields Eq. (15).

$$p_t = \frac{Q^{-x_t}}{\sum_{k=1}^{T} Q^{-x_k}} \tag{15}$$

On the other hand, Eq. (16) is obtained from Eq. (14) and Eq. (15).

$$-\sum_{t=1}^{T} \left(\frac{Q^{-x_t}}{\sum_{k=1}^{T} Q^{-x_k}} \right) \log \left(\frac{Q^{-x_t}}{\sum_{k=1}^{T} Q^{-x_k}} \right) - C = 0 \tag{16}$$

Therefore, Q, which satisfies Eq. (16), is numerically obtained and p_t, which minimizes φ in Eq. (12) (inventory management energy E), is obtained by substituting the Q into Eq. (15). By substituting the p_t into Eq. (10), it is possible to estimate m_t (production quantity at month t).

Comparing Eq. (6), which is the solution of the base model, with Eq. (15), which is the solution of the proposed model of this study, it turns out that the same result is obtained in the form. Fukao (1987) pointed out that the dual problem based on the minimum average-energy principle, which exchanged the objective function and the constraint of the maximum entropy principle, gives the same result in the form, and this result suggests the duality of the proposed model to the base model.

4. Analysis by simple numerical examples

4.1 Analysis method

We will try to analyze simple numerical examples based on sales result data of domestic home air conditioners with large fluctuations in demand (Ministry of Economy, Trade and Industry "Monthly Sales Statistics Survey of Mass Retailers of Home Electric Appliances," 2011) and on Kurihara and Yamashita (2013).

Therefore, for the three months (May - July) where the demand rises sharply towards the peak, the demand quantity d_t is set to the same value as the previous study of Kurihara and Yamashita (2013), and a numerical example is set, as shown in Table 1. In addition, for the purpose of simplicity, the initial inventory a_0 and the ending inventory a_T are set to 0, and the inventory management energy coefficient α is set to 1. Then, in order to focus on analysis of the influence on the minimum value of inventory management energy by the difference in smoothing coefficient γ, three cases are set, as shown in Table 2. At that time, the constant (production allocation) entropy in Table 2 is calculated based on Eq. (11) under the demand pattern in Table 1.

Table 1. Demand per month (From May to July)

Month	May (t=1)	June (t=2)	July (t=3)
Demand quantity (d_t)	36	69	127

Table 2. Case Setting

Case	Smoothing Coefficient γ	Constant Entropy C
Case 1	1	1.0986
Case 2	0.8	1.0939
Case 3	0.5	1.0694

4.2 Results and discussion

Table 3 shows the results of the minimum value of the inventory management energy for each case under numerical examples, as shown in Table 1 and Table 2. Further, Table 4 shows the results of the production quantity determined for Case 2. In addition, Table 5 shows the result of obtaining the maximum value of the production allocation entropy for each case, assuming the inventory management energy determined in Table 3 to be constant, based on the base model (Kurihara and Yamashita 2013).

First, we found that the minimum value of inventory management energy decreases as the smoothing coefficient decreases. Production with a small smoothing coefficient means "agile production" with high capability to follow demand fluctuation, and can reduce inventory quantity, so it will be a reasonable result.

Next, it is understood that the dual relationship between the proposed model of this study and the base model (Kurihara and Yamashita 2013) exists, comparing the relationship between the constant production allocation entropy and the minimum inventory management energy in Table 3 and the relationship between the constant inventory management energy and the maximum production allocation entropy in Table 5. From the above, the validity of the proposed model of this study is suggested.

Table 3. Minimum Inventory Management Energy E

Case	Smoothing Coefficient γ	Constant Production Allocation Entropy C	Minimum Inventory Management Energy E
Case 1	1	1.0986	91
Case 2	0.8	1.0939	72
Case 3	0.5	1.0694	46

Table 4. Demand/Production/Inventory per month in Case 2

Month	May (t=1)	June (t=2)	July (t=3)	Total
Demand quantity (d_t)	36	69	127	232
Production quantity (m_t)	68	77	87	232
Inventory quantity (a_t)	32	40	0	72

Table 5. Maximum Production Allocation Entropy H

Case	Constant Inventory Management Energy C	Maximum Production Allocation Entropy H
Case 1	91	1.0986
Case 2	72	1.0939
Case 3	46	1.0694

5. Conclusion

In this paper, we proposed a dual approach to the harmonized model between inventory reduction and production leveling ("heijyunka") based on the minimum average-energy principle, and confirmed the validity of the proposed approach and model. By the proposed model, under the current production facilities and capabilities (given state of production leveling, that is, production allocation entropy associated with the smoothing coefficient), it will be possible to seek the optimal production quantity that minimizes the inventory management workload (energy) associated with inventory quantity. Also, multifaceted analysis on harmonized level between "heijyunka" and inventory reduction, including minimization of inventory management energy to maximization of production allocation entropy, will be possible.

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