# Monitoring a cold rolling process using $\bar{X}$ &EWMA charts

**Mohammad Shamsuzzaman[a]**
Sustainable Engineering Asset Management (SEAM) Research Group
Department of Industrial Engineering and Engineering Management
University of Sharjah
Sharjah 27272, United Arab Emirates
[a]mshamsuzzaman@sharjah.ac.ae

**Salah Haridy[b]**
Sustainable Engineering Asset Management (SEAM) Research Group.
Department of Industrial Engineering and Engineering Management,
University of Sharjah, Sharjah 27272, United Arab Emirates.
Benha Faculty of Engineering, Benha University, Benha, Egypt.
[b]sharidy@sharjah.ac.ae

**Hamdi Bashir[c] and Imad Alsyouf[d]**
Sustainable Engineering Asset Management (SEAM) Research Group
Department of Industrial Engineering and Engineering Management
University of Sharjah
Sharjah 27272, United Arab Emirates
[c]hbashir@sharjah.ac.ae; [d]ialsyouf@sharjah.ac.ae

## Abstract

This article presents an optimization design of the $\bar{X}$ &EWMA chart for monitoring a cold rolling process producing galvanized steel coils. Using real data from industry, the results show that the detection effectiveness of the $\bar{X}$ &EWMA charts is better than the optimal $\bar{X}$ chart and the optimal EWMA chart in terms of Expected Average Number of Observation required to identify an out-of-control case (*EANOS*) by about 36% and 10%, respectively. The optimization design of the $\bar{X}$ &EWMA charts ensures that extra inspection resources will not be necessary, and the false alarm rate of the charts will not be increased.

**Keywords**. Cold rolling process, Online monitoring, Statistical process control (SPC), $\bar{X}$ &EWMA charts.

## 1. Introduction

Cold rolling is a metal forming process in which sheet metal or strip stock passes through rollers at a temperature below its recrystallization temperature, and then compressed and squeezed to give it the desired shape/value (Figure 1). Cold rolling increases the yield strength and hardness of a metal by introducing defects into the metal's crystal structure. The aim of the rolling process is to reduce the thickness of a strip to the desired value with good dimensional accuracy, surface finish, and excellent mechanical properties. During the cold rolling process, there are uncertainties and external disturbances such as eccentricity arise, which usually appears on the strip periodically and causes variation in the thickness of the strip (Koofigar *et al*. 2011, Haridy *et al*. 2011). Thus, an online monitoring system is essential to identify the excessive variation in the thickness of the strip at the earlier stage, and Statistical Process Control (SPC) charts can play an important role in this respect.

The SPC charts are popular choices in manufacturing and service industries for monitoring process variation over time so that timely action can be taken to identify the root causes of the excessive variability, and as a result, reducing the defective rate of the process. In recent years, many new charts and their applications have been proposed and studied by researchers in engineering, management, and statistics (Jarrett and Pan 2007, Messaoud *et al*. 2008, Shu *et al*. 2008, Shamsuzzaman *et al*. 2009, Zhang *et al*. 2011, Haridy *et al.* 2017).

Amongst all, the Shewhart $\bar{X}$ (or $X$), and $\bar{X}\,\&\,R$ (or $\bar{X}\,\&\,S$) charts are used most widely in industries. Many extensions to the Shewhart $\bar{X}$ and $\bar{X}\,\&\,R$ charts have been proposed, and their applications have been explained. For instance, Haridy et al. (2017) investigated the effect of sample size on the performance of $\bar{X}\,\&\,R$ and $\bar{X}\,\&\,S$ charts and explained their applications in monitoring a cold rolling process. However, the Shewhart-type charts are efficient in identifying large shift, but not very efficient in identifying smaller and moderate shifts. Two types of control charts are primarily used to detect shifts of small and moderate sizes, namely, cumulative sum (CUSUM) charts (Page 1954), and exponentially weighted moving average (EWMA) charts (Roberts 1959).

Nonetheless, the EWMA chart is a popular choice over the CUSUM chart, as the EWMA scheme is easier in design and operation (Montgomery, 2013). Ou et al. (2014) presented an optimization design of the EWMA chart for monitoring a cold rolling process. However, even though the EWMA chart is efficient in identifying small changes in process shifts, it suffers from inertia problem in detecting large shifts (Yashchin 2018, Woodall and Mahmoud 2005). In SPC, inertia refers to the resistance of a chart to signaling a particular process shift (Woodall and Mahmoud 2005). Several authors suggested using a combined $\bar{X}$&EWMA chart in order to avoid the inertia problem (Capizzi 2003; Reynolds and Stoumbos 2006). Besides, the process shift (e.g., mean shift $\delta$) is a random variable, and in most of the practical applications, the actual sizes of the process shifts are usually unknown. In such a case, researchers also recommend combining $\bar{X}$ chart and EWMA chart so that the charting scheme will perform efficiently over a wide range of process shifts (Tolley and English, 2001; Simoes *et al*., 2010; Lin and Chou, 2011, Shamsuzzaman *et al.* 2016).

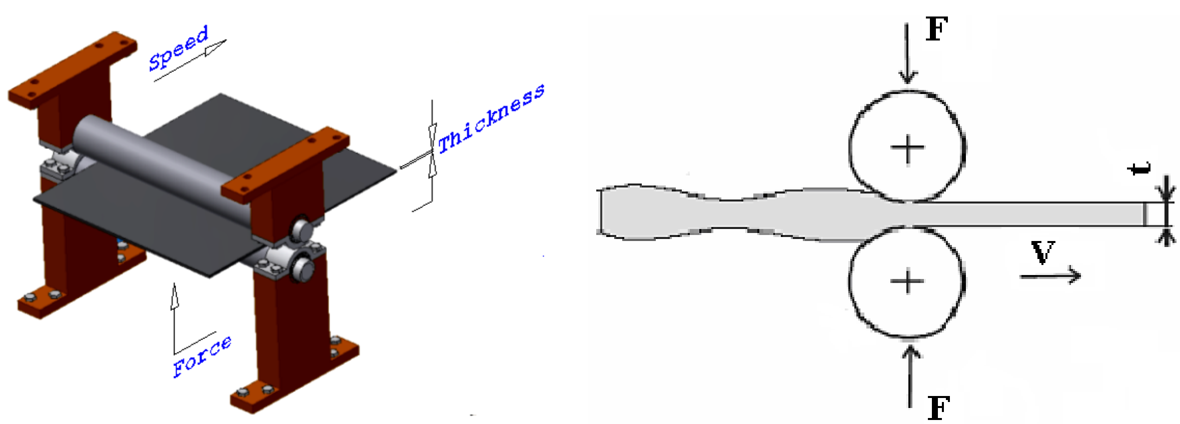


Figure 1. Cold rolling process (reproduced from Haridy and Wu (2009))

The current study aims to design an $\bar{X}$&EWMA chart for monitoring a cold rolling process, where the sample size ($n$), sampling interval ($h$), weighting parameter ($\lambda$), and control limits of the charting scheme are all optimized. According to Montgomery (2013), the values of weighting parameter $\lambda$ are widely chosen within the interval ($0.05 \leq \lambda \leq 0.25$), with 0.05, 0.1, and 0.2 being the popular choices. Thus, in the current study, the optimal value of the weighting parameter $\lambda$ is searched in the interval of ($0.05 \leq \lambda \leq 0.25$) so that the inertia problem can be avoided, and the charting scheme is more acceptable from a practical viewpoint. In designing the proposed $\bar{X}$&EWMA charts, it is assumed that the quality characteristic $x$ is normally and independently distributed with known in-control mean $\mu_0$ and standard deviation $\sigma_0$. When a mean shift occurs, the process mean $\mu$ will change accordingly, that is,

$$\mu = \mu_0 + \delta\sigma_0 \tag{1}$$

where $\delta$ is the mean shift in terms of $\sigma_0$. When the process is in control, $\delta = 0$. In this study, the shift in the standard deviation is not taken into consideration (i.e., $\sigma \equiv \sigma_0$). For the sake of convenience, the design of the proposed $\bar{X}$&EWMA charts are carried out under the standard condition ($\mu_0 = 0$ and $\sigma_0 = 1$).

## 2. Optimization design

## 2.1 Specifications

The following three parameters are needed in designing the proposed $\bar{X}$&EWMA charts:

- $\tau$ minimum allowable in-control Average Time to Signal, $ATS_0$
- $R$ maximum allowable inspection rate;
- $\mu_\delta$ mean of the mean shifts $\delta$

The value of $\tau$ is decided according to the requirements on the false alarm rate and the detection power. The inspection rate $R$ is defined as the number of inspected units per unit time when the process is in control. Its value is decided according to the available resources (operators and measuring instruments) and can be estimated from the field test during the pilot runs. The value of $\mu_\delta$ can be estimated from the historical data of the out-of-control cases (Wu et al. 2004). Suppose the $m$ sample values of mean shifts $\delta$ (denoted by $d_1$, $d_2$, ..., $d_m$) are obtained from the observations of $m$ out-of-control cases during the operation of the control chart. Then, the value of $\mu_\delta$ can be calculated by,

$$\mu_\delta = \frac{\sum_{i=1}^{m} d_i}{m} \tag{2}$$

## 2.2 Optimization model

The statistics $\bar{X}_i$ and $S_i$ to be plotted and updated for the $\bar{X}$&EWMA charts.

$$S_i = \lambda \cdot \bar{X}_i + (1-\lambda)\cdot S_{i-1} \qquad i = 1,\ 2, ..., \tag{3}$$

where $\lambda$ ($0.05 \le \lambda \le 0.25$) is the weighting parameter. The sample mean $\bar{X}_i$ is the average of the measurements in the $i$th sample. The value of $S_0$ (i.e. at $i = 0$) is the process target (i.e. $S_0 = \mu_0$). The $\bar{X}$&EWMA combination will produce an out-of-control signal if $S_i$ falls beyond the control limits of the EWMA chart and/or the current value of the sample average $\bar{X}_i$ exceeds the control limits of the $\bar{X}$ chart. The design algorithm of the $\bar{X}$&EWMA charts is formulated by the following optimization model:

Minimize: *EANOS*, (4)
Subject to: $ATS_0 \cong \tau$; (5)
$r \cong R$ (6)
Design variables: $\lambda$, $n$, $h$, $H$, $UCL$.

where, $r$ is the actual (or resultant) inspection rate. The constraint on inspection rate $r$ ensures that the use of the optimization model will not need extra inspection resources (Shamsuzzaman and Wu 2012). For simplicity, the focus of this article is on the studies of the combination of the $\bar{X}$&EWMA chart for detecting increasing process shifts in the mean. As a result, an upper-sided EWMA chart with an upper control limit ($H$) and an $\bar{X}$ chart with an upper control limit ($UCL$) are combined. A symmetrical $\bar{X}$&EWMA chart for detecting decreasing mean shifts can be designed straightforwardly. Among the five design variables, $\lambda$, $n$, $h$, $H$ and $UCL$, the parameters $\lambda$ and $n$, are independent design variables. The sampling interval $h$ depends on $n$,

$$h = n / R. \tag{7}$$

Equation (7) ensures that the constraint on the inspection rate $r$ (constraint (6)) is satisfied and the available inspection resources are fully utilized. The control limits $H$ and $UCL$ are determined so that the resultant in-control $ATS_0$ of the $\bar{X}$&EWMA combination is equal or very close to $\tau$ (constraint (5)). The objective function *EANOS* is the Expected Average Number of Observations to Signal. It is the average number of items to be inspected by the charting scheme to signal an out-of-control condition. The smaller the *EANOS*, the better is the performance of a charting scheme. For a given value of mean shift $\delta$, the objective function, *EANOS* can be estimated by (Saha et al. 2018),

$$EANOS = \int_0^\infty n \cdot ATS(\delta) / h \cdot f(\delta) \cdot d\delta \tag{8}$$

where $ATS(\delta)$ is the out-of-control $ATS$ for a given mean shift $\delta$, and $f(\delta)$ is the probability density function of $\delta$. The random mean shift $\delta$ is assumed to follow a Rayleigh distribution (Wu et al. 2004). The probability density function $f(\delta)$ in Equations (8) can be determined as follows,

$$f(\delta) = \frac{\pi\delta}{2\mu_\delta^2}\exp\left(-\frac{\pi\delta^2}{4\mu_\delta^2}\right) \tag{9}$$

which is characterized by a single parameter $\mu_\delta$. The integration in Equation (8) can be computed accurately by a numerical method, such as the Legendre-Gauss Quadrature. The $ATS(\delta)$ of the $\bar{X}$&EWMA chart is calculated by Markov-chain approach.

### 2.3 Optimization process

The optimization design of $\bar{X}$&EWMA charts is conducted through a three-level search as described below (Shamsuzzaman *et al*. 2016). At the first level, the optimal value of $n$ is searched from 1 with a step size of one (for a given value of $n$, the sampling interval $h$ is calculated by Equation (9)). It ensures the satisfaction of constraint (6). At the second level, the optimal value of $\lambda$ is searched in the range of ($0 < \lambda < 1$). At the third level, for a given set of values of ($n$, $h$, $\lambda$), the optimal value of $UCL$ is searched with a starting value of $UCL_{\bar{X}}$, which is the upper control limit of an individual $\bar{X}$ chart that meets ($ATS_0 \cong \tau$). Next, for a given set of values of ($n$, $h$, $\lambda$, $UCL$), the value of $H$ is determined that ensures the satisfaction of the constraint of ($ATS_0 \cong \tau$) by the $\bar{X}$&EWMA chart. At the end of the entire three-level search, the optimal $\bar{X}$&EWMA scheme that produces the minimum $EANOS$ and satisfies the constraints ($ATS_0 \cong \tau$) and ($r \le R$), is identified. The corresponding optimal values of ($n$, $h$, $\lambda$, $UCL$, and $H$) are also finalized. The design of the $\bar{X}$&EWMA charts is conducted under standard condition ($\mu_0 = 0$, $\sigma_0 = 1.0$.), the actual control limits using real values of $\mu_0$ and $\sigma_0$ are calculated as follows:

$$\begin{aligned} H_{actual} &= \mu_0 + \sigma_0 \cdot H \\ UCL_{actual} &= \mu_0 + \sigma_0 \cdot UCL \end{aligned} \tag{10}$$

## 3. Example

The case study in Haridy et al. (2017) has been adopted here in order to demonstrate the performance of the proposed charting scheme. The case study focuses on the quality of galvanized coils produced by a metal forming industry in Egypt. The galvanized cold rolled steel coils are obtained by passing the hot rolled coil (as a raw material) through a sequence of processes. The thickness $x$ of the galvanized coil is a critical dimension that has to be monitored by an SPC chart for detecting a wide range of unknown shifts. The thickness $x$ is specified as 1.50 ± 0.015 mm. When the process was in-control, the quality control (QC) engineer collected 25 samples of size 5 to estimate the in-control process mean $\mu_0$ and in-control process standard deviation $\sigma_0$ of the thickness $x$. The normal probability plot of the data on $x$ confirms that the data is well approximated by a normal distribution (p-value = 0.106) with $\mu_0 = 1.52$ and $\sigma_0 = 0.005343$. The available resources allowed the QC engineer to use an inspection rate of 10. Based on the historical record on out-of-control cases, the mean of the mean shifts $\mu_\delta$ was calculated (Equation (2)) and found to be 0.75. The minimum in-control $ATS_0$, $\tau$ is set at 400 hours. The following three control charts are designed, and their performances are compared.

(1) Optimal $\bar{X}$ chart (Haridy et al. 2017). The charting parameters, $n$, $h$, and $UCL$ of the $\bar{X}$ chart are optimized following the optimization process presented in this article.

(2) Optimal EWMA chart (Ou et al. 2014). The charting parameters, $n$, $h$, and $H$ of the EWMA chart are optimized following the optimization process presented in this article.

(3) Optimal $\bar{X}$&EWMA charts. The charting parameters, $n$, $h$, $\lambda$, $UCL$, and $H$ of the $\bar{X}$&EWMA charts are all optimized according to the optimization process presented in this article.

Based on the collected information on the design specifications ($R = 10$, $\mu_\delta = 0.75$, $\tau = 400$), the above mentioned three charts are designed, and their parameters and the performance indexes are listed below.

Optimal $\bar{X}$ chart:
$n = 57$, $h = 5.7$, $UCL_{actual} = 1.522$, $ATS_0 = 400.0$, $EANOS = 58.4$

Optimal EWMA chart:
$n = 4$, $h = 0.4$, $\lambda = 0.05$, $H_{actual} = 1.521$, $ATS_0 = 399.2$, $EANOS = 47.3$

Optimal $\bar{X}$&EWMA charts:

$n = 19$, $h = 1.90$, $\lambda = 0.135$, $UCL_{actual} = 1.524$, $H_{actual} = 1.521$, $ATS_0 = 397.1$, $EANOS = 43.0$

Even though all three charting schemes required the same inspection resources ($R$) and their false alarm rates ($ATS_0$) are almost the same, the expected average number of observations, *EANOS* required to signal an out-of-control case of the three schemes are quite different. The values of *EANOS* of the three charts indicate that the conventional EWMA chart outperforms the conventional $\bar{X}$ chart. However, the optimal $\bar{X}$ &EWMA charts outperform the conventional $\bar{X}$ chart and the conventional EWMA chart by about 36% and 10%, respectively. The combination of the $\bar{X}$ chart and EWMA chart, and the optimization design makes the proposed $\bar{X}$ &EWMA charts more effective from an overall viewpoint.

## 4. Conclusion

This article presents optimal $\bar{X}$ &EWMA charts, and explains the application of the proposed charts for monitoring a cold rolling process producing steel coils. The application of SPC charts can help to take timely action to reduce the number of defective items. The optimization design of the $\bar{X}$ &EWMA charts considers a wide range of mean shifts in the cold rolling process, which is more realistic in practice. The effectiveness of the proposed scheme is investigated through an example, which shows that the proposed optimal $\bar{X}$ &EWMA charts outperform (in terms of *EANOS*) the conventional $\bar{X}$ chart and the conventional EWMA chart by about 36%, and 10%, respectively. In the proposed design, the mean shift is characterized by a Rayleigh distribution. Future research could consider the effectiveness of the charts under different shift distributions such as Beta distributions.

## References

Capizzi G. and Masarotto G., An adaptive exponentially weighted moving average control chart, *Technometrics,* vol. 4, pp. 199-207, 2003.

Haridy S. and Wu, Z., Univariate and multivariate control charts for monitoring dynamic-behavior processes: a case study, *Journal of Industrial Engineering and Management*, vol. 2, no. 3, pp. 464-498, 2009.

Haridy S., Wu Z. and Castagliola P., Univariate and Multivariate Approaches for Evaluating the Capability of Dynamic-Behavior Processes (Case Study), *Statistical Methodology*, vol. 8, pp. 185-203, 2011.

Haridy, S., Maged, A., Kaytbay, S., and Araby, S., Effect of sample size on the performance of Shewhart control charts, *International Journal of Advanced Manufacturing Technology,* vol. 90, no. 1-4, 1177-1185, 2017.

Jarrett, J. E. and Pan, X., The quality control chart for monitoring multivariate autocorrelated processes, *Computational Statistics & Data Analysis,* vol. 51, pp.3862-3870, 2007.

Koofigar, H. R., Sheikholeslam, F., and Hosseinnia, S., Unified gauge-tension control in cold rolling mills: a robust regulation technique, *International Journal of Precision Engineering and Manufacturing*, vol. 12, no. 3, pp. 393-403, 2011.

Lin, Y. C. and Chou, C. Y., Robustness of the EWMA and the combined $\bar{X}$-EWMA control charts with variable sampling intervals to non-normality, *Journal of Applied Statistics*, vol. 38, no. 3, pp. 553-570, 2011.

Messaoud, A., Weihs, C. and Hering, F., Detection of chatter vibration in a drilling process using multivariate control charts, *Computational Statistics & Data Analysis,* vol. 52, pp.3208-3219, 2008.

Montgomery, D. C., *Introduction to statistical quality control*: John Wiley & Sons, 2013.

Ou, Y., Hu, J., Li, X., and Haridy, S., An Incipient on-line anomaly detection approach for the dynamic rolling process, *International Journal of Precision Engineering and Manufacturing*, vol. 15, no. 9, pp. 1855-1864, 2014.

Page, E. S., Continuous inspection scheme, *Biometrika*, vol. 41, pp.100-114, 1954.

Reynolds, M. R. Jr., Stoumbos, Z. G., Comparisons of some exponentially weighted moving average control charts for monitoring the process mean and variance, *Technometrics*, vol. 48, pp. 550-567, 2006.

Roberts, S. W., Control chart tests based on geometric moving averages, *Technometrics*, vol. 1, pp.239-250, 1959.

Shamsuzzaman, M. and Wu, Z., Design of EWMA control chart for minimizing the proportion of defective units. *International Journal of Quality & Reliability Management,* vol. 29, no. 8, pp. 953-969, 2012.

Shamsuzzaman, M., Khoo, M. B. C., Haridy, S., and Alsyouf, I., An optimization design of the combined Shewhart-EWMA control chart. *The International Journal of Advanced Manufacturing Technology,* vol. 86, no. 5, pp. 1627-1637, 2016.

Shamsuzzaman, M., Wu, Z., and Elias, M. R. S., Designs of $\bar{X}$ &$S$ control charts with optimal manpower deployment, *Computers & Industrial Engineering*, vol. 56, pp.1589-1596, 2009.

Shu, L., Jiang, W., and Wu, Z., Adaptive CUSUM procedures with Markovian mean estimation, *Computational Statistics & Data Analysis*, vol. 52, pp.4395-4409, 2008.
Simoes, B. F. T., Epprecht, E. K., and Costa, A. F. B., Performance comparison of EWMA control chart schemes, *Quality Technology and Quantitative Management,* vol. 7, pp. 249-261, 2010.
Tolley, G. O. and English, J. R., Economic designs of constrained EWMA and combined EWMA-$\bar{X}$ control schemes, *IIE Transactions*, vol. 33, pp.429-436, 2001.
Woodall W. H. and Mahmoud M. A., The inertial properties of quality control charts, *Technometrics*, vol. 47, pp. 425-436, 2005.
Wu, Z., Shamsuzzaman, M. and Pan, E. S., Optimization designs of the control charts based on taguchi's loss function and random process shifts, *International Journal of Production Research*, vol. 43, pp. 379-390, 2004.
Yashchin, E., Some aspects of the theory of statistical control schemes, *IBM Journal of Research and Development*, vol. 31, pp. 199-205, 1987.
Zhang, H. Y., Shamsuzzaman, M., Xie, M., and Goh, T. N., Design and application of exponential chart for monitoring time-between-events data under random process shift, *International Journal of Advanced Manufacturing Technology*, vol. 57, pp.849-857, 2011.

## Biographies

**Mohammad Shamsuzzaman** currently is an Associate Professor in the Department of Industrial Engineering and Engineering Management at University of Sharjah, UAE. He obtained his PhD in Systems and Engineering Management in 2005 from Nanyang Technological University, Singapore. His current research focuses on statistical process control, lean six sigma, reliability, simulation and multi-criteria decision-making. He is a member of the American Society for Quality.

**Salah Haridy** is an Assistant Professor in the Department of Industrial Engineering and Engineering Management at University of Sharjah, UAE. He received his M.Sc. and Ph.D. degrees from Benha University, Egypt and Nanyang Technological University, Singapore in 2008 and 2014, respectively. He is the recipient of the 2013 Mary G. and Joseph Natrella Scholarship awarded by the American Statistical Association (ASA) and the 2014 Richard A. Freund International Scholarship awarded by the American Society for Quality (ASQ). His research interests cover quality engineering, statistical process control and design of experiments.

**Hamdi Bashir** received his PhD degree in 2000 from McGill University, Montreal, Canada. Currently, he is an Associate Professor of Industrial Engineering and Engineering Management at the University of Sharjah. Prior to joining this university, he held faculty positions at Sultan Qaboos University, University of Alberta, and Concordia University. His research interests are in the areas of project management, manufacturing systems, quality management, and healthcare management. He is a senior member of the Institute of Industrial and Systems Engineers (IISE).

**Imad Alsyouf** is an associate professor of Industrial Engineering, employed by University of Sharjah, UAE. He is the founder and coordinator of the Sustainable Engineering Asset Management (SEAM) Research Group. He has produced more than 67 conference and journal papers. He has about 30 years of industrial and academic experience in various positions in Jordan, Sweden and UAE. His research interests include reliability, quality, maintenance, and optimization. He has developed and taught more than 25 post and undergrad courses. He delivered training courses in Kaizen, TQM, and organizational excellence.