Optimization design of economic EWMA control chart based on random process shifts

Mohammad Shamsuzzaman[a]
Sustainable Engineering Asset Management (SEAM) Research Group
Department of Industrial Engineering and Engineering Management
University of Sharjah
Sharjah 27272, United Arab Emirates
[a]mshamsuzzaman@sharjah.ac.ae

Salah Haridy[b]
Sustainable Engineering Asset Management (SEAM) Research Group.
Department of Industrial Engineering and Engineering Management,
University of Sharjah, Sharjah 27272, United Arab Emirates.
Benha Faculty of Engineering, Benha University, Benha, Egypt.
[b]sharidy@sharjah.ac.ae

Imad Alsyouf[c] and Hamdi Bashir[d]
Sustainable Engineering Asset Management (SEAM) Research Group
Department of Industrial Engineering and Engineering Management
University of Sharjah
Sharjah 27272, United Arab Emirates
[c]ialsyouf@sharjah.ac.ae; [d]hbashir@sharjah.ac.ae

Abstract

This study presents a model for the optimization design of an economic exponentially weighted moving average (EWMA) control chart based on random process shifts in the process mean. The optimization model aims to minimize the total expected cost involved with the charting scheme. The optimization process ensures that the false alarm rate of the charting scheme will not be increased, and no extra inspection resources will be required. The performance of the proposed optimal economic EWMA chart is compared with that of the traditional economic $\bar{X}$ chart, optimal economic $\bar{X}$ chart, and traditional economic EWMA chart. The results reveal that the total expected cost can be reduced by the optimal economic EWMA chart compared to the other three charts by about 109%, 23%, and 95%, respectively.

Keywords. Statistical process control (SPC), EWMA control chart, random process shifts

1. Introduction

In this era of ever-growing competition, it has become necessary for a company to keep a continuous watch over the quality of the goods produced. A continuous quality improvement program is the only tool to achieve this goal. A successful quality improvement effort may lead to lower costs, higher productivity, increased customer satisfaction, higher market share, and ultimately the higher profits for the company. In order to produce high-quality products, it is necessary for a company to keep the variation in the quality of its products as minimum as possible, and to make it as the main task in quality control. The need for quality control arises from the fact that even after the quality standards have been met, some variation in quality is unavoidable. The control chart is the most widely used statistical process control (SPC) tool in manufacturing industries that helps in ensuring that the variation in the quality of the produced products is acceptable.

Traditionally, control charts are designed in two directions, that is, the statistical designs and the economical designs. In statistical designs, the performance of a control chart is measured based on out-of-control Average Time to Signal (ATS) (or Average Run Length, ARL). However, the statistical designs do not directly measure the costs (or quality losses) resulting from the out-of-control cases. On the other hand, the economic designs of the control charts measure the chart performance in terms of monetary value. Even though the design procedures of the traditional control charts (either statistical or economic designs) are easier, they are not very effective in identifying out-of-control cases. Many authors proposed optimization designs of the traditional control charts in order to improve their performances. The advantages of the optimal models over the traditional models are that they select the chart parameters in such a way so that the out-of-control *ATS* (or *ARL*) or total cost associated with the SPC scheme is minimized while the false alarm rate (in-control ATS_0 (or ARL_0)) is maintained at a specified level.

Following pioneer model proposed by Duncan (1956), many authors (Chen and Yu, 2001; Castagliola *et al.*, 2007; Torng *et al.*, 2009; Yu *et al.*, 2010; Niaki *et al.*, 2011; Francoa *et al.*, 2014) developed advanced model for the economic designs of the control charts based on a single or a few process shifts. However, process shift is a random variable and different processes may have different probability distributions for the shift. Thus, some authors (for instances, see Wu *et al*. 2004; Wu *et al*. 2005; Shamsuzzaman and Wu, 2006; Shamsuzzaman *et al*. 2009, Haridy *et al*. 2017) designed control charts based on random process shifts in order to make them efficient over a wide range of process shifts, where only the associated quality costs are optimized. However, the objective of the economic designs is to minimize total cost, including inspection cost, quality cost, cost of false alarms, and cost of detecting out-of-control signals. Recently, Shamsuzzaman *et al.* (2018) developed an optimization model for the economic design of the $\overline{X}$ chart based on random process shifts for monitoring electric power loss through transmission and distribution system. However, $\overline{X}$ charts are efficient in identifying large shifts, but they are less sensitive to small shifts.

On the other hand, the Exponentially Weighted Moving Average (EWMA) control charts are very efficient in identifying small or moderate shifts. Thus, the objective of this study is to develop an optimization model for the economic design of the EWMA chart based on random process shifts (i.e., mean shift δ). The design model optimizes chart parameters such as smoothing factor (λ), sample size (n), sampling interval (h), and upper and lower control limits (*UCL* and *LCL*) of the EWMA control chart in order to minimize the total cost associated with the SPC scheme on condition that the false alarm rate (ATS_0) will not be increased, and no extra inspection resources (R) will be required.

2. Model formulation

2.1 Assumptions

Few assumptions are used in developing the proposed model.

(1) The quality characteristic x is normally and independently distributed with in-control mean μ_0 and standard deviation σ_0. The occurrence of an assignable cause will change the in-control mean μ_0 to out-of-control mean μ_1,

$$\mu_1 = \mu_0 \pm \delta\sigma_0 \tag{1}$$

where ($\pm\delta$) is the amount of mean shift incurred by an assignable cause. When the process is in control, $\delta = 0$. For simplicity, the shift in standard deviation is not taken into consideration (i.e., $\sigma \equiv \sigma_0$).

(2) The process starts with an in-control state.

(3) Only a single assignable cause is considered when an out-of-control case occurs. The occurrence of the assignable cause follows a homogenous Poisson process with mean λ_a.

(4) The process continues to run during the search for the assignable cause.

2.2 Specification

The following parameters are required for the design of the proposed economic EWMA chart (Shamsuzzaman *et al*., 2018),

ζ Minimum allowable in-control ATS_0
R Maximum allowable inspection rate
μ_δ Mean of the shifts in process mean δ

λ_a Rate of occurrence of the assignable cause
e Time to estimate an observed data of a sample of power loss
D Time from the detection of an out-of-control state to the location and removal of the assignable cause
b Fixed component of sampling cost
c Variable component of sampling cost
W Cost of finding and fixing an assignable cause
T Cost of examining a false alarm
Q Production rate
C Average cost for scarping or reworking an out-of-specification product

2.3 Optimization model

Let z_t is the statistic for tth sample to be plotted and updated for the EWMA chart, i.e.

$$z_t = \lambda \bar{x}_t + (1-\lambda) z_{t-1} \tag{2}$$

where λ ($0 < \lambda < 1$) is the smoothing factor and $\bar{x}_t$ is the mean of measurements in tth sample. The starting value of z_t (i.e., at $t = 0$) is the in-control process mean (i.e., $z_0 = \mu_0$). The parameter z_t is plotted on the control chart. If the plotted point falls within *LCL* and *UCL*, it indicates that the process is in the state of statistical control, and no action is warranted. If the plotted point falls beyond *UCL* and/or *LCL*, it indicates that the process is out-of-control due to an assignable cause. Thus, an action should be taken to identify and remove the assignable causes. The design algorithm of the EWMA chart is described by the following optimization model,

Minimize: EC (3)
Subject to: $ATS_0 \cong \zeta$ (4)
$r \cong R$, (5)
Design variables: λ, n, h, *LCL*, *UCL*.

where, r is the actual (or resultant) inspection rate. The optimization model optimizes λ, n, h, *LCL* and *UCL* to minimize *EC*, total expected cost per unit time during an operational cycle, on condition that the constraints on ATS_0 and r are all satisfied.

Among the five design variables, λ, n, h, *LCL*, and *UCL*, the sample size n and smoothing factor λ are the only independent variables. The sampling interval h depends on n, that is,

$$h = n / R. \tag{6}$$

Equation (6) ensures that the constraint on the inspection rate r is satisfied and Equation (5) ensures that the available resource is fully utilized. When the inspection rate R is given, an optimal combination of n and h will result in the minimum value of *EC*. The control limits *LCL* and *UCL* are determined so that the resultant in-control ATS_0 is equal or very close to ζ (constraint (4)). The objective function *EC* can be computed by,

$$EC = \int_0^\infty \left[TC(\delta) \cdot f_\delta(\delta) \right] \, d\delta \tag{7}$$

where, $TC(\delta)$ is the total cost per unit time for a given mean shift of δ. The total cost $TC(\delta)$ can be estimated based on Duncan's (1956) model,

$$TC(\delta) = \frac{b + cn}{h} + \frac{\left[ATS_1 - \frac{h}{2} + \frac{\lambda_a h^2}{12} + en + D \right] QC\lambda_a + \frac{T}{ATS_0} + \lambda_a W}{1 + \lambda_a \left[ATS_1 - \frac{h}{2} + \frac{\lambda_a h^2}{12} + en + D \right]} \tag{8}$$

where the values of ATS_1 and ATS_0 are calculated using Markov chain approach. Finally, the probability density function $f(\delta)$ in Equation (7) can be obtained from Rayleigh distribution. The probability density function of the Rayleigh distribution is given by

$$f_\delta(\delta) = \frac{\pi\delta}{2\mu_\delta^2} \exp\left(-\frac{\pi\delta^2}{4\mu_\delta^2} \right) \tag{9}$$

which is characterized by a single parameter— the mean value μ_δ of δ.

2.4 Optimization process

The optimization design of the economic EWMA chart is carried out in a two-level search, in which the optimal values of the independent variables n and λ are sought, each through a single variable search. At the first level, the optimal value of the sample size n is searched (the sampling interval h is dependent on n) from 1 with a step size of one until EC cannot be further reduced. At the second level, the optimal value of the smoothing factor λ $(0 < \lambda < 1)$ is searched. In this level, the upper control limit UCL ($LCL = -UCL$) is also optimized based on the specified value of ζ. The ultimate objective of both first and second level searches is to minimize the EC. At the end of the entire search, the optimal EWMA chart with the minimum EC is determined and the corresponding optimal values of λ, n, h, LCL and UCL are also determined.

3. Comparative studies

The following four control charts are designed and compared in this section:

(1) A traditional economic $\bar{X}$ chart– It is an economic $\bar{X}$ chart using a sample size of five ($n = 5$) (Montgomery, 2013).
(2) An optimal economic $\bar{X}$ chart (Shamsuzzaman et al. 2018)– The charting parameters n, h, LCL, and UCL of this economic $\bar{X}$ chart are optimized in order to minimize EC.
(3) A traditional economic EWMA chart– This economic EWMA chart uses a fixed smoothing factor of λ = 0.1. The widely used values of λ are 0.05, 0.1, and 0.20. However, these choices are somewhat subjective (Montgomery, 2013). The sample size n is set at 1 ($n = 1$) as the EWMA scheme with a sample size of one is considered to be highly successful from a general viewpoint (Reynolds and Stoumbos, 2004).
(4) An optimal economic EWMA chart. It is an economic EWMA chart that optimizes the charting parameters, λ, n, h, LCL, and UCL in order to minimize EC.

To facilitate the comparison, a normalized EC_{normal} values for each chart is calculated by using the EC value of the optimal economic EWMA chart as the norm.

$$EC_{normal} = \frac{EC}{EC_{opt}} \tag{10}$$

Obviously, if the value of EC_{normal} of a chart is larger than one, the performance of this chart will be inferior to that of the optimal economic EWMA chart, and *vice versa*. Without losing generality, the in-control process mean μ_0 and standard deviation σ_0 are set at 0 and 1, respectively.

The above-mentioned four control charts are designed using the following hypothetical values of the design specifications:

- ζ (minimum allowable in-control average time to signal ATS_0, hour) = 400
- R (maximum allowable inspection rate, per hour) = 10
- μ_δ (mean of the mean? shifts δ) = 0.5
- λ_a (rate of occurrence of the assignable cause, occurrences per hour) = 0.01
- D (time period from the detection of the out-of-control state to the location and removal of the assignable cause, hour) = 2.0
- e (time to estimate an observed data of a sample, hour) = 0.01
- b (fixed component of sampling cost, \$) = 1.0
- c (variable component of sampling cost, \$) = 0.1
- W (cost of finding and fixing an assignable cause, \$) = 50.0
- T (cost of examining a false alarm, \$) = 100.0

The parameters of the four control charts along with their expected costs are calculated and listed below:

Traditional economic $\bar{X}$ chart:
$n = 5$, $h = 0.50$, $UCL = 1.3521$, $LCL = -1.3521$, $ATS_0 = 400.02$, $EC = 27.75$
$EC_{normal} = 2.094$

Optimal economic $\bar{X}$ chart:
$n = 19$, $h = 1.9$, $UCL = 0.5950$, $LCL = -0.5950$, $ATS_0 = 400.01$, $EC = 16.33$
$EC_{normal} = 1.232$

Traditional economic EWMA chart:
$\lambda = 0.10$, $n = 1$, $h = 0.1$, $UCL = 0.7883$, $LCL = -0.7883$, $ATS_0 = 399.76$, $EC = 25.90$
$EC_{normal} = 1.955$

Optimal economic EWMA chart:
$\lambda = 0.03$, $n = 4$, $h = 0.40$, $UCL = 0.1655$, $LCL = -0.1655$, $ATS_0 = 399.86$, $EC = 13.25$
$EC_{normal} = 1.000$

The results show that both optimal $\bar{X}$ and EWMA charts exhibit superior performance compared to their traditional versions, which highlights the importance of the optimization design of the control charts. However, the optimal economic EWMA chart significantly reduced the total expected cost compared to the other three charts. The values of EC_{normal} indicate that the optimal economic EWMA chart outperforms the traditional economic $\bar{X}$ chart, optimal economic $\bar{X}$ chart, and traditional economic EWMA chart by about 109%, 23%, and 95%, respectively.

4. Conclusions

This study presents an optimization model for the design of an economic EWMA control chart. The optimization process considers random process shifts instead of a single or a few shift values, which is more realistic from a practical viewpoint. The optimization process also ensures that the false alarm rate of the designed chart will not be increased, and extra inspection resources will not be needed. The proposed optimal economic EWMA chart outperforms (in terms of total expected cost per unit time) the traditional economic $\bar{X}$ chart, the optimal economic $\bar{X}$ chart, and the traditional economic EWMA chart by about 109%, 23%, and 95%, respectively, which highlights the importance of optimization design of the control charts. In this study, the optimal economic EWMA chart is designed for monitoring shifts in the process mean. Future research can consider shifts in both process mean and variance, and the randomness of the process shift can also be characterized by using other distributions such as uniform distribution instead of Rayleigh distribution.

References

Castagliola, P., Celano, G., and Fichera, S. Economic-statistical design of an S-EWMA control chart for monitoring process variability. *Journal of Quality in Maintenance Engineering*, vol. 13, no. 3, pp.304-320, 2007.

Chen, Y., & Yu, F. (2001). Economic Design of a Moving Average Control Chart with Multiple Assignable Causes when Two Failures Occur. *Asian Journal on Quality*, vol. 2, no. 1, pp.69-86, 2001.

Duncan, A. The economic design of X chart used to maintain current control of a process. *Journal of The American Statistical Association*, vol. 51, pp. 228-42, 1956.

Franco, B. C., Celano, G., Castagliola, P., and Costa, A. F. Economic design of Shewhart control charts for monitoring autocorrelated data with skip sampling strategies. *International Journal of Production Economics*, vol. 151, pp.121-130, 2014.

Montgomery, D. C. *Introduction to statistical quality control*: John Wiley & Sons, 2013.

Niaki, S., Malaki, M., and Ershadi, M. A particle swarm optimization approach on economic and economic-statistical designs of MEWMA control charts. *Scientia Iranica*, vol. 18, no. 6, pp.1529-1536, 2011.

Reynolds, M. R., & Stoumbos, Z. G. Should Observations Be Grouped for Effective Process Monitoring? *Journal of Quality Technology,* vol. 36, no. 4, pp.343-366, 2004.

Shamsuzzaman, M., and Wu, Z. Control chart design for minimizing the proportion of defective units. *Journal of Manufacturing Systems*, vol. 25, no. 4, pp.269-278, 2006.

Shamsuzzaman, M., Haridy, S., Alsyouf, I., and Rahim, A. Design of economic $\bar{X}$ chart for monitoring electric power loss through transmission and distribution system. *Total Quality Management & Business Excellence*, pp. 1-21, 2018.

Shamsuzzaman, M., Wu, Z., and Elias, M. R. Designs of $\bar{X}$&S control charts with optimal manpower deployment. *Computers & Industrial Engineering*, vol. 56, no. 4, pp.1589-1596, 2009.

Torng, C., Lee, P., and Liao, N. An economic-statistical design of double sampling control chart. *International Journal of Production Economics*, vol. 120, no. 2, pp.495-500, 2009.

Wu, Z., Shamsuzzaman, M., and Pan, E. S. Optimization design of control charts based on Taguchi's loss function and random process shifts. *International Journal of Production Research*, vol. 42, no 2, pp.379-390, 2004.

Wu, Z., Shamsuzzaman, M., and Wang, Q. Designs of control charts with optimal manpower deployment. *International Journal of Production Research*, vol. 44, no. 11, pp.2119-2132, 2005.

Yu, F., Chin, H., Huang, K., and Wu, Z. An economic-statistical design of $\bar{X}$ control charts with multiple assignable Causes. *Journal of Quality*, vol. 17, no. 4, pp.327-337, 2010.

Haridy S., Rahim M. A., Shokri Z. Selim, Wu Z. and Benneyan J. (2017) EWMA Chart with Curtailment for Monitoring Fraction Nonconforming. *Quality Technology & Quantitative Management*, 14, 412-428

Biographies

Mohammad Shamsuzzaman currently is an Associate Professor in the Department of Industrial Engineering and Engineering Management at University of Sharjah, UAE. He obtained his PhD in Systems and Engineering Management in 2005 from Nanyang Technological University, Singapore. His current research focuses on statistical process control, lean six sigma, reliability, simulation and multi-criteria decision-making. He is a member of the American Society for Quality.

Salah Haridy is an Assistant Professor in the Department of Industrial Engineering and Engineering Management at University of Sharjah, UAE. He received his M.Sc. and Ph.D. degrees from Benha University, Egypt and Nanyang Technological University, Singapore in 2008 and 2014, respectively. He is the recipient of the 2013 Mary G. and Joseph Natrella Scholarship awarded by the American Statistical Association (ASA) and the 2014 Richard A. Freund International Scholarship awarded by the American Society for Quality (ASQ). His research interests cover quality engineering, statistical process control and design of experiments.

Imad Alsyouf is an associate professor of Industrial Engineering, employed by University of Sharjah, UAE. He is the founder and coordinator of the Sustainable Engineering Asset Management (SEAM) Research Group. He has produced more than 67 conference and journal papers. He has about 30 years of industrial and academic experience in various positions in Jordan, Sweden and UAE. His research interests include reliability, quality, maintenance, and optimization. He has developed and taught more than 25 post and undergrad courses. He delivered training courses in Kaizen, TQM, and organizational excellence.

Hamdi Bashir received his PhD degree in 2000 from McGill University, Montreal, Canada. Currently, he is an Associate Professor of Industrial Engineering and Engineering Management at the University of Sharjah. Prior to joining this university, he held faculty positions at Sultan Qaboos University, University of Alberta, and Concordia University. His research interests are in the areas of project management, manufacturing systems, quality management, and healthcare management. He is a senior member of the Institute of Industrial and Systems Engineers (IISE).