Moderately-skilled	0.8
Less-skilled	0.7

The problem can thus be identified as to develop efficient metaheuristic for the multi-objective multi-skill resource-constrained project scheduling problem (MO-MSRCPSP) taking skill proficiencies into account. We have to minimize the total makespan as well as total time elapsed with less-skilled resource assignments as explained in section 2.1 what follows next.

2.1 Mathematical model

To accommodate the different proficiency levels in skills, a bi-objective mathematical model is formulated for the MSRCPSP. The notations and constraints of the model are adapted from Joshi et. al (2019) with modification in the objective function as shown below:

Parameters	**Definition**
N	number of non-dummy activities in the project
A , $i \in \{1, \ldots \ldots, N+2\}$	set of activities
p_i	processing time of activity A_i
K	total number of skills available
P	total number of available staff members
S , $k \in \{1, \ldots, K\}$	set of skills
P, $m \in \{1, \ldots\ldots, P\}$	set of staff members
$S_{m,k}$	greater than 0 if staff member P_m possesses skill S_k, 0 otherwise
$b_{i,k}$	number of staff members with skill S_k required by activity i
t_i	start time of activity i

Decision variables:

$x_{i,m,t}$ = 1; if staff member m starts an activity i at time t, 0 otherwise
$y_{i,m,k}$ = 1; if staff member m starts an activity i with skill k, 0 otherwise
$Z_{i,t}$ = 1; if activity i is started at time t, 0 otherwise

Objective function:

$$\text{Min. } Z_1 = t_{N+2} \tag{1}$$
$$\text{Min. } Z_2 = \sum_{i \in A} \sum_{k \in S} \sum_{m \in P} (y_{i,m,k} \cdot p_i (1 - S_{m,k})) \tag{2}$$

Besides the regular objective of minimizing the project *makespan* (Z_1), the second objective aims to minimize the total time elapsed with less-skilled resource assignments defined as Skill Divergence Span (*SDS*) (Z_2).*SDS* is basically the product of processing time of an activity and the corresponding amount of penalty attracted due to the low skill attained by staff member in performing that activity. Mathematically,

$$\text{Skill Divergence Span } (SDS) = y_{i,m,k} \cdot p_i \cdot (1 - S_{m,k}) \tag{3}$$

2.2 An illustrative example

To understand the nature of the MO-MSRCPSP under investigation, consider the following project instance. It comprises of four non-dummy activities linked by precedence relations as shown in Figure 1. Activity number 1 and 6 are dummy activities i.e. they do not consume any time or resource for their execution. For other activities, at least one resource is required for their execution. It is important to note that the skills attained by the staff members have been assumed to have different proficiencies or expertise level (Table 2). The value '1' indicates that a staff member masters that particular skill at an expert level while value less than 1 signifies low proficiency to exhibit the skill relative to the expert level. One can observe that staff member 1 is expert in exhibiting skill S_2 while has only 70 % proficiency in skill S_3 and so on.

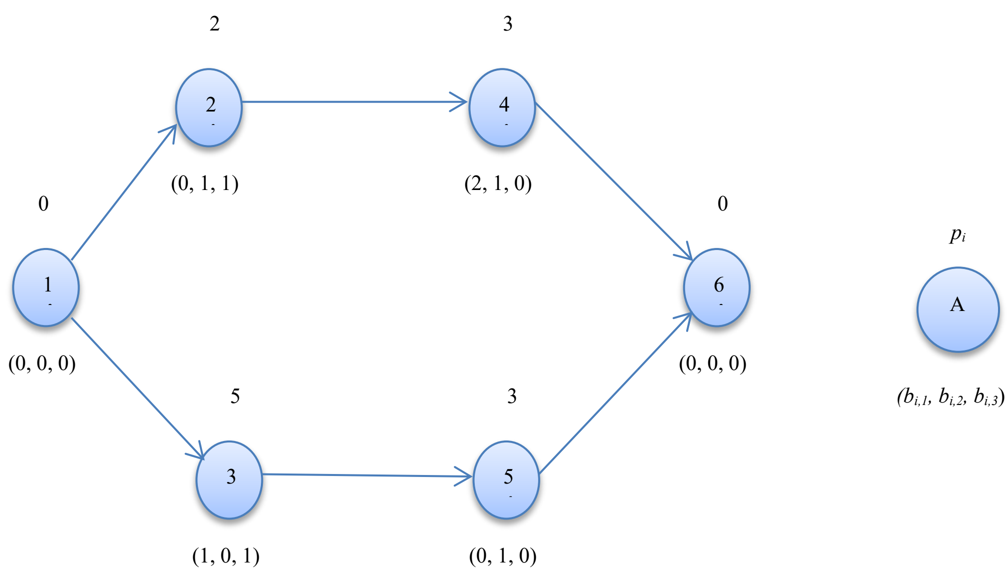

Figure 1: Precedence graph of the illustrative project

Table 2: Staff-Skill Proficiency Matrix

Staff	Skills attained		
	S_1	S_2	S_3
1	0	1	0.7
2	0.9	0	1
3	1	0	0
4	1	0	0.8

1	3	2	4	5	6
	3	1	1	1	

4	2	2	
		3	

Figure 2: A solution (encoded individual) of illustrative project

An individual for the MSRCPSP can be conveniently encoded into two parts as shown in Figure 2 (Gürbüza, E. , 2010). The first part is in the form of a top horizontal row that determines the relative priorities of the activities which is more popularly known as activity list (AL). The second part comprises of vertical columns corresponding to each activity such that the number of elements in each column is equal to the total number of resources required for the particular activity.

The procedure of computing the values of two objective functions for this particular instance is illustrated now. In order to calculate the makespan, the modified serial schedule generation scheme (Joshi et. al, 2019) is employed. Using this procedure the value of Z_1 i.e. makespan is calculated as 11 time units. To calculate the value of second objective function i.e. total time elapsed in less-skilled resource assignments, the *SDS* is computed corresponding to each resource using equation (3) as mentioned below:

Skill Divergence Span (*SDS*) = $y_{i,m,k}.p_i.(1 - S_{m,k})$
For this particular problem, i=1,2,3,4,5,6; k =1,2,3; m=1,2,3,4)
Thus, $Z_2 = \sum_i^{N+2} SDS$ = 0+0+ 1+ 0.3+0+ 0=1.3 time units.

The value of Z_2 quantifies the total amount of time during which resources with skill proficiency less than 1 are engaged in performing project activities. A fraction value is justified by the fact that proficiencies of staff members have been assumed on a continuous scale rather than discrete or hierarchical levels as found in literature. This value of Z_2 has to be minimized in the algorithm along with Z_1 i.e. project makespan.

3. Proposed algorithms for solving the MO-MSRCPSP

In this work we employ one of the intuitive and direct approach of multi-objective optimization namely weighted-sum or scalarization method to solve the MO-MSRCPSP. More specifically, the two objectives are combined by assigning suitable weights and the MO-MSRCPSP is converted into a single-objective problem. To elaborate further, a weighted-sum or scalarization method for an n-objective problem assigns weight w_i to the i^{th} objective function $f_i(x)$ and minimizes a positively weighted sum of all the objectives. Mathematically,

$$Z_3 = \text{Min.} \sum_{i=1}^{n} w_i . f_i(x) \quad (4)$$
$$\sum_{i=1}^{n} w_i = 1 \quad (5)$$
$$w_i > 0, i = 1, \ldots\ldots, n \quad (6)$$

Equation (4) presents a unique objective function denoted by Z_3. In comparison to minimization of *SDC,* a higher consideration is given to minimization of makespan and hence the weights of the two objective functions have been specified as w_1=0.7 and w_2=0.3. Nevertheless, these weights have been chosen arbitrarily and a decision maker can suitably alter their values depending upon his/her preference of the objective function to be minimized. In the next section the details of the metaheuristic approaches employed to solve the problem under hand have been presented.

3.1 A multi-objective TLBO for the MO-MSRCPSP

In recent years, metaheuristics have been an indispensable choice for solving a lot of NP-hard problems (Kumar et. al, 2019 (a) (b)). As mentioned earlier, looking to the promising results of the TLBO algorithm obtained for various discrete and optimization problems (Rao et. al, 2011, Kumar et. al, 2018) we employ the same for the multi-objective MSRCPSP. Table 3 summarizes the important parameters used in the TLBO developed for the MO-MSRCPSP (Joshi et. al., 2019(a), (b)):

Table 3: Summary of the MO-TLBO algorithm

Architecture of the TLBO developed for the MO-MSRCPSP	
Encoding scheme	Activity List (AL) with vertical columns having index of staff members as resource assignment
Decoding scheme	Modified serial schedule generation scheme
Initial population	RBRS sampling method with LFT priority rule
Teacher and Learner phase	Using 2-point crossover mechanism
Self-study phase	Using Boctor's mutation (BM)
Examination phase	Elitism
Test Instances	Generated by using methodology proposed by Almeida et al. (2016) with Staff-Skill Matrix replaced by Staff-Skill Proficiency Matrix.
Parametric details of the TLBO for the MO-MSRCPSP	
Size of initial population (*Class_size*)	40
Probability of self-study (*SS_prob*)	10%
Percentage of elite learners (*Elite_per*)	10 %
Number of instance tested	36
Number of schedules generated per instance	5000

3.2 A multi-objective GA for the MO-MSRCPSP

A MO-GA is also developed as an alternative metaheuristic primarily for comparing the results with the proposed TLBO. The basic scheme, operators and implementation framework of the proposed GA on the MO-MSRCPSP are depicted below (Table 4):

Table 4: Summary of the proposed MO-GA

Architecture of the GA developed for the MO-MSRCPSP	
Encoding scheme	Activity List (AL) with vertical columns having index of staff members as resource assignment
Decoding scheme	Modified serial schedule generation scheme
Initial population	RBRS sampling method with LFT priority rule
Crossover mechanism	2-point crossover mechanism
Mutation mechanism	Boctor's mutation (BM)
Elitism	Ranking based
Selection mechanism	2-tournament method for parents selection
Test Instances	Generated by using methodology proposed by Almeida et al. (2016) with Staff-Skill Matrix replaced by Staff-Skill Proficiency Matrix.
Parametric details of the GA for the MO-MSRCPSP	
Size of initial population (*Class_size*)	40
Crossover probability (*Cross_prob*)	0.80
Mutation Probability (*Mut_prob*)	0.10
Percentage of elite learners (*Elite_per*)	0.10
Number of instance tested	36

Number of schedules generated per instance	5000

4. Computational results

To test the behaviour of the two algorithms the algorithms have been coded in MATLAB 7 environment with Core i3 processor having Windows 8.1 and 4GB RAM. Using the methodology of Almeida et. al (2016) 36 different instances induced by different values of network complexity (*NC)*, skill factor (*SF*) and modified resource strength (*MRS*) are created. However, staff-skill matrix is modified accordingly to incorporate the different levels of skill proficiencies. It is ensured that each staff member is 'expert' in at least one skill type. For other skills attained by him, the proficiency level is varied using a random number *r* such that $r \in \{0.7, 0.8, 0.9\}$.

A total of 5000 schedules have been generated for both the algorithms as stopping criterion. It is important to note that fitness (objective) function in these problem set is modified by combining two objective functions by selecting a weight vector as $w = (0.7, 0.3)$. As the optimum solutions of these problems are not known, the percentage deviation from critical path based lower bound is calculated which is given as: % DEV = $(Z^H - Z^{CP})/Z^{CP} * 100$, where Z^H is the heuristic solution provided by the algorithm and Z^{CP} is the critical path duration.

The computational results are shown in Table 5. On the basis of these results some useful observations can be derived as follows. The average % deviation from ctitical path based lower bound obtained is comparatively lower for the MO-TLBO as compared to the MO- GA. It is 62.17% for the proposed MO-TLBO while for MO-GA its value is 75.11%. The average percent deviation increases with increase in skill factor (SF) which can be attributed to the fact that due to the increased proportion of resource requirements by activities, number of possible resource combinations increases accordingly.

Table 5.: Comparison of MO-TLBO and MO-GA

SF	NC	MRS	*P*	MO-TLBO (AVG. % DEV)	MO-GA (AVG. % DEV)
0.5	1.5	0.0667	8	68.28%	86.05%
		0.075	9	62.47%	81.40%
		0.0917	11	53.49%	77.42%
	1.8	0.0667	8	59.09%	78.79%
		0.075	9	56.06%	62.88%
		0.0917	11	40.91%	51.52%
	2.1	0.0667	8	60.21%	65.67%
		0.075	9	47.76%	62.69%
		0.0917	11	35.82%	49.76%
0.75	1.5	0.0667	12	79.07%	90.70%
		0.0778	14	65.12%	88.05%
		0.0944	17	62.79%	87.23%
	1.8	0.0667	12	69.70%	84.85%
		0.0778	14	62.12%	66.67%
		0.0944	17	60.61%	64.09%
	2.1	0.0667	12	61.19%	70.15%
		0.0778	14	52.24%	56.72%
		0.0944	17	40.30%	60.22%
1	1.5	0.0667	16	90.70%	116.28%
		0.075	18	76.74%	100.00%
		0.0917	22	74.42%	93.02%
	1.8	0.0667	16	78.79%	96.97%
		0.075	18	75.76%	77.27%

		0.0917	22	71.21%	65.15%
	2.1	0.0667	16	65.67%	77.61%
		0.075	18	56.72%	68.66%
		0.0917	22	53.73%	62.69%
var.	1.5	0.0667	12	79.07%	93.02%
		0.0778	14	67.44%	83.72%
		0.0944	17	53.49%	79.07%
	1.8	0.0667	12	72.70%	83.30%
		0.0778	14	66.67%	72.73%
		0.0944	17	62.12%	65.64%
	2.1	0.0667	12	60.21%	72.15%
		0.0778	14	52.24%	61.19%
		0.0944	17	43.30%	50.75%
Avg.				**62.17%**	**75.11%**

With increase in network complexity (NC), the number of precedence relations also increases. This means less number of activities are available that can be processed simultaneously, i.e. the degree of parallelization decreases. This in turn results in low values of percent deviation.

5. Conclusions

This paper investigates a scarcely treated work in literature about multi-objective multi-skilled resource-constrained project scheduling problem (MO-MSRCPSP) by considering mixed skill proficiencies. A multi-objective mathematical formulation is presented for this problem which aims to minimize two time estimates; the project makespan and the total time elapsed with less-skilled resource assignments which has been conceptualized as total skill divergence span (*SDS*). To solve this complex problem, a priori-approach based on weighted-sum or scalarization method is used. The weights given to makespan and *SDS* function are 0.7 and 0.3 respectively which can be arbitrarily modified by decision maker. To solve this problem, two metaheuristics have been proposed namely MO-TLBO and MO-GA. The test instances developed for the MSRCPSP have been suitably modified with mixed proficiency levels of the staff members. The comprehensive test results reveal that the MO-TLBO has performed significantly better than the MO-GA and can be an effective metaheuristic for solving such real life problems.

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