

# An Investigation on Assessing Process Capability Indices under Weibull Distribution

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## Abstract

Process capability indices (PCIs) are widely-used statistical quality control tools for measuring process performance. In manufacturing industries, PCIs can be used to help companies stay competitive by providing understanding of process capability and guidance for quality improvement. However, the normality assumption for traditional PCIs, such as  $C_p$  and  $C_{pk}$ , might be violated in real-world manufacturing processes, and this can lead to erroneous conclusions of process capability and eventually financial losses for companies. Hence, many researchers modify existing PCIs or construct new PCIs to evaluate the performance of non-normal processes. Given various non-normal PCIs that have been proposed, there is an interesting in knowing how well these methods are, and there are demands for a proper comparative analysis.

In this study, we investigate four non-normal PCIs,  $C_{Npk}$ ,  $C_s$ ,  $C_y$ , and  $C_{py}$ . We first compare these PCIs in terms of their capability to reflect true process yields under different parameter combinations of Weibull distribution, then examining their magnitude of estimation biases when implemented in different sample sizes. The result shows that  $C_y$  is the most exceptional among the four PCIs in the above two aspects, and has a recommended sampling size of 50 or more. Moreover, we employ a well-known inference approach, Bayesian inference with Markov-chain Monte Carlo (MCMC), to review the sampling variation derived from parametric estimation of the Weibull distribution. A series of simulations for establishing MCMC credibility intervals is conducted on  $C_y$  with the adaptive rejection Metropolis sampling (ARMS) algorithm. The coverage rates and average widths of credibility intervals are adopted as two criteria to assess the accuracy and precision of the MCMC method, respectively. It turns out that the MCMC credibility intervals perform satisfactorily in different sample sizes and under various parameter combinations of Weibull distribution.

## Keywords

Process capability indices; Markov-chain Monte Carlo; Weibull distribution; Interval estimation

## Biographies

**Chun-Yi Chang** is a graduate student in Industrial Engineering and Engineering Management from National Tsing Hua University, Taiwan(ROC). Mr. Chang holds a Bachelor of Science degree in Industrial Engineering and Engineering Management from National Tsing Hua University. His research

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**Chien-Wei Wu** is currently a Professor in the Department of Industrial Engineering and Engineering Management at National Tsing Hua University (NTHU), Taiwan. Dr. Wu received his Ph.D. degree in Industrial Engineering and Management with Outstanding Ph.D. Student Award from National Chiao Tung University in 2004 and the M.S. degree in Statistics from NTHU in 2002. He is serving as one of Editors-in-Chief of Quality Technology and Quantitative Management (QTQM) and editorial board members for various international journals. His research interests include quality engineering and management, statistical process control, process capability analysis and data analysis.