

# **Integrated optimization problem of Maintenance and Lot-Sizing with Risk assessment**

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## **Abstract**

This paper presents an integrated maintenance and lot-sizing problems under energy constraints with risk assessment in the case of unforeseeable circumstance. The considered production system is composed of identical parallel machines that must meet a random demand with a required service level. The lot-sizing optimization consists in determining the number of operating machines and produced quantity to meet demand and/or replenish the stock. Then, an analytical model is developed to determine a common maintenance plan for all machines while minimizing the total maintenance cost. The failure rate, of each machine, assumed that increases according to both manufactured size and the cumulative failure rate. In the second part of this paper, a novel risk assessment is presented in order to overcome the unforeseeable circumstance making some machines unavailable. The conducted experiments on generated instances show that the proposed approach is efficient and provide good solutions for maintenance plan, lot-sizing plan and risk assessment.

## **Keywords**

Lot-sizing, maintenance policy, energy constraints and risk assessment.

## **1. Introduction**

Nowadays, the dependability plays a very important role in all industry sectors and even other activities, particularly in the defense, aeronautics, space, nuclear, telecommunications and transport. It consists of assessing the potential risks, predicting the occurrence of unforeseen failures and attempting to minimize the consequences of catastrophic situations when they arise. In that unforeseen situation, risk assessment (Lee et al. (2017), Mossa et al. (2016)) is a key part in decision support for industrial stakeholders. In this paper, the risk assessment for a production system aims to satisfy the customer orders, their financial consequences and the condition of choosing the outsourcing solution.

In addition to risk assessment, the industrial stakeholders are responsible for effectively determining the produced lot-size and the maintenance policy adopted. To ensure increased competitiveness, the industrialists must manage these two parts simultaneously and in a complementary way. In this context, several studies have been conducted how to combine the lot-sizing and maintenance after several searches in which those two activities are treated independently. The study of maintenance policy is started with Barlow and Hunter (1960) and followed by others several works as they are cited in Pierskalla and Voelker (1976). However, in the real world, a study of maintenance without considering the lot-sizing problem can only lead to bankruptcy. The need for studying novel integrated maintenance and lot-sizing strategies became useful. Brandolese et al. (1996) presented a policy for maintaining a multi-machines production system. The planning consists in determining the date of each job and the machine that should perform it. They introduced the preventive maintenance actions in the lot-size planning by placing them close to optimal maintenance periods. Chelbi and Rezg (2006) elaborated a programming model in order to establish both the stock level and the preventive maintenance actions for a production system composed of one unit. The optimization of preventive maintenance and stock level also treated by Radhoui et al. (2010) with imperfect production system that can move to a state called out-of-control after a time considered random. In addition, the authors assumed that stock level and production rate are considered as decision variables. Rezg et al. (2008) was addressed the joint inventory control and

preventive maintenance, by developing an analytical model in which the failure of production system is random. Recently, Kammoun and Rezg (2018b) developed a decision-support system of selective maintenance for any multi-component production system based on supervised failure. They obtained an optimal preventive maintenance decision, aiming for replace the component by taking into account the real age of component, the spare-part stock, the available number of repairman and the constraint of break time. The authors are used the k-means algorithm and the Apriori algorithm of datamining approach to solve the problem.

In the same perspective, others works have added constraints of real industry. Zied et al. (2017) developed an analytic model for integrated maintenance and lot-sizing problem with the outsourcing solution in situation in which the main production system cannot satisfy the random demand. The authors proved the notable profit of company at maintenance actions and produced lot-sizing by using the outsourcing solution. In the same vein of integration maintenance and lot-sizing problem with outsourcing constraint, Nourelfath et al. (2010-2012) developed a decision-support system for a production system made-up of a set of parallel components, which supposed the presence of economic dependence and common cause failures in the production system. In the integrated maintenance and lot-sizing policies literature, few papers have attempted to solve the problem for parallel identical machines under energetic constraint. Moreover, the consideration of production rate influences on system degradation in which the demand is random per period. The research presented by Hajej et al. (2011) proposed a lot-sizing plan minimizing the stock level taking into account the outsourcing constraint. The optimal maintenance policy is then established by using the obtained lot-sizing plan.

Firstly, this work aims to determine sequentially the best lot-size and maintenance plan of machines under energy constraints. To avoid shortage due to the manufacturing system unavailability following uncertain events, the outsourcing can be a solution provided that it does not result an excessive risk to the company's profitability. Within this framework, the second objective is to establish suitable solution for company between the choice of a subcontractor or not following a risk assessment. The production system of outsourcing is assumed available and has a constant production rate, and its unit production cost is well-known. The remainder of the paper is organized as follows. Section 2 presents the problem statement. Section 3 provides the analytical lot-sizing model. Section 4 is dedicated to the development of analytical maintenance model. The novel risk assessment in order to avoid shortage is presented in Section 5. A numerical example is illustrated in Section 6 to demonstrate the proposed approach. Finally, Section 6 concludes the paper and provides future works.

## **2. Problem statement**

### **2.1 Notations**

The notations used throughout this work are following:

- $H, \Delta_t$  : production horizon;
- $U$  : production rate;
- $p_t$  : number of operating machines at period  $t$ ;
- $T_{kt}$  : overall working time of machine  $k$  at period  $t$ ;
- $eq_t$  : amount of energy needed to produce one unit;
- $s_t$  : stock level at the end of period  $t$ ;
- $d_t$  : random demand at period  $t$ ;
- $\gamma$  : service level to meet the demand;
- $c_p$  : unit cost of production;
- $cs_t$  : unit storage cost per period;
- $c_{egy}$  : unit cost of energy;
- $W_{kt}$  : failure of machine  $k$  at production period  $t$ ;
- $W_n$  : nominal failure rate of a given machine;
- $\beta_k$  : preventive maintenance period for machine  $k$ ;
- $N$  : number of preventive maintenance actions;
- $c_p$  : unit cost of preventive maintenance;
- $c_c$  : unit cost of corrective maintenance.

## 2.2 Problem description and assumption

The production system considered in the present paper is made up of several identical machines operated in parallel (Fig. 1). The production rate of machines varies from one period to the next according to customer demand rate, which is considered in this work as random. Indeed, at each production period the number of functional machines must be decided in accordance with the request quantity. Besides, the operating time, during a period, is variable from one machine to another, which is depends on the quantity of consumed energy. In other words, the total operating time of machines is related to consume energy that should be economical. In order to meet the random request given that a required inventory service level and failure constraint of production system, a stock is built. The first objective is to determine sequentially the best lot-size and maintenance plan of machines, during the production horizon, while optimizing the costs incurred; production, maintenance and energy costs. It is important to note that the best lot-size plan stands for a good distribution of production on machines, as well as a good adjustment of working time per period.

To avoid shortage due to the manufacturing system unavailability following uncertain events, the outsourcing can be a solution provided that does not result an excessive risk to the company's profitability. The second objective is to establish suitable solution for company between the choice of a subcontractor or not following a risk analysis study. The production system of outsourcing is assumed available and has a constant production rate, and its unit production cost is well-known.

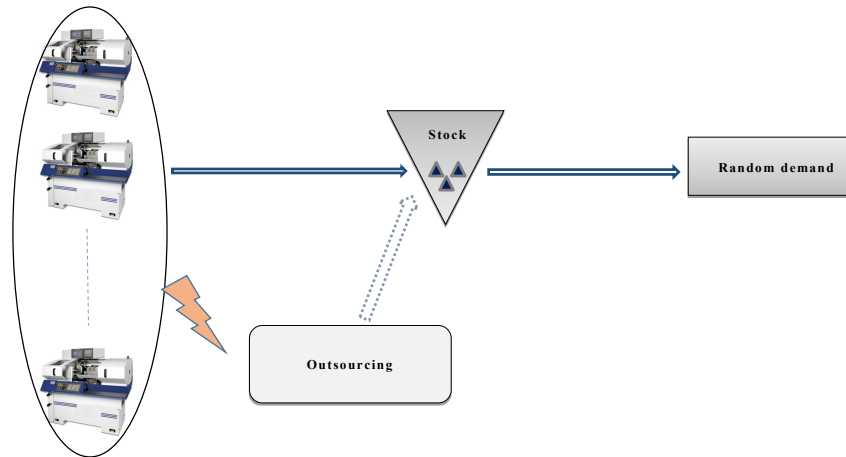


Figure 1. The environment of the study

## 3. Analytic lot-sizing model

### 3.1 Variables of analytical model

We recall that manufactured size relies on both production rate and operating time of machines working in parallels. Let  $U$  the production rate of machine,  $p_t$  and  $T_{kt}$  are, respectively, the number of operating machines and overall working time of machine  $k$  at period  $t$ , the total manufactured size over a production horizon  $H$  can be formulated as follows:

$$MS = U \times \sum_{t=1}^T \sum_{k=1}^{p_t} T_{kt} \quad (1)$$

The stock level at the end of production horizon is calculated at the beginning of period  $(T - 1)$ , since the last period is not intended for production. The inventory balance equation, over horizon  $H$ , formulated by Eq (2) in which depends of inflows ( $U \times \sum_{k=1}^{p_t} T_{kt}$ ) and outflows ( $d_t$ ) of stock.

$$S = \sum_{t=1}^T (s_{t-1} + U \times \sum_{k=1}^{p_t} T_{kt} - d_t) \quad (2)$$

Where  $s_t = s_0$  is the initial stock for  $t = 1$ .

A service level is required by company in order to meet the demands at each period, which is expressed as follows:

$$P[s_t \geq 0] \geq \gamma \quad \forall t \in \{1 \dots T\} \quad (3)$$

The consumed energy  $CE$  over production horizon  $H$  is proportionally used with the operating time and the production rate. Let  $eq_t$  is the energy quantity to produce one unit, the total energy consumed over production horizon  $H$  can be formulated as follows:

$$CE = eq_t \times U \times \sum_1^T \sum_{k=1}^{p_t} T_{kt} \quad (4)$$

### 3.2 Deterministic model

This section provides a transformation of stochastic cost of production model into an equivalent deterministic model. The goal of this transformation is to facilitate resolution while keeping the same main model properties. For this, we note by  $E[X] = \hat{X}$  the mean of random variable  $X$ ,  $V[X] = \sigma_X$  its variance. Also,  $E[Y] = Y$  and  $V[Y] = 0$  whether  $Y$  is constant within each period.

The stochastic total production cost over production horizon can be expressed by Eq. (5), where  $c_p$  is the production cost per time unit of one unit.

$$c_p \times \sum_{t=1}^T E\left(\sum_{k=1}^{p_t} T_{kt}\right)^2 \quad (5)$$

As  $E\left(\sum_{k=1}^{p_t} T_{kt}\right)^2 = \left(\sum_{k=1}^{p_t} T_{kt}\right)^2$ , then the equivalent deterministic cost as shown by Eq. (6).

$$c_p \times \sum_{t=1}^T \left(\sum_{k=1}^{p_t} T_{kt}\right)^2 \quad (6)$$

The stochastic cost of storage is defined by Eq. (7). To avoid the shortage of stock we fixe the minimum stock by the expression  $(l_1 + l_2 \times d_t)$ , where  $l_1$  and  $l_2$  are positive number that defined by the production service.  $cs_t$  is the storage cost unit per period.

$$cs_t \times \sum_1^T E\left[\left(s_t - (l_1 + l_2 \times d_t)\right)^2\right] \quad (7)$$

The development of Eq. (7) given the equivalent deterministic cost as following:

$$cs_t \times \sum_1^T \left(\hat{s}_t - (l_1 + l_2 \times \hat{d}_t)\right)^2 + T^2 \times \sigma_{d_t}^2 \times (1 + l_2)^2 \quad (8)$$

Given the consumed energy  $CE$  defined by Eq. (4), the total energy cost consumed over production horizon can be expressed by Eq. (9), where  $c_{egy}$  is the unit cost of energy.

$$c_{egy} \times eq_t \times U \times \sum_1^T \sum_{k=1}^{p_t} T_{kt} \quad (9)$$

The determinist constraints per period of inventory balance equations and the required service level are respectively defined by Eq. (10-11). We denoted by  $\varphi^{-1}$  the function of inverse distribution.

$$\hat{s}_t = \hat{s}_{t-1} + U \times \sum_{k=1}^{p_t} T_{kt} - \hat{d}_t \quad (10)$$

$$U \times \sum_{k=1}^{p_t} T_{kt} \geq \sigma_{d_t} \times \varphi^{-1}(\gamma) + \hat{d}_t - s_{t-1} \quad (11)$$

The optimal production plan is the optimal couple, at each period, of number of machines used,  $p_t$ , and operating time of each one  $T_{kt}$ , which minimizing the incurred costs of production, storage and energy. Indeed, the establishment of optimal production plan amounts to solving the deterministic programming model  $Q_1$ .

$Q_1$ :

$$\begin{aligned} \text{Min}_{(p_t, T_{kt})} \left\{ c_p \times \sum_{t=1}^T \left( \sum_{k=1}^{p_t} T_{kt} \right)^2 + cs_t \times \sum_{t=1}^T \left( \widehat{s}_t - (l_1 + l_2 \times \widehat{d}_t) \right)^2 + T^2 \times \sigma_{d_t}^2 \times (1 + l_2)^2 \right. \\ \left. + c_{egy} \times eq_t \times U \times \sum_{t=1}^T \sum_{k=1}^{p_t} T_{kt} \right\} \end{aligned}$$

Subject to:

$$\begin{aligned} \widehat{s}_t &= \widehat{s}_{t-1} + U \times \sum_{k=1}^{p_t} T_{kt} - \widehat{d}_t & \forall t \in [1, \dots, H] \\ 1 &\leq p_t \leq K & \forall t \in [1, \dots, H] \\ U \times \sum_{k=1}^{p_t} T_{kt} &\geq \sigma_{d_t} \times \varphi^{-1}(\gamma) + \widehat{d}_t - s_{t-1} & \forall t \in [1, \dots, H] \\ 0 &\leq T_{kt} \leq T^{max} & \forall t \in [1, \dots, H] \\ U \times \sum_{t=1}^T \sum_{k=1}^{p_t} T_{kt} &\leq E^{max} & \forall t \in [1, \dots, H] \end{aligned}$$

### 3.3 Optimization of deterministic lot-sizing problem

The resolution of  $Q_1$ , to determine the optimal manufacturing lot-sizing ( $p_t, T_{kt}$ ), using an ordinary exact optimization method seems very difficult given the problem complexity. The complexity of lot-sizing problem, studied in this paper, stems from the number of machines operating in parallel and the huge number of constraints related to each one. In other words, it is difficult to avoid the explosion of solutions space characterizing this combinatorial optimization problem.

Therefore, to reduce the number of generated solutions, this paper proposes a resolution approach based on two optimization methods which are Branch and Bound and random exploration. The random exploration method performs a restricted random draw of solution candidates, among the space solutions. The feasibility of solution candidates is then checked against the constraints of  $Q_1$ : stock level, service level, number of operated machines, energy constraint and working time. The remaining solutions are used, in a third step, as input data to establish the sub-optimal solution by applying the Branch and Bound algorithm. The reader can refer to Kammoun et al. (2014-2018a) for more details about the Branch and Bound and random exploration algorithms.

## 4. Analytic maintenance model

### 4.1 Maintenance policy

The maintenance policy adopted in this paper is a preventive maintenance that is considered as a perfect in which, after each maintenance action, the machine is regarded with the status “as good as new”. About the corrective maintenance that apply between two preventive maintenance, a minimal repair action is executed. We note that the duration of preventive or corrective maintenance is supposed negligible. The purpose is to establish the optimal number of preventive maintenance actions  $N^*$  over horizon production  $H \cdot \Delta_t$  for  $M$  machines as shown by Fig. (2).

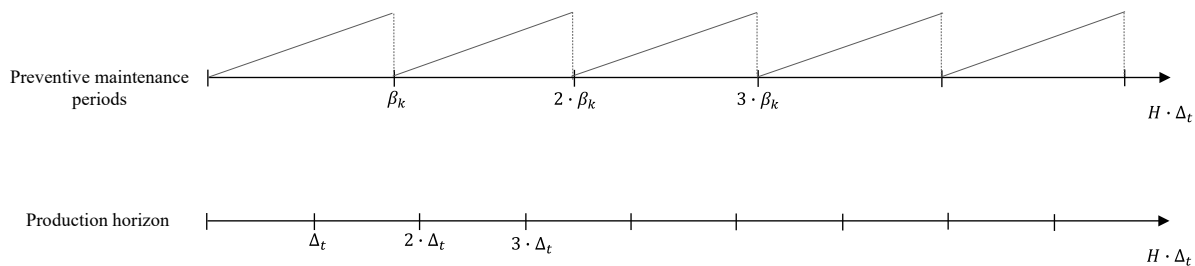


Figure 2. Production horizon and preventive maintenance periods

The optimal number of preventive maintenance actions  $N^*$  is achieved by optimization of total maintenance cost  $C_M$  that is expressed as follows:

$$C_M = c_p \times N + c_c \times NF_M \quad (12)$$

Each production period is distinguished by its own manufactured size. The failure rate increases, on the one hand, according to the manufactured size adopted in each period, and in terms of the failure rate cumulated at each period the end of the previous period, on the other hand. Besides, the failure rate is considered in this paper as linear at each production period for all machines, which we formulated as follows:

$$W_{kt}(j) = y_{kt} \times W_{kt-1}(\Delta_j) + \frac{\sum_{k=1}^{p_t} T_{kt}}{T^{max} \times \Delta_t} \times W_n(j) \quad (13)$$

Where:

$$y_{kt} = \begin{cases} 1 & \text{if the machine } k \text{ is exploited in } (t - 1) \\ 0 & \text{else} \end{cases}$$

We suppose that  $W_{k0}(j) = 0$ . With a proof by recurrence from  $t = 1$  to  $t = T$ , we can reformulated the failure rate expression as following:

$$W_{kt}(j) = \left[ \sum_{b=s, \beta_k}^t \frac{\sum_{k=1}^{p_t} T_{kt}}{T^{max} \times \Delta_j} \right] \times W_n(j) + \frac{\sum_{k=1}^{p_t} T_{kt}}{T^{max} \times \Delta_t} \times W_n(j) \quad \forall j \in [0, \Delta_j] \quad (14)$$

We recall that the average number of failure for machine  $k$  during  $\Delta_j$  is a cumulative number of failure rate of machine, which is expressed as follows:

$$NF_k = \int_0^{\Delta_j} W_{kt}(j) dj$$

Therefore, the average number of failure for machine  $k$  over horizon production  $H, \Delta_t$  is:

$$NF_k = \sum_{s=1}^{PM_k} \left[ \begin{aligned} & \sum_{t=In\left(\frac{\beta_k}{\Delta_j}\right)+1}^{In\left(\frac{\beta_k}{\Delta_j}\right)} \int_0^{\Delta_j} W_{kt}(j) \cdot dj \\ & + \int_0^{s \cdot \beta_k - In\left(\frac{\beta_k}{\Delta_j}\right) \cdot \Delta_j} W_k\left(In\left(\frac{\beta_k}{\Delta_j}\right)+1\right)(k) \cdot dj \\ & + \int_0^{In\left(\frac{\beta_{k+1}}{\Delta_j}\right) \cdot \Delta_j - s \cdot \beta_k} \frac{T_k\left(In\left(\frac{\beta_{k+1}}{\Delta_j}\right) \cdot \Delta_j\right)}{T^{max}} \cdot dj \end{aligned} \right] \quad (15)$$

For  $M$  identical parallel machines, the total average number over horizon production  $H, \Delta_t$  by using is Eq. (15) is as follows:

$$NF_M = \sum_{k=1}^M \left[ \sum_{s=1}^{PM_k} \left[ \begin{aligned} & \sum_{t=\ln\left(\frac{s \cdot \beta_k}{\Delta_j}\right)+1}^{\ln\left(\frac{s \cdot \beta_k}{\Delta_j}\right)} \int_0^{\Delta_j} W_{kt}(j) \cdot dj \\ & + \int_0^{s \cdot \beta_k - \ln\left(\frac{s \cdot \beta_k}{\Delta_j}\right) \cdot \Delta_j} W_{k\left(\ln\left(\frac{s \cdot \beta_k}{\Delta_j}\right)+1\right)}(j) \cdot dj \\ & + \int_0^{\ln\left(\frac{s \cdot \beta_{k+1}}{\Delta_j}\right) \cdot \Delta_j - s \cdot \beta_k} \frac{T_{k\left(\ln\left(\frac{s \cdot \beta_{k+1}}{\Delta_j}\right) \cdot \Delta_j\right) / \Delta_j}}{\tau_{max}} \cdot dj \end{aligned} \right] \right] \quad (16)$$

## 4.2 Optimization of maintenance actions

The goal of optimizing maintenance actions is to find a common maintenance plan for all machines. Indeed, a common maintenance plan makes it possible to avoid the interruption of production several times, in which at each break, in common plan, the preventive maintenance is applied to all machines. Then this optimizes the maintenance cost and increases the productivity.

Therefore, the proposed optimization approach is to determine, at first time, for each machine  $k$  the optimal maintenance period  $\beta_k^*$ . Then, at second time, the set of optimal maintenance periods,  $\beta_k^*$  with  $1 \leq k \leq K$ , is used to establish the average maintenance period,  $\bar{\beta}^*$ , for all machines where the maintenance action is applied for all machines.

## 5. Risk assessment

The company can confront an unforeseen situation, which creates the risk of malfunction of some machines. In order to deal with this new situation, the company can call on outsourcing to meet the demands provided it does not affect the financial plan of company. For this purpose, a risk assessment is studying in this section to make the best choice for company.

We define by  $\hat{p}_t$  the number of available operating machines at period  $t$  that is lower than scheduled operating machines  $p_t$ , which is defined by Eq. (17).

$$\hat{p}_t = \theta_t \cdot p_t \quad (17)$$

With  $\theta_t \in [0,1]$  is a random variable.

The quantity of products  $\delta_p$  over production horizon, which cannot be manufactured since the reduction of available machines, that is actually  $\hat{p}_t$ , is calculated as follows:

$$\delta_p = U \times \sum_{t=1}^T \sum_{k=1}^{p_t} T_{kt} - U \times \sum_{t=1}^T \sum_{k=1}^{\hat{p}_t} T_{kt} \quad (18)$$

The risk assessment aims to determine the optimal unit cost of outsourcing  $c_o$  that is profitable for company to transfer the quantity  $\delta_p$ . Eq. (19) sets the quantity transfer condition  $\delta_p$  to subcontractor, where  $c_u$  the unit cost of product sale,  $p_{s,t}$  the transferred lot-size to subcontractor,  $p_{l,t}$  is the lost lot-size and  $\rho$  is the penalty cost of dissatisfaction.

$$c_o \leq \frac{c_u \times (\sum_{t=1}^H p_{s,t} - \sum_{t=1}^H p_{l,t}) - \rho \cdot \delta_p}{\delta_p} \quad (19)$$

## 6. Numerical application

We consider a manufacturing system contains  $K = 5$  machines operated in parallel over a production horizon  $H$ .  $\Delta_t = 6$  months with  $\Delta_t = 1$  month. The production rate of each machine is  $U = 175$ , the service level defined by

company is  $\gamma = 0.95$ . According to random demand  $d_t$ , the sub-optimal solutions of lot-sizing problem are given by Tab. (1) and Tab. (2) whilst optimizing  $Q_1$  by means of Branch and Bound and random exploration methods.

Table 1. The sub-optimal lot-size plan under the random demand

$t$	$d_t$	$p_t$	$U \times \sum_{k=1}^{p_t} T_{kt}$	$s_t$
1	35638	5	36400	1061
2	35072	4	30450	3566
3	35535	4	27825	2870
4	36383	3	29050	2643
5	35628	4	28000	2291
6	36732	3	28700	1384

Table 2. Obtained lot-size per machine at each production period

$t$	$U \times T_{1t}$	$U \times T_{2t}$	$U \times T_{3t}$	$U \times T_{4t}$	$U \times T_{5t}$
1	4200	3675	4200	4200	4200
2	3150	4025	4200	4025	3500
3	3675	4025	3850	4200	3325
4	3850	4200	3850	4025	4025
5	4200	4200	3675	4025	3325
6	3675	3500	4200	3150	3850

The maintenance plan of five machines is shown by Tab. (3), which is represented by the number of preventive maintenance  $PM_k$  and the period  $\beta_k$  for each machine  $k$ . Obviously, the results of maintenance plan are obtained by using the established lot-size as input to the maintenance model.

Table 3. The maintenance plan for five machines

<i>machines</i>	$PM_k$	$\beta_k$ (months)
1	2	2,7
2	2	2,2
3	3	1,8
4	2	2,9
5	3	1,6

Thereafter, the common maintenance plan of the five machines is computed to avoid the interruption of production several times and increase the productivity. The average maintenance period by using the five maintenance plan is  $\bar{\beta}^* = 2,3$ , which corresponds to two preventive maintenance actions ( $PM = 2$ ) for the five machines.

The unforeseen situation considered with  $\theta_t = [0,4; 0,5; 0,7; 0,3; 0,9; 0,8]$  leads to malfunction of some machines as exhibited by Tab. (4). For example, for  $\theta_6 = 0,8$  only the machine one is available. In such situation, the total lot-size cannot be produced, by applying the Eq. (18), is  $\delta_p = 170800$ . The risk assessment approach explained above is called to help company to choose between the transfer of  $\delta_p = 170800$  to outsourcing or not satisfying the customer. Indeed the outsourcing decision, by using Eq. (19) with  $\rho = 0,12 mu$  and  $c_u = 2,5 mu$ , is only released whether the unit cost of subcontractor is less than or equal  $c_o = 2,35 mu$ .

Table 4. The unforeseen situation

$t$	$m_1$	$m_2$	$m_3$	$m_4$	$m_5$
1	0	1	0	1	0
2	0	1	0	1	0
3	1	0	0	1	0
4	0	1	1	1	0
5	0	0	0	0	0
6	1	0	0	0	0



## 7. Conclusion

In this paper, we studied an integrated maintenance and lot-sizing problems with risk assessment in the case of unforeseeable circumstance. The manufacturing system composed of identical parallel subject to energy consumption constraints and random failures. Besides, the production system may have an unforeseen impact that may affect the availability of certain machines at specific periods. With the aims of decreasing the machines failure rate, a common periodic preventive maintenance for all machines is proposed with minimal repair for corrective maintenance. A random demand over a finite planning horizon that must be satisfied by production system with a required service level defined by the stakeholders. With this in mind, analytical models have been developed to establish combined sub-optimal lot-sizing and preventive maintenance plans that optimize the total costs of lot-sizing, maintenance, consumed energy and inventory with regard to different constraints. Besides, to overcome the unforeseeable circumstance making some machines unavailable and unable to meet demands, a novel risk assessment is presented to assist the company to choose the appropriate subcontractor cost. The proposed approach has been illustrated clearly through a numerical example with five machines over six months of production horizon. In the future, we investigate to address the lot-size and maintenance plan optimization in the case of new returned products and their influences on different costs.

## References

- Barlow, R., and Hunter, L., Optimum preventive maintenance policies. *Operations research*, vol. 8, no. 1, pp. 90-100, 1960.
- Brandolese, M., Franci, M., and Pozzetti, A., Production and maintenance integrated planning. *International Journal of Production Research*, vol. 34, no. 7, pp. 2059-2075, 1996.
- Chelbi, A., and Rezg, N., Analysis of a production/inventory system with randomly failing production unit subjected to a minimum required availability level. *International Journal of Production Economics*, vol. 99, no. 1-2, pp. 131-143, 2006.
- Hajej, Z., Sofiene, D., and Nidhal, R., Optimal integrated maintenance/production policy for randomly failing systems with variable failure rate. *International Journal of Production Research*, vol. 49, no. 19, pp. 5695-5712, 2011.
- Hajej, Z., Rezg, N., and Gharbi, A., Improved preventive maintenance in the framework of forecasting problem under subcontractor constraint. *International Journal of Production Research*, vol. 55, no. 16, pp. 4557-4600, 2017.
- Kammoun, M. A., Rezg, N., and Achour, Z., New approach for air traffic management based on control theory. *International Journal of Production Research*, vol. 52, no. 6, pp. 1711-1727, 2014.
- Kammoun, M. A., and Rezg, N. An efficient hybrid approach for resolving the aircraft routing and rescheduling problem. *Journal of Air Transport Management*, vol. 71, pp. 73-87, 2018a.
- Kammoun, M. A., and Rezg, N., Toward the optimal selective maintenance for multi-component systems using observed failure: Applied to the FMS study case. *The International Journal of Advanced Manufacturing Technology*, vol. 96, no. 1-4, pp. 1093-1107, 2018b.
- Lee, J., Jin, C., and Bagheri, B., Cyber physical systems for predictive production systems. *Production Engineering*, vol. 11, no. 2, pp. 155-165, 2017.
- Mossa, G., Boenzi, F., Digiesi, S., Mummolo, G., and Romano, V. A., Productivity and ergonomic risk in human based production systems: A job-rotation scheduling model. *International Journal of Production Economics*, vol. 171, pp. 471-477, 2016.
- Nourelfath, M., Fitouhi, M. C., and Machani, M., An integrated model for production and preventive maintenance planning in multi-state systems. *IEEE Transactions on Reliability*, vol. 59, no. 3, pp. 496-506, 2010.
- Nourelfath, M., and Châtelet, E., Integrating production, inventory and maintenance planning for a parallel system with dependent components. *Reliability Engineering & System Safety*, vol. 101, pp. 59-66, 2012.
- Pierskalla, W. P., & Voelker, J. A., A survey of maintenance models: the control and surveillance of deteriorating systems. *Naval Research Logistics Quarterly*, vol. 23, no. 3, pp. 353-388, 1976.
- Radhoui, M., Rezg, N., and Chelbi, A., Joint quality control and preventive maintenance strategy for imperfect production processes. *Journal of Intelligent Manufacturing*, vol. 21, no. 2, pp. 205-212, 2010.
- Rezg, N., Dellagi, S., and Chelbi, A., Joint optimal inventory control and preventive maintenance policy. *International Journal of Production Research*, vol. 46, no. 19, pp. 5349-5365, 2008.