Optimal EWMA Chart for Monitoring Failure Rate

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Abstract

This research proposes an optimal Exponentially Weighted Moving Average (EWMA) control chart for monitoring the failure rate of buses in a transport company to improve the quality of service, avoid negative impacts and enhance the customers' satisfaction. The charting parameters of the EWMA chart, including the weighting parameter and the control limit, are optimized to achieve the best detection effectiveness. The proposed control chart is compared with the optimal NP chart in terms of the Average Number of Failures (*ANF*) since the shift occurs until the control chart can detect it. Failure data were obtained from the company for the implementation of the control charts. The results of the comparative study reveal that the EWMA chart substantially outperforms the NP chart, especially for detecting small and moderate shifts. It is very beneficial for the company to use an effective monitoring tool to guarantee continuous improvement and a high standard of efficiency.

Keywords
Control chart, Average number of failures, Average time to signal, EWMA chart.

1. Introduction
Exponentially Weighted Moving Average (EWMA) chart is one of the most powerful control charts to reduce the variation and improve the quality of manufacturing systems and service sectors. This chart is widely used to detect small and moderate shifts. To detect an upward p shifts, a statistic C_t is updated and plotted for the tth sample in an EWMA chart

$$\begin{aligned} &C_0 = 0 \\ &C_t = \lambda(d_t - d_0) + (1-\lambda)C_{t-1} \end{aligned} \tag{1}$$

where d_t is the number of nonconforming units found in the tth sample, d_0 is the in-control value of $d_t = n \times p_0$ (where n is the sample size and p_0 is the in-control fraction nonconforming), and λ ($0 < \lambda < 1$) is a weighting parameter. An out-of-control signal is produced when C_t exceeds the control limit H of the EWMA chart. On the other hand, the NP chart is one of the most popular techniques for monitoring the number of defectives. It has only one charting parameter, which is the Upper Control Limit (*UCL*). The NP chart will give a signal if d_t is larger than *UCL*.

Gibra (1978) presented two production models for maintaining the current control of an industrial process under the surveillance of an NP control chart. Reynolds and Arnold (2001) studied the statistical characteristics of the Variable Sample Size (VSS) and Variable Sample Interval (VSI) control charts. They found that the VSI feature usually improves the detection capability more than the VSS feature. Woodall et al. (2004) discussed some general issues that involve using control charts to monitor process and product quality profiles and reviewed the Statistical Process Control (SPC) literature. Yu and Hou (2006) developed an economic model and a numerical example for VSI control charts with multiple assignable causes. Jing-Rong (2007) presented a new approach where forecasting stock market volatility results are combined based on a weight that reflects the inverse of the EWMA of the Mean Absolute Percentage Error (MAPE) of each individual prediction model. Niaki (2008) studied the problem of a high false alarm rate and the increase in the probability of not detecting defects when the process is monitored by a set of independent unit-attribute control charts. Zheng et al. (2010) developed a new Cycle Forecasting EWMA (CF-EWMA) approach to deal with the problem of large deviations in the first few runs of each cycle for producing mixed-products in semiconductor manufacturing processes. The study showed that the proposed approach is effective for this purpose.

Wang (2016) proposed a residual-based EWMA control chart to monitor and detect faults in air handling units. Shrivastava et al. (2016) proposed an integrated model for joint optimization of preventive maintenance and Cumulative Sum (CUSUM) control chart parameters. Sanusi et al. (2017) emphasized that the Shewhart-type control chart plays an essential role in monitoring the existing variance in industrial sectors. It is crucial to explore the necessary changes that could lead to improved processes. Mansouri et al. (2018*a*) developed a novel technique for power systems monitoring using a new monitoring technique, called wavelet optimized EWMA chart. Mansouri et al. (2018*b*) further presented a new monitoring scheme that merges the advantages of Midpoint-radii Principal Component Analysis (MRPCA) method with the EWMA chart to enhance fault detection of the process of air quality monitoring. Robert (2018) applied the VSI EWMA chart at Monsanto's nylon fiber plant. The objective was a significant decrease in laboratory cost with little adverse impact on control chart performance compared with the fixed-interval EWMA chart. This objective was achieved by employing a fixed time with the VSI scheme.

The speed of signaling an out-of-control condition is usually measured by the Average Time to Signal (*ATS*), that is, the average time required to signal an out-of-control condition after its occurrence. Charts are more effective when the out-of-control *ATS* is smaller as the problem can be signaled earlier. The smaller the *ATS* is, the easier it is to determine when the p shift occurs (Pignatiello and Samuel 2001). Thus, the minimization of *ATS* improves the capability of the control chart to diagnose the cause.

In this article, an optimal EWMA chart is proposed to monitor the failure rate p of buses in a transport company. This research assumes the random number d (the number of failed buses) follows a binomial distribution with a known in-control failure rate p_0. The p shift is assumed to follow a uniform distribution. The performance of the optimal EWMA chart and NP chart is compared in terms of the Average Number of Failures (*ANF*), which is an overall performance of the control charts. Control charts for attributes are usually used to detect an increase in fraction nonconforming (Lucas 1985, Reynolds and Stoumbos 1999). Therefore, the focus of this research is to detect increasing shifts in failure rate.

The paper is organized as follows. Section 2 explains the design of the optimal EWMA chart. In section 3, a case study is presented and the optimal EWMA chart is compared with the optimal NP chart for monitoring the failure rate. Finally, the conclusion is drawn in the last section.

2. Design of the optimal EWMA chart

Four specifications are required to design the optimal EWMA chart:

(1) The permitted minimum value τ of the in-control average time to signal (ATS_0). This value is set based on the tolerable false alarm.
(2) The in-control failure rate p_0. It is estimated from the data noticed during the pilot runs in phase I.
(3) The maximum shift $\delta_{\max}$ in failure rate p. It is the maximum increasing shift in the failure rate, which the user is interested in detecting.
(4) The sample size n. It is mainly determined based on the available resources and managerial considerations.

When an increasing shift in the failure rate occurs, failure rate p will change to:

$$p = \delta \times p_0 \tag{2}$$

where δ ($1 \le \delta \le \delta_{max}$) is the increasing p shift in terms of p_0.

In this article, the *ANF* is used to measure the overall performance of a control chart.

$$ANF = N\int_{1}^{\delta_{max}} ATS(\delta)\times f_{\delta}(\delta)\ \ d\delta \tag{3}$$

where N is the number of buses in operation, $ATS(\delta)$ is the out-of-control ATS produced by a control chart at δ. N in Equation (3) is constant and can be removed so that the ANF can be further simplified as follows:

$$ANF = \int_{1}^{\delta_{max}} ATS(\delta)\times f_{\delta}(\delta)\ \ d\delta \tag{4}$$

The integration in Equation (4) can be evaluated by Legendre–Gauss Quadrature method. The random shift δ in p is assumed to follow a uniform distribution (Castagliola et al. 2011, Haridy et al. 2013) with a probability density function $f_{\delta}(\delta)$ of:

$$f_{\delta}(\delta) = \frac{1}{\delta_{max}-1} \tag{5}$$

In this article, ANF is used as the objective function to be minimized. The design of the optimal EWMA chart is carried out using the following model:

$$\begin{array}{ll} \text{Objective:} & \text{Minimize } ANF \\ \text{Constraint:} & ATS_0 \geq \tau \\ \text{Design variables:} & \lambda, H. \end{array} \tag{6}$$

The optimization design aims at determining the optimal values of λ and H that minimize ANF over a shift range of ($1 < \delta \leq \delta_{max}$) and meanwhile ensure that $ATS_0 \geq \tau$. Minimizing ANF in turn, will lead to a smaller out-of-control ATS over the entire range of p shifts. The optimization design can be described as follows:

(1) Decide the design specifications τ, p_0, δ_{max} and n.
(2) Initialize ANF_{min} as a very large number (ANF_{min} is used as the minimum value of ANF).
(3) Search the optimal value of λ within the range $0 < \lambda < 1$. For a given λ,
 (3.1) Find the H that satisfies $ATS_0 \geq \tau$.
 (3.2) When the values of the two charting parameters, λ and H, are preliminarily determined, the ANF is determined by Equation (4).
 (3.3) If the calculated ANF is smaller than the current ANF_{min}, set $ANF_{min} = ANF$ and the current values of λ and H are stored as a temporary optimal solution.
(4) The optimal EWMA chart that produces the smallest ANF and satisfies $ATS_0 \geq \tau$ is identified. The corresponding optimal λ and H are also identified.

3. Case Study

In this section, we analyze the data that we collected from the company that represent the failure rate of buses over two years. The main objective is to detect the increasing shift in the failure rate of buses as soon as it occurs using different types of control charts and to take the required corrective actions in order to minimize it and improve the quality of service in that company. Figure 1 shows the steps of constructing and evaluating the performance of the control charts.

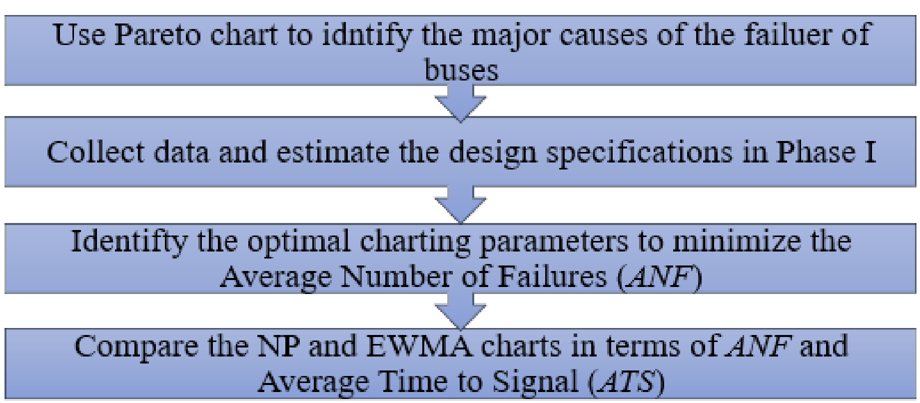

Figure 1. Steps of constructing and evaluating the performance of the control charts

Based on the information received from the transport company, the Pareto chart was constructed, as shown in Figure 2, to identify the main causes of the failure of the buses. Figure 2 indicates that the failures of tyres & rim, air pressure and engine cooling are the major contributors to the failure of the buses.

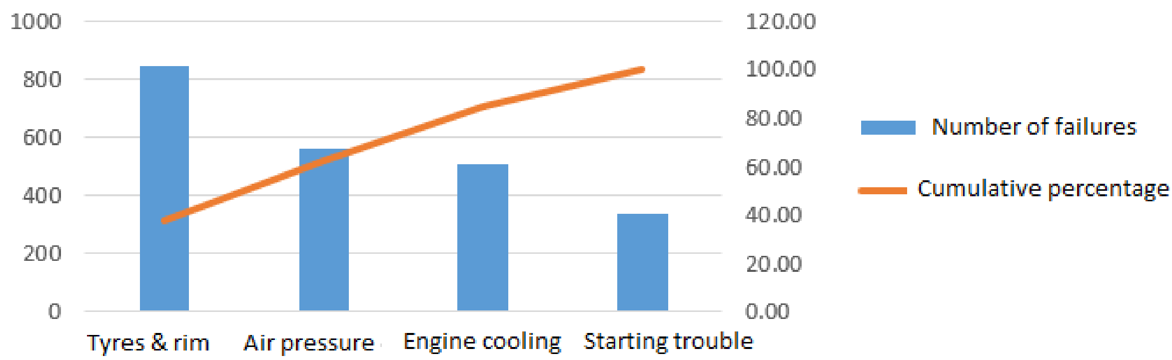

Figure 2. Pareto chart for the causes of failure of buses

To estimate the in-control failure rate, the number of failed buses out of 150 operating buses (n = 150) over 25 days was determined and the in-control failure rate of buses (p_0) was found to be 0.8%. This pilot study is considered as Phase I for the design of the control chart. After the discussion with the quality engineer, the value of τ was decided to be 500 based on the false alarm rate the company can handle. The maximum shift in the failure rate the company is interested in detecting is 10 (δ_{max} = 10).

Using the aforementioned design specifications (τ = 500, n = 150, p_0 = 0.008 and δ_{max} = 10), a C program was coded based on the optimization model explained in section 2 to identify the optimal charting parameters for both NP and EWMA charts.

The control limit (*UCL*) of the NP chart is optimized to minimize the objective function *ANF* and in the meantime, to satisfy the constraint τ = 500. Similarly, the control limit (*H*) and the weighting parameter (λ) of the EWMA control chart are optimized to minimize the objective function *ANF* and to satisfy the constraint τ = 500.

The charting parameters and *ANF* for the NP chart are as follows:

UCL = 5, *ANF* = 0.10.

The charting parameters and *ANF* for the EWMA chart are as follows:

H = 1.512, λ = 0.275, *ANF* = 0.06.

The NP and EWMA charts are compared based on *ANF* which is a measure of the overall performance. We calculated the ratio of *ANF*(NP)/*ANF*(EWMA) = 0.10/0.06 = 1.67. This means that the overall detection speed of the EWMA chart is better than the NP chart by 67% and consequently, it will perform better than the NP chart for detecting a wide range of shifts.

To have a good idea about the performance of the control charts against different shifts within the range ($1 < \delta \leq 10$), the NP and EWMA charts are also compared in terms of the out-of-control *ATS* and the *ATS* values are shown in Table 1.

Table 1. *ATS* values of NP and EWMA charts

δ	*ATS* (NP)	*ATS* (EWMA)	*ATS* (NP)/*ATS* (EWMA)
1	710.89	500.15	1.42
2	28.55	9.70	2.94
3	6.01	3.07	1.95
4	2.37	1.73	1.36
5	1.29	1.18	1.09
6	0.86	0.89	0.97
7	0.67	0.72	0.93
8	0.58	0.62	0.93
9	0.53	0.56	0.95
10	0.51	0.50	1.03

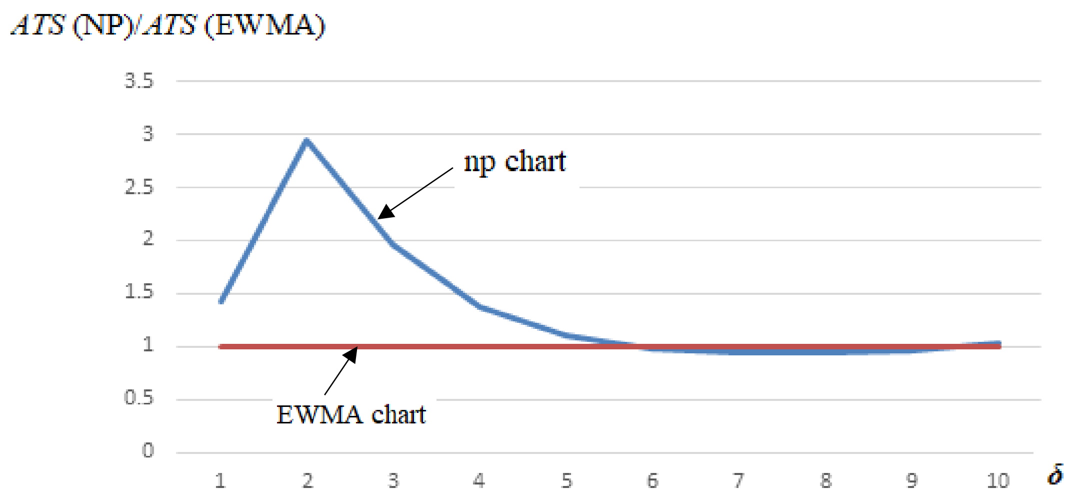

Figure 3. Values of *ATS* (NP)/*ATS* (EWMA)

As shown in Table 1, when there is no shift ($\delta = 1$), both control charts have a value of *ATS* larger than or close to the false alarm rate, which is 500. This means that the process is in control and the comparison is fair. This also ensures that the requirement on the false alarm rate is satisfied. It is noted that the ATS_0 value of the EWMA chart (=500.15) is fairly closer to the predetermined τ (=500) because this chart has more than one charting parameter that can fit the constraint ($ATS_0 \geq \tau$) and as a result, its potential effectiveness can be better utilized. On the other hand, the NP chart with only one integral parameter (*UCL*) generates an in- control ATS_0 (=710.89) much larger than τ (=500).
Figure 3 shows that the EWMA chart outperforms the NP chart for detecting small and moderate shifts ($\delta \leq 5$) to a significant degree, but the NP chart detects the large shifts ($\delta \geq 6$) faster than EWMA chart with a very slight superiority.

4. Conclusion

This research proposes an optimization design of the EWMA chart. The design algorithm optimizes the charting parameters of the EWMA chart so that the best overall performance can be achieved and, in the meantime, the constraint on the false alarm rate is satisfied. The main objective is to detect the increasing shifts in the failure rate as soon as possible. This will definitely help the decision-makers take a quick action to analyze the failure and recommend the required solutions.
A performance assessment has been carried out between the optimal NP and EWMA charts. The results of the comparative study reveal that the optimal EWMA chart outperforms the optimal NP chart substantially for monitoring the failure rate of buses. Specifically, The EWMA chart is more effective than the NP chart by 67% in terms of *ANF*. This study highlights the importance of optimization design in control charts. The optimization design has convincingly achieved higher overall effectiveness for the EWMA chart.

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