(5) Otherwise (i.e., if $E_t > H$), the process is considered out of control and should be terminated as quickly as possible for identifying the assignable causes and making a corrective action.

3. Design of the *w*EWMA Scheme

If a shift occurs in the process, the fraction nonconforming p will shift to:

$$p = \delta \times p_0 \tag{5}$$

where δ ($1 \leq \delta \leq \delta_{max}$) represents the shift with an in-control status when $\delta = 1$ (i.e., $p = p_0$) and an out-of-control status when ($1 < \delta \leq \delta_{max}$). The maximum possible shift is δ_{max}.

A sound measure of the overall performance of attribute schemes is the Average Number of Defectives (*AND*) (Haridy et al. 2013).

$$AND = \int_{1}^{\delta_{max}} \delta \times p_0 \times ATS(\delta) \times f_\delta(\delta) \quad d\delta \tag{6}$$

where $ATS(\delta)$ represents the out-of-control *ATS* value at a shift δ, and $f_\delta(\delta)$ is the probability density function of δ. The index *AND* is actually a weighted average of *ATS* that uses δ as the weight. The scheme producing a smaller *AND* is considered to be more effective over different shifts δ. The random shift δ in p is presumed to follow a uniform distribution (Sparks 2000) with a probability density function $f_\delta(\delta)$ of:

$$f_\delta(\delta) = \frac{1}{\delta_{max} - 1} \tag{7}$$

Prior to the design of the *w*EWMA scheme, four parameters τ, n, p_0 and δ_{max} should be determined. τ is the minimum value of ATS_0. In order to lower the false alarm rate when managing the Type I error is costly, a larger τ is used. However, a large τ may cause a reduction in the effectiveness of the control scheme (Wu et al. 2006). The parameter n is determined with regards to the availability of resources and the required sensitivity of the scheme in detecting a shift, while the value of p_0 is specified based on the data obtained in Phase I. Additionally, the quality engineer can choose δ_{max} according to the maximum possible p shift or the interested shift range.

The *w*EWMA scheme can be designed using the following model:

Objective: Minimize *AND*
Constraint: $ATS_0 \approx \tau$ (8)
Design variables: w, λ, H

where w and λ are the independent charting parameters, while H depends on w, λ and the specified τ. The main goal of the model is to determine the best values of w and λ that produce the smallest *AND* across a shift range of ($1 < \delta \leq \delta_{max}$) and in the meantime, H is tuned so that $ATS_0 \approx \tau$.

4. Comparative Studies

4.1. Comparison under One Case

This section studies and compares the performance of the EWMA and *w*EWMA schemes. First, the two schemes are studied under one case. The design specifications are the following:

$\tau = 740$, $p_0 = 0.008$, $\delta_{max} = 6$ and $n = 125$

The two schemes are designed and the results are as follows:

EWMA control scheme: $w = 1$, $\lambda = 0.485$, $H = 2.155$, $AND = 0.113$.
*w*EWMA control scheme: $w = 0.90$, $\lambda = 0.245$, $H = 1.167$, $AND = 0.097$.

The values ATS_0 (where $\delta = 1$) and *ATS* (where $1 < \delta \leq 6$) of both schemes are calculated within the range of the p shift, and the results are shown in Table 1. Figure 1 illustrates the normalized *ATS* (i.e., $ATS_{\text{EWMA}}/ATS_{w\text{EWMA}}$) of both schemes. The normalized *ATS* shows the percentage by which the *w*EWMA scheme outperforms the traditional EWMA in terms of *ATS* at a particular δ. For example, if $ATS/ATS_{w\text{EWMA}} = 1.66$ at $\delta = 1.5$, then the *ATS* value of the *w*EWMA scheme is less than that of the traditional EWMA scheme by 66% at $\delta = 1.5$. The following can be observed from Table 1 and Figure 1:

(1) The two schemes generate an ATS_0 value very near to τ (constraint (8)) when there is no shift in p. This shows that the two schemes have almost the same false alarm rate and ensures a fair comparison.
(2) The *w*EWMA scheme is superior to the traditional EWMA scheme when $\delta \leq 3.5$, while the traditional EWMA scheme outperforms the *w*EWMA scheme when $\delta > 3.5$.
(3) The superiority of the *w*EWMA scheme over the traditional EWMA scheme when $\delta \leq 3.5$ is more observable than that of the latter over the former when $\delta > 3.5$. This reflects that the *w*EWMA is more likely to exhibit better overall detection speed than the EWMA scheme over the entire range of p shifts (where $1 < \delta \leq 6$).

Table 1. *ATS* values of the EWMA and *w*EWMA schemes

δ	*ATS*	
	EWMA scheme (w = 1)	wEWMA scheme (w = 0.9)
1	730.7376	727.6995
1.5	67.6361	40.8391
2	17.6465	11.6891
2.5	7.6551	6.0543
3	4.4467	3.9919
3.5	2.9686	2.9585
4	2.2489	2.3410
4.5	1.7548	1.9567
5	1.4242	1.6589
5.5	1.2186	1.4531
6	1.0536	1.2920

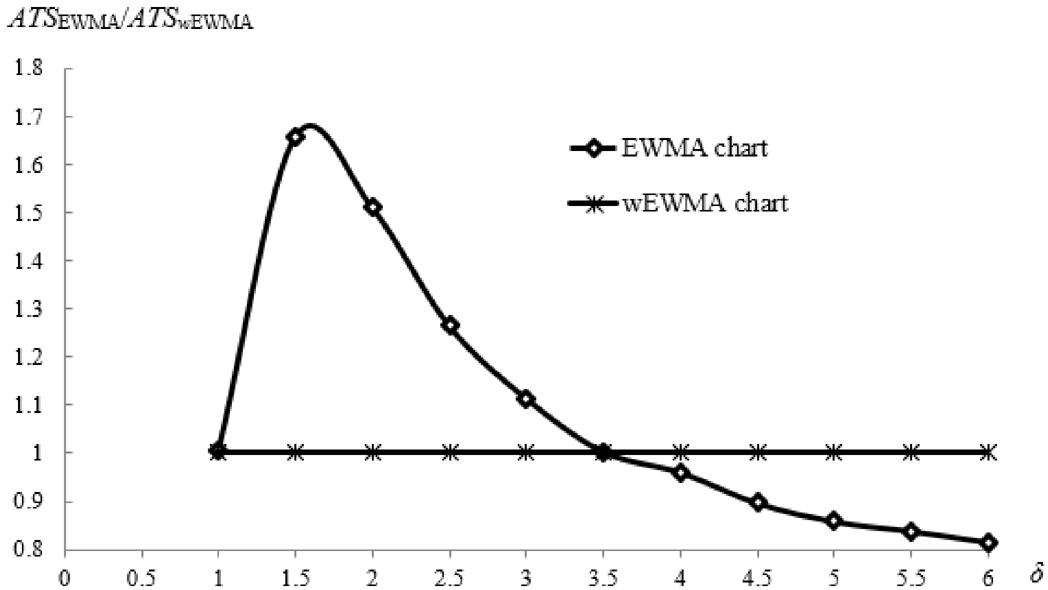

Figure 1. Normalized *ATS* of the EWMA and *w*EWMA schemes

The values of *AND* (Equation (6)) of both schemes are also evaluated, and the ratio of AND_{EWMA}/AND_{wEWMA} is found to be 0.113/0.097 = 1.16. This value indicates that, for this case (where τ = 740, p_0 = 0.008, δ_{max} = 6 and n = 125), the mean number of defects resulted when using the *w*EWMA scheme is 16% less than that produced by the traditional EWMA scheme against the entire range of *p* shifts.

4.2. Comparison under Different Cases

The EWMA and *w*EWMA schemes are further studied in additional different conditions for which the design specifications (τ, p_0, δ_{max} and n) are utilized as the input factors, and all of them has two levels as follows:

τ:	500,	1500.
p_0:	0.005,	0.01.
δ_{max}:	3,	8.
n:	50,	200.

The levels are determined with reference to those that many authors commonly use (Bourke 2008; Haridy et al. 2019). The different combinations of τ, p_0, δ_{max} and n resulted in six cases as shown in Table 2. For all cases, the *w*EWMA and EWMA schemes are designed, and both produce an ATS_0 close to τ. Reflecting *AND,* the overall performance and the charting parameters (w, λ and H) for the six cases are summarized in Table 2. The ratio of AND_{EWMA}/AND_{wEWMA} is always larger than 1. This demonstrates that the *w*EWMA scheme always performs better than or as equal as the traditional EWMA scheme. Of note is that the *w*EWMA scheme adopts a relatively large w when p_0 and δ_{max} are large. This finding is consistent with the fact concluded by previous studies (Reynolds and Stoumbos 2004; Wu et al. 2008) that a large w is preferred to detect large shifts.

Finally, the grand mean $\overline{AND_{\text{EWMA}}/AND_{w\text{EWMA}}}$ representing the mean of the AND_{EWMA}/AND_{wEWMA} values over all the six cases in Table 2 is calculated. The result of $\overline{AND_{\text{EWMA}}/AND_{w\text{EWMA}}}$ is 1.35. This shows that, from an overall viewpoint considering various values of τ, n, p_0 and δ_{max}, the *w*EWMA scheme is outperforming the EWMA scheme by 35%.

In this context, it is worth mentioning that the traditional EWMA scheme is unlikely to possess a better overall detection speed than the *w*EWMA scheme as the EWMA scheme is merely a special case of the *w*EWMA scheme. If w of a *w*EWMA scheme is equal to 1 and its λ and H are equal to those of an EWMA scheme, then this *w*EWMA scheme will work exactly as the EWMA scheme. Consequently, a *w*EWMA scheme can be always designed so that it will act either better than or at least equal to the EWMA scheme.

Table 2: Comparison of the EWMA and *w*EWMA schemes under different conditions

Case	τ	p_0	n	δ_{max}	Scheme	w	λ	H	AND	AND_{EWMA} / $AND_{w\text{EWMA}}$
1	500	0.005	50	3	EWMA	1	0.365	0.956	0.3230	1.470
					*w*EWMA	1.20	0.065	0.306	0.2193	1.000
2	500	0.005	200	8	EWMA	1	0.485	2.043	0.0528	1.121
					*w*EWMA	1.35	0.170	1.346	0.0471	1.000
3	500	0.01	50	3	EWMA	1	0.485	1.544	0.4113	1.578
					*w*EWMA	0.90	0.125	0.516	0.2606	1.000
4	1500	0.005	200	8	EWMA	1	0.245	1.423	0.0612	1.034
					*w*EWMA	1.35	0.155	1.516	0.0592	1.000
5	1500	0.01	50	3	EWMA	1	0.395	1.527	0.6246	1.813
					*w*EWMA	1.05	0.080	0.501	0.3445	1.000
6	1500	0.01	200	8	EWMA	1	0.410	2.738	0.0680	1.056
					*w*EWMA	1.50	0.170	2.959	0.0644	1.000

5. Conclusion

This article proposes a *w*EWMA control scheme for monitoring increasing shifts in proportion p of nonconforming items in a binomial model. The traditional binomial EWMA chart is just a special case of the proposed scheme. Our monitoring statistic uses $(d_t - d_0)^w$ instead of difference $(d_t - d_0)$. By applying this salient feature, the *w*EWMA is able to considerably outperform its traditional counterpart for detecting p shifts much quickly in terms of AND in different scenarios. The binomial EWMA scheme is, in general, not as efficient as its generalized version, namely, the *w*EWMA scheme. As a result, the *w*EWMA scheme should replace the EWMA scheme for all SPM applications.

This article assumes that the number of nonconforming items d_t follows a binomial distribution with a known p_0, and the shift δ follows a uniform distribution. Thus, studying the performance of the *w*EWMA scheme when d_t and δ follow other distribution rather than the binomial and uniform, respectively, is worthwhile. Another prospective future work would be to investigate how the detection speed of the *w*EWMA scheme is affected when p_0 is estimated from a limited number of samples.

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