Compartive Study of Queuing Systems of Medical Out Patient Departments of Two Public Hospitals

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Abstract

Delayed healthcare service delivery is major highlighted problem in healthcare centers and is caused due to poor design of queuing system or its mismanagement. This study was conducted to suggest the optimum number of receptionists and doctors at the study areas in order to optimize the performance of existing queuing systems at the out-patient departments (OPDs). The most congested OPD i.e. medical OPD was selected for the study at the case hospital 1 and then same OPD was selected in another public sector hospital (case hospital 2). Both hospitals were the tertiary care hospitals of the different districts of Sindh Pakistan. Data was collected for two weeks: data collection parameters were; arrival rate, service rate of patients, number of servers, salaries of the servers and associated waiting cost of patients. Arrival and service distribution of the patients were verified as per assumptions of the multi-server queuing model (M/M/c) by using input analyzer of Rockwell Arena 14.5. Performance measures of the queuing system were calculated by using TORA optimization software. For cost calculation and graph plots MS excel was used. According to the results, one receptionist and doctor was suggested to be increased at both of the OPDs for the minimization of congestion of patients and their waiting times. In this way, they will have to wait for less time as the compared to the existing scenario; their waiting cost would certainly be decreased which will be the good point on the side of healthcare providers.

Keywords

Queue, waiting cost, serving cost, servers.

INTRODUCTION AND LITERATURE

Pakistani public healthcare system is largely dispersed and its access is given to the citizens along with the well trained doctors and staff. Inconsistencies in healthcare system are due to the unavailability of the doctors and medicine and the poor managerial constraints are beyond these crises (Callen et al., 2013). Research was conducted to reveal the problem which are faced by the patients when they need to visit public hospitals. Frustrated response of staff was reported by 41.8% of the patients, 72.7% patients had the opinion that poor patients were not treated well the hospital, on the same time, doctors` preference to their relatives and known patients was also reported by 96.4% (Ahmad et al., 2013). I was suggested in one of the research that the healthcare should be focused in the context of infrastructure, medical equipment and check and balance so that problems can be overcome (Naz et al., 2012). Moreover, there are number of problems which are usually faced by the patients but delay is one of the highlighted issue; delay is one of the major threat which is observed to be increasing day by day (Obulor & B.O, 2016). Delay is not the new problem in hospitals, awaiting patients can be found in laboratories of the hospitals, emergencies, surgical departments etc. (Green, 2006). Overcrowding of patients at the hospitals is the highlighted issue. Recently, it has been the teething concern for healthcare providers (Ikwunne & Onyesolu, 2016). Practically, queuing theory is an application for simplification of overcrowding problems (Olorunsola et al., 2014): This theory was formulated by Danish telephone engineer Agner Krarup Erlang in 1913 (Bastani, 2009)(Kissani & Rifai, 2015). In 20th century, He was only scientist who treated the queuing problem in the context of telephone exchange (Mwangi & Ombuni, 2015). Since, queuing theory is developed for simplifying the problems of waiting lines therefore, it is called as theory of waiting lines/overcrowding (Adaora .D., 2013). The applications of theory are being applied in service

sector industries in order to resolve the problems of waiting lines (Olorunsola et al., 2014). This application can also be applied in the healthcare setting for simplification of overcrowding issue (Priyan, 2017) (Ghimire et al., 2017). Patient arrives the facility and cause the formation of queue and served as per the followed queue discipline then he/she leaves the system after getting served (Fitzsimmons et al., 2008). Waiting lines of the patients are related with the waiting cost of awaiting patients (Kembe et al., 2012) therefore, service time and total system time (time spent by the patients in the system) of entities/patients is computed (Varma, 2016). In quantification of waiting lines` phenomenon, queuing theory is used and it is used by the calculation of performance measures of corresponding queuing system (Adaora .D., 2013). Around 40 queuing models are there which are used for the systemic study of waiting lines (Bastani, 2009; Kandemir-Cavas & Cavas, 2007).

Dynamic leadership and governance is needed desperately for designing and enforcing evidence-based policies, programs and the way to take care of the system (Kumar & Bano, 2017). This study was conducted in order to find out the optimum service level of medical OPDs of two public sector hospitals of Sindh, Pakistan by using the multi-server queuing model. On the same time, the performance of both OPDs was also compared.

RESEARCH OBJECTIVE

Objective of this study was the comparison of the performance of medical OPDs of two public sector hospitals of Sindh, Pakistan.

RESEARCH METHODOLOGY

3.1 Selection of Study Area

This study is based on the OPDs of two public sector hospitals. The hospital was consisted of 22 out-patient departments OPDs. Data i.e. number of arriving patients at the various OPDs was collected from the department of statistics of ABC hospital. The arrival records of patients of three months i.e. January, February and March were collected from the department of statistics of the concerned hospital. Frequency distribution of the collected data revealed the most congested OPD i.e. medical OPD (see Fig. 1); which was selected for the study to be conducted. On the basis of maximum arrival rate of patients, medical OPD and this was the highest arrival rate of patients per day among all the OPDs of the hospital.

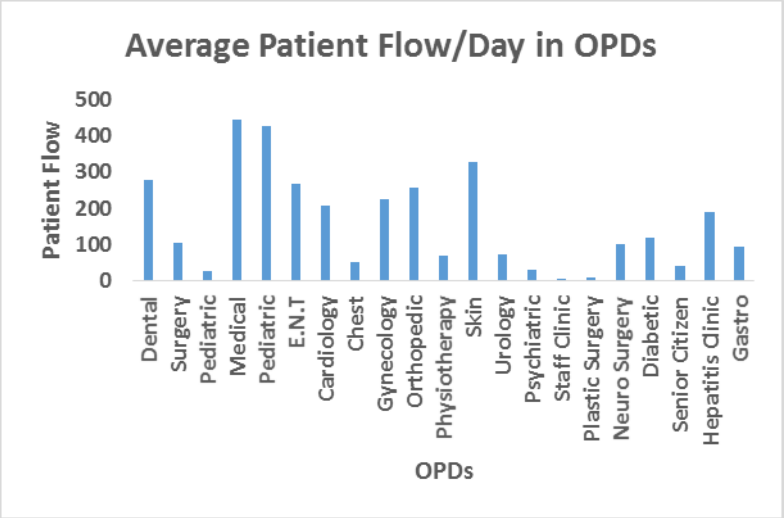


Fig.1: Average per day arrival or patients according to several present OPDs at Civil hospital

It was the medical OPD case hospital 1 which required the research attention and optimization because when the OPD was visited the research team confronted the problem of long queues and long waiting times. Since the waiting time of patients was associated with the opportunity cost thus it was required to study and optimize the queuing system of medical OPD.

3.2 Multi-server queuing model

When there are more than one service channels in the queuing system such system is called as the multi-server queuing system. It is necessary to be known, before applying the multi-server model application on any queuing system that arrival and service pattern of entities must follow the poison and exponential distribution respectively. Patients were served on the basis of First come first served queuing discipline (FCFS).

3.2.1 Performance measures of the multi-server queuing model

μ = Service rate of patients
λ = arrival rate of patients
S = Number of service channels.
P_o = probability of when there are zero customers/patients in the system.
Ls= Number of the patients in the system
Wq = Expected time of the patient that he/she spends while remaining in queue.
Lq = Number of patients in the queue
Wq = Expected time of the patient that he/she spends while remaining in queue.
Ws = Expected time of the patient that he/she spends while remaining in the system.
E(Cw) = Expected waiting cost
E (S_c) = Expected Service Cost

3.2.2 Assumptions of multi-server queuing model

1. Arrival distribution of patients should follow the poison distribution
2. Service distribution of patients should be exponential.
3. It was assumed that the servers were operating at their full capacity
4. Patients were being served on the basis of first come, first served (FCFS).
5. In the present case, servers are meant to be either doctor or receptionist.
6. It was assumed that service rate was not dependent queue length; because the servers were not going faster even there were long queues in the system
7. The average arrival rate (**λ**) of entities was supposed to be greater than service rate of patients
8. It was also supposed that there were infinite entities in the queue

3.2.3 Assumptions of this Research

1. Only the salaries of doctors were considered as the service cost
2. The waiting cost of patients was only computed for their total waiting time at OPD i.e. reception and doctor
3. The patients coming to the OPD have somehow relevant medical problems.

3.2.4 Cost Calculations

Three cost were incorporated in the study i.e. service cost, waiting cost and total system cost.

1. **Expected Service cost:** Salaries of doctors they are paid by the hospital for rendering their services.
2. **Expected Opportunity cost:** The cost of patients while being at the hospital because of doing nothing to earn.
3. **Expected total System cost:** This cost is the addition of expected service cost and expected waiting cost.

Economic analysis of the cost will assist the hospital administration to line up the balance between the waiting cost, the service cost that is increase of service cost by providing better service and decreasing waiting cost of the patients that incurs for waiting in the hospital.

$$E(S_c) = SC_s \qquad (1)$$

Cost paid by the customers / patients due to waiting in the system

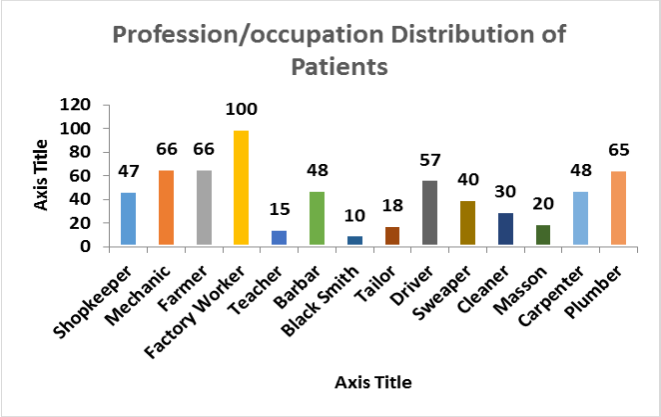
$$E(W_c) = (\lambda W_s)*C_w \qquad (2)$$

3.3 Data collection

For data collection, medical OPDs of two public sector hospitals were selected: Those hospitals were named as case hospital 1 and 2 respectively. Data collection variables were arrival rate and service rate of patients, salaries of receptionist, doctors and patients. On the same time, professions of the patients were also inquired along with the income.

3.3.1 Profession Distribution and Associated Waiting Cost of the Patients

Before collecting the data regarding monthly income of patients, their professions were collected. Patients who arrived at the OPD belonged to 14 different professions (see Fig. 1). It can be seen in the fig. 1 that maximum number of patients were factory workers i.e. 100. Since, the workers belonged to different profession; hence the waiting cost for each profession was different (see Fig. 2). When the patients were in the system, they were idle in terms of earning money; therefore, they were supposed to be charged with what they could have earned, if they would have worked out of the hospital.



(*Profession distribution of the patients arriving at the medical OPD of respective hospital*)

The mean waiting cost of patients was computed to be Rs.227.98 (see fig. 3). The highest waiting cost among all the costs i.e. Rs.1100 was of meson because of remaining idle for the whole day; that's why his whole day cost was considered in the study.

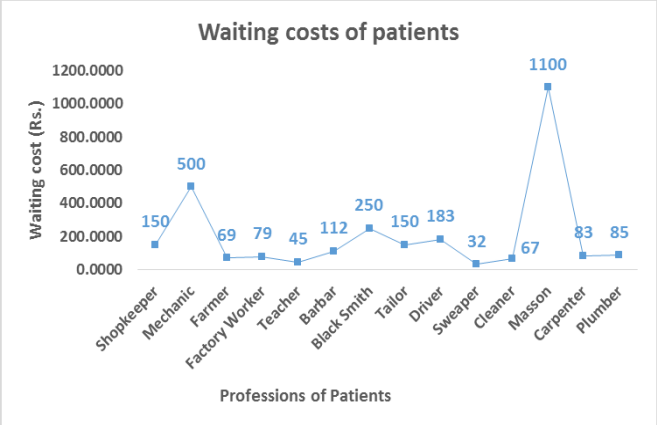


Fig. 1. *Waiting cost of patients arriving at the OPD*

People did not only visit the hospital when they get ill themselves, perhaps they also came with relatives, friends and family. In any case, they were supposed to be charged with the opportunity cost.

3.3.2 Service Cost/Salary of Resources

In this study the service cost was considered as the cost which could be affected after the suggestion of optimal scenario; and it was only the salaries of increased doctors. Thus, the salaries of available doctors at medical OPD of the case hospital 1 were considered as expected service cost. There were five doctors at the OPD and their salaries are presented in table I.

TABLE I. DOCTORS` SALARIES AT THE CASE HOSPITAL 1

S.No	Salary of doctors Per month (Rs.)	Salary per day (Rs.)	Salary per Hour (Rs.)
1	210650.00	7021.6667	1170.27778
2	225190.00	7506.3333	1251.05556
3	170415.00	5680.5	946.75
4	190130.00	6337.67	1056.27778
5	145340.00	4844.67	807.444444
Average	188345.00	6278.17	1046.36111

Average of the monthly salaries of doctors came out to be Rs.188345.

3.3.3 OPD selection at the case hospital 2

Medical OPD was the most congested OPD at the case hospital 1; therefore, the OPD of same type was selected at the case hospital 2.

3.3.4 Patients` profession distribution

Profession distribution of patients at the case hospital 2 is presented in the fig 3

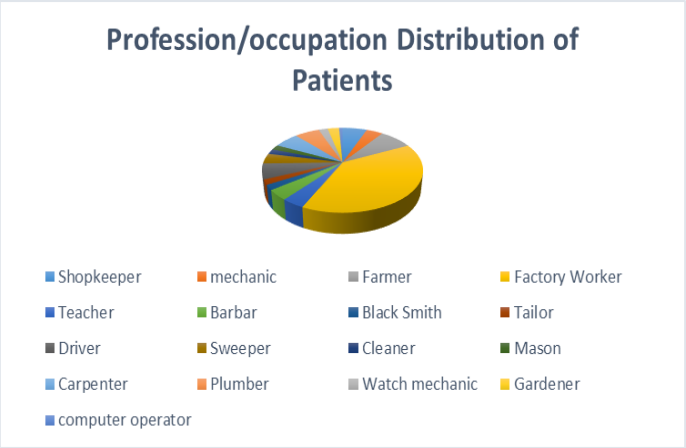


Fig. 3 Profession/occupation distribution of patients at case hospital 2

At the same time of collection of data regarding the profession of patients their monthly salaries were also collected; hence the income of patients is presented in the fig 4.

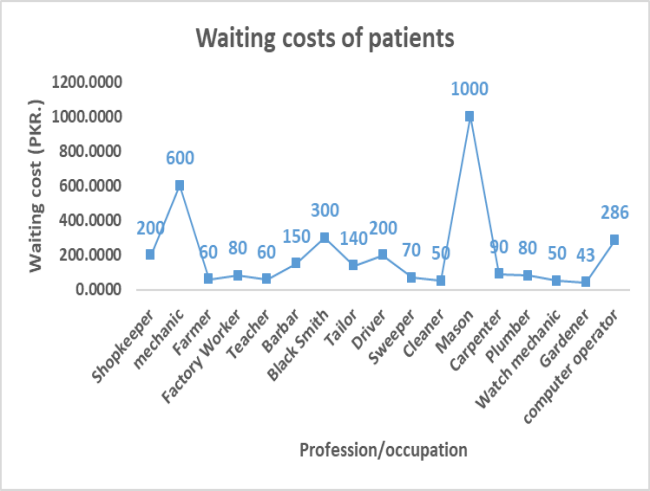


Fig. 4. Waiting cost of patients at the case hospital 2

3.3.5 Salaries of doctors at the case hospital 2

There were three doctors at the medical OPD of case hospital 2 and their salaries are presented in the table 2 given as under.

Table II. Doctors` salaries at the case hospital 2

S.No	Salary/Month (PKR.)	Salary/day (PKR.)	Salary/hour (PKR.)
1	140590.	4686	781
2	110370	3679	613
3	80925	2697	449
Average	128333	128333	614

Average monthly salary of doctors came out to be Rs.128333.

3.4 Data Analysis

The data which was collected from the OPD of the hospital was put into the input analyzer; so that, the arrival and service distributions could be revealed. It is the assumption of the multi-server queuing model that arrival of the entities in the system should follow the poison distribution and the service should be exponentially distributed. These distribution are discussed in the below given headings.

3.4.1 Arrival Distribution of Patients at Medical OPD of Case Hospital 1

After putting the arrival data into input analyzer, it was revealed that the arrival of patients followed the poison distribution as shown in the fig 5 and 6.

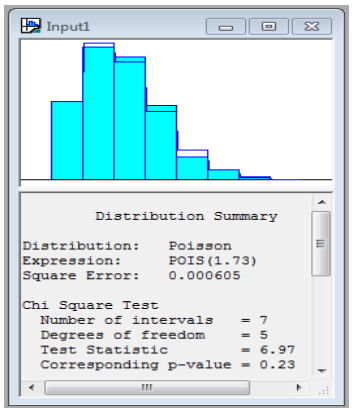


Fig. 5. Distribution fit for the arrival of patient

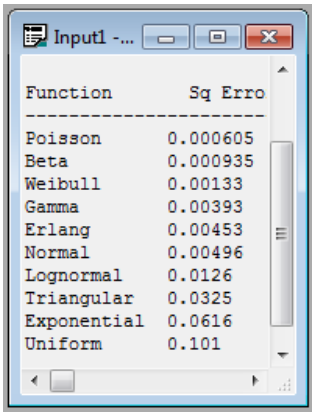


Fig. 6. Summaries of square error of various Applied distribution functions

Arrival of patients at the reception counter of the OPD was POIS (1.73) per minute.

3.4.2 Service Distribution of Patients at the Medical OPD of Case Hospital 1

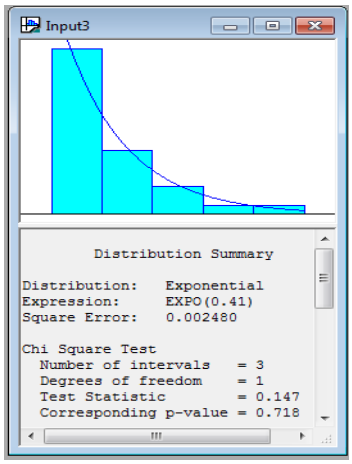


Fig. 7. Distribution fit for the service of patients by the doctors

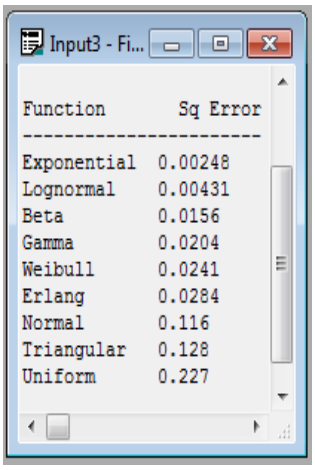


Fig. 8. Summaries of square error of various Applied distribution functions

At the reception counter, service of patients was exponentially distributed. It was revealed by the fig 7 and 9 that each patient was served in 0.41 minutes (EXPO (0.41)) at reception counter.

3.4.3 Service distribution of patients by doctors at case hospital 1

Patients` service distribution by the doctors at the case hospital 1 was taken out and it was revealed that each of the patient was served in the 0.99+EXPO(3.01) minutes on an average by each of the doctor. Furthermore, the distribution of the patients is given in the fig 7 and 8 respectively

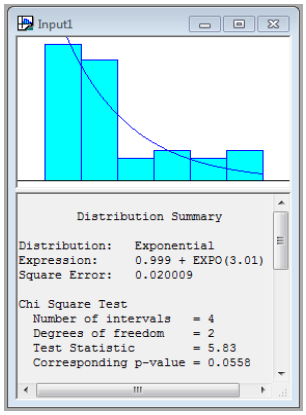


Fig. 7. Distribution fit for the service of patients by the doctor at the OPD

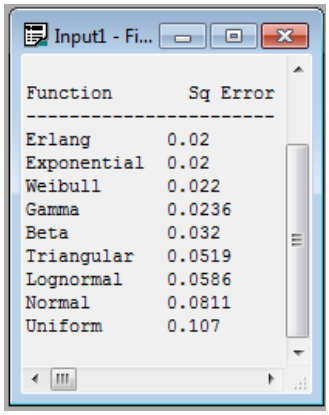


Fig. 8. Summaries of square error of various Applied distribution functions

3.4.4 Distribution of Patients` Arrival at the Case Hospital 2

Similarly, as per the assumption of the multi-server queuing model i.e. arrival of patients must follow the poison distribution; therefore, in order to confirm this assumption, the arrival data was checked by using the input analyzer of the Rockwell arena software. It was revealed that, 1.29 patients arrived at the reception per minute (POIS (1.29)) as shown in the fig 9 and 10.

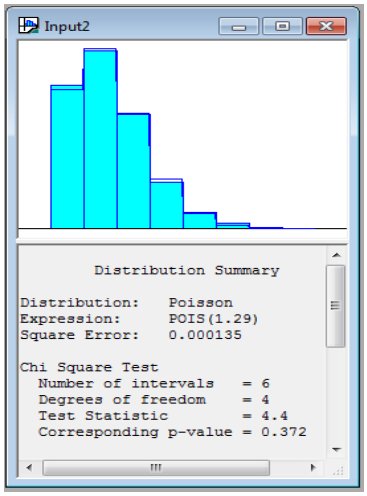


Fig. 9. Distribution fit for the arrival of patient

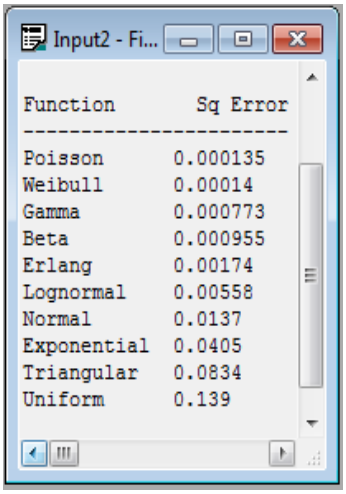


Fig. 10. Summary of square error as generated by Arena software

3.4.5 Service Distribution of Patients at the Medical OPD of Case Hospital 2

After checking the assumption regarding the service distribution of the patients (exponential distribution), it was revealed that each patient took 0.45 (EXPO (0.45)) minutes to be served by the receptionist as shown in the fig 11 and 12.

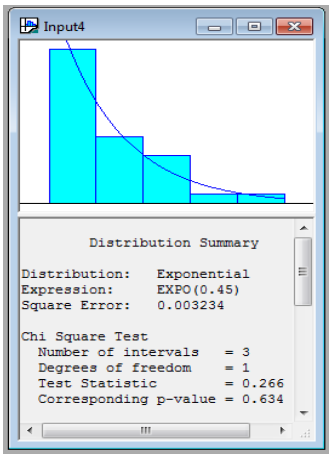


Fig. 11 Distribution fit for the service of patients by the doctors

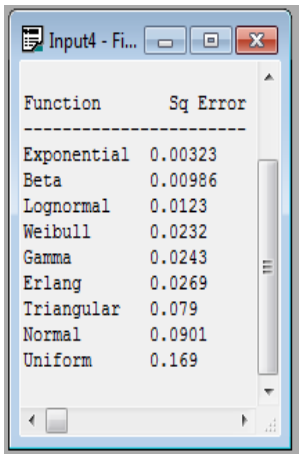


Fig. 12. Summary of the square error for various distribution functions

3.4.6 Service distribution of patients by doctor at case hospital 2

Service of entities must be exponentially distributed as per the assumption of the assumption of the multi-server queuing model. The expression for the distribution came out to be 0.99 + EXPO (2.8) as shown in the fig 13.

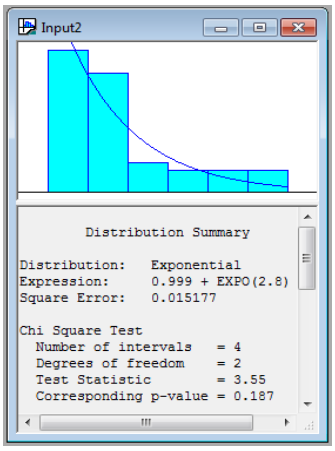


Fig. 13. Distribution fit for the service of patients by the doctor at the OPD

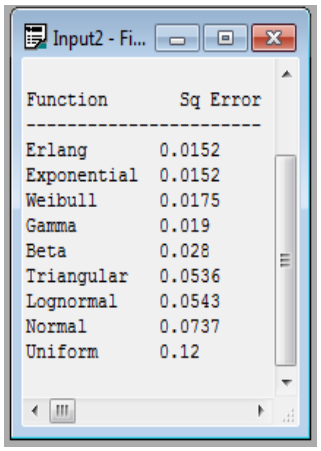


Fig. 14. Summaries of square error of various Applied distribution functions

4. RESULT

Performance measures of the queuing systems i.e. reception and OPDs are presented and discussed under this heading.

4.1 Case Hospital 1

There were two queues in the OPD: first at the reception counter and second queue was outside of the doctors hall which is named as 'OPD' in the discussion. Multi-server queuing model was applied on both the queues separately so that both the queuing systems could be optimized.

4.1.1 Performance Measures of the Reception of OPD of Case Hospital 1

The medical OPD was ranked the first in terms of arrival of patients for medical care at the case hospital 1. The performance measures of the queuing system of reception of medical OPD of case hospital 1 is presented in table III. The calculation of performance measures is based on five scenarios; so that the optimum number of receptionists can be calculated in order to optimize the waiting time of patients at the reception counter. In each scenario, one receptionist was assumed to be increased with same arrival and service rate of patients.

The average arrival rate of patients/hour was collected to be 101 having one receptionist. 106 patients/hour was the service rate of patients. On calculating the utilization factor, it came out to be 95%; the queue was filled with the 19.5 patients waiting to get registered and the system (i.e. reception) was crowded with 20.20 patients. Moreover, the waiting time of patients in the queue and system was 11.43 and 23 minutes respectively. The expected service cost/hour, expected waiting cost/hour and total system cost/hour were calculated to be PKR. 167, PKR.3285 and 3452 respectively (see fig.15).

Table III. Performance measures of the queuing system of reception of medical OPD at case hospital 1

Scenarios	1	2	3	4	5
Receptionist	1	**2**	3	4	5
Lambda (Arrival Rate)/hour	101	**101**	101	101	101
Mue (Service rate)/server	106	**106**	106	106	106
Expected (Service cost PKR.)/hour	166.67	**333.34**	500.01	666.68	833.35
Expected Waiting Cost (PKR.)/hour	3284.52	**200.36**	161.11	155.85	155.03
Expected total system cost (PKR.)/hour	3451	**534**	661	823	988
L'daeff	101.0	**101.0**	101.0	101.0	101.0
(Rho) Utilization factor	95%	**48%**	32%	24%	19%
p0	0.04717	**0.35463**	0.382	0.3852	0.3856
Ls (Number of patients in the system)/hour	20.20	**1.23**	0.99	0.96	0.9536
Lq (Number of patients in the queue)/hour	19.25	**0.28**	0.04	0.01	0.00
Ws (Waiting time of patients in the system in hours)	0.20	**0.01**	0.01	0.01	0.01
Wq Waiting time of patients in the queue in hours)	0.19	**0.00**	0.00	0.00	0.00

Suppose, two receptionists are appointed, the utilization factor will come down from 95% to 48%; which means the server will be 47% less busy as compared to scenario 1; furthermore, 0.28 patients will be left awaiting in the queue and 1.23 patients in the system. The expected service cost/hour, expected waiting cost/hour and total system cost/hour were calculated to be PKR. 333, PKR.200 and 534 respectively (see fig.15). The performance measures calculated for second scenario were considered to be optimum because the expected total system cost was seen to be minimum (see fig.15).

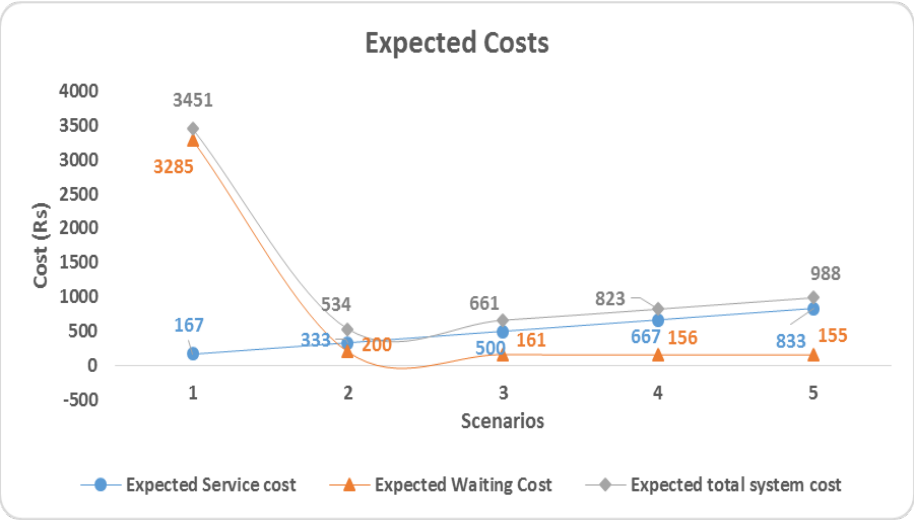


Fig. 15. Cost calculations of the reception of medical OPD at case hospital 1

In the gravity of the consideration of the calculated results, one receptionist at the reception counter of medical OPD of case hospital 1 was suggested to be increased so that the problem of long waiting line can be simplified.

3.4.2 Performance Measures of the OPD of Case Hospital 1

The performance measures of the queuing system of medical OPD of case hospital 1 is presented in table IV. The calculation of performance measures is based on five scenarios; so that the optimum number of doctors can be calculated in order to optimize the waiting time of patients at the OPD. In each scenario, one doctor was assumed to be increased with same arrival and service rate of patients.

The average arrival rate of patients/hour was collected to be 101 having 5 doctors. 107.5 patients/hour was the service rate of patients. On calculating the utilization factor, it came out to be 94%; the queue was filled with the 13.256 patients waiting to get registered and the system (i.e. OPD) was crowded with 17.953 patients. Moreover, the waiting time of patients in the queue and system was 7.8 and 10.8 minutes respectively. The expected service cost/hour, expected waiting cost/hour and total system cost/hour were calculated to be PKR. 5232, PKR.2919 and 8151 respectively (see fig.16).

Table IV. Performance measures of the queuing system of medical OPD of case hospital 1

Scenarios	1	2	3	4	5
Doctors	5	**6**	7	8	9
Lambda (Arrival Rate)/hour	101	**101**	101	101	101
Mue (Service rate)/server per hour	107.5	**107.5**	107.5	107.5	107.5
Expected Service cost (PKR.)/hour	5231.806	**6278.167**	7325	8370.889	9417.25
Expected Waiting Cost (PKR.)/hour	2919.121	**1047.599**	848.9	793.048	774.16
Expected total system cost (PKR.)/hour	8150.926	**7325.766**	8173	9163.937	10191.41
L'daeff	101	**101**	101	101.000	101
(Rho) Utilization factor	94%	**78%**	67%	59%	52%
p0	0.003	**0.007**	0.008	0.009	0.009
Ls (Number of patients in the system)	17.953	**6.443**	5.221	4.878	4.761
Lq (Number of patients in the queue)/hour	13.256	**1.745**	0.523	0.180	0.063
Ws (Waiting time of patients in the system in hours)/hour	0.18	**0.06**	0.05	0.05	0.05
Wq (Waiting time of patients in the queue in hours)	0.13	**0.02**	0.01	0.00	0.00

Suppose, six doctors are appointed, the utilization factor will come down from 94% to 78%; which means the doctors will be 16% less busy as compared to scenario 1; furthermore, 1.745 patients will be left awaiting in the queue and 6.443 patients in the system. The expected service cost/hour, expected waiting cost/hour and total system cost/hour were calculated to be PKR. 6278, PKR.1048 and 7326 respectively (see fig.16). The performance measures calculated for second scenario were considered to be optimum because the expected total system cost was seen to be minimum (see fig.16).

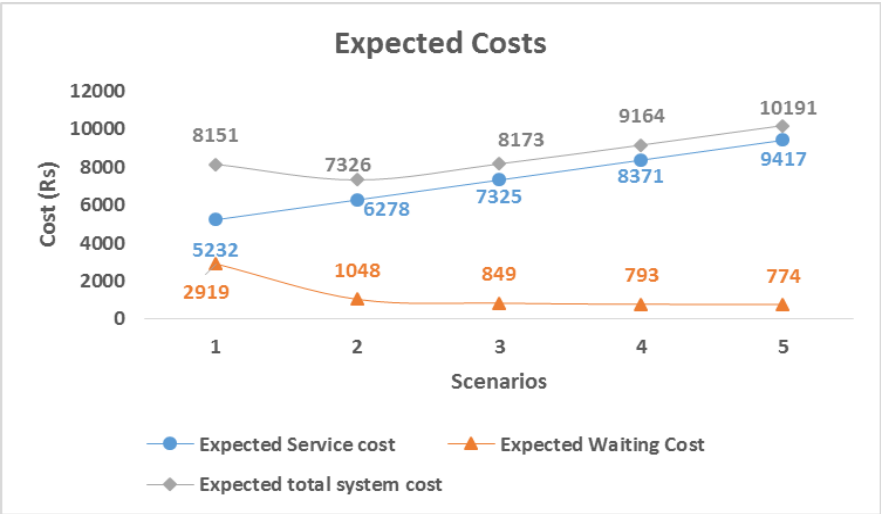


Fig. 16. Expected Cost calculations of the queuign system of medical OPD of hospital of case hospital 2

In the gravity of the consideration of the calculated results, one doctor at the medical OPD of case hospital 1 was suggested to be increased so that the problem of long waiting line can be simplified.

4.2 Case Hospital 2

4.2.1 Performance Measures of the Reception of OPD of Case Hospital 2

The performance measures of the queuing system of reception of medical OPD of case hospital 2 is presented in table V. The calculation of performance measures is based on five scenarios; so that the optimum number of receptionists can be calculated in order to optimize the waiting time of patients at the reception counter. In each scenario, one receptionist was assumed to be increased with same arrival and service rate of patients.

The average arrival rate of patients/hour was collected to be 70 having one receptionist. 75.5 patients/hour was the service rate of patients. On calculating the utilization factor, it came out to be 92%; the queue was filled with the 11.80 patients waiting to get registered and the system (i.e. reception) was crowded with 12.7273 patients. Moreover, the waiting time of patients in the queue and system was 10.2 and 10.8 minutes respectively. The expected service cost/hour, expected waiting cost/hour and total system cost/hour were calculated to be PKR. 156, PKR.2580 and 2745 respectively (see fig.17).

Table V. Performance measures of the queuing system of reception of medical OPD at case hospital 2

Scenarios	1	2	3	4	5
Receptionist	1	**2**	3	4	5
Lambda (Arrival Rate)/hour	70	**70**	70	70	70
Mue (Service rate)/server per hour	75.5	**75.5**	75.5	75.5	75.5
Expected Service cost (PKR.)/hour	155.56	**311.12**	466.68	622.24	777.8
Expected Waiting Cost (PKR.)hour	2589.219	**240.23829**	195.5229	189.542	1886.87
Expected total system cost (PKR.)/hour	2745	**551**	662	812	2665
L'daeff	70.0	**70**	70.0	70.0	70.0
(Rho) Utilization factor	92.%	**46%**	31%	23%	18.5%
p0	0.0729	**0.3665**	0.3923	0.3953	0.3956
Ls (Number of patients in the system)/hour	12.7273	**1.1809**	0.9609	0.9319	0.9278
Lq (Number of patients in the queue)/hour	11.80	**0.25**	0.03	0.00	0.00
Ws (Waiting time of patients in the system in hours)	0.18	**0.02**	0.01	0.01	0.13
Wq Waiting time of patients in the queue in hours)	`0.17	**0.00**	0.00	0.00	0.00

Suppose, two receptionists are appointed, the utilization factor will come down from 92% to 46%; which means the server will be 46% less busy as compared to scenario 1; furthermore, 0.25 patients will be left awaiting in the queue and 1.1809 patients in the system. The expected service cost/hour, expected waiting cost/hour and total system cost/hour were calculated to be PKR. 311, PKR.240 and 551 respectively (see fig.17). The performance measures calculated for second scenario were considered to be optimum because the expected total system cost was seen to be minimum (see fig.17).

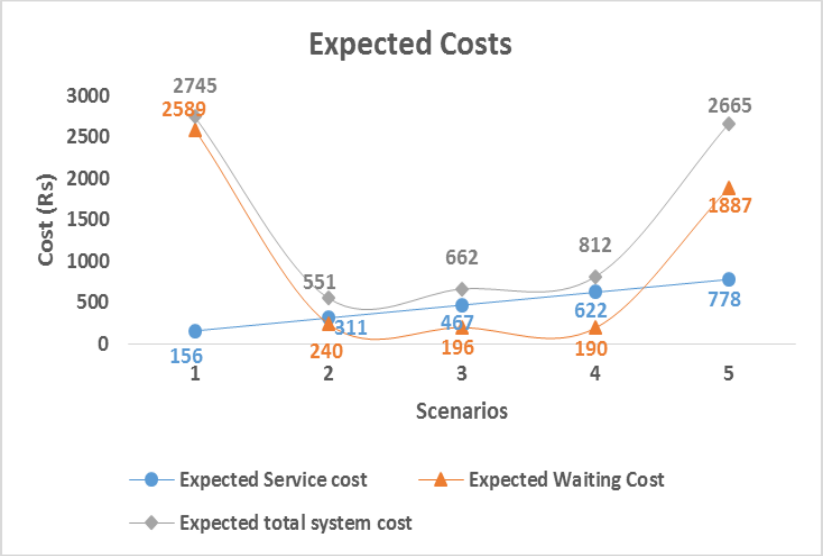


Fig. 17. Calculation of expected cost of the queuing systems of the reception of medical OPD of case hospital 2

In the gravity of the consideration of the calculated results, one receptionist at the reception counter of medical OPD of case hospital 2 was suggested to be increased so that the problem of long waiting line can be simplified.

4.2.2 Performance Measures of the OPD of Case Hospital 2

The performance measures of the queuing system of medical OPD of case hospital 1 is presented in table VI. The calculation of performance measures is based on five scenarios; so that the optimum number of doctors can be calculated in order to optimize the waiting time of patients at the OPD. In each scenario, one doctor was assumed to be increased with same arrival and service rate of patients.

The average arrival rate of patients/hour was collected to be 7 having 3 doctors. 76.5 patients/hour was the service rate of patients. On calculating the utilization factor, it came out to be 92%; the queue was filled with the 9.09 patients waiting to get registered and the system (i.e. OPD) was crowded with 11.83 patients. Moreover, the waiting time of patients in the queue and system was 7.8 and 10.2 minutes respectively. The expected service cost/hour, expected waiting cost/hour and total system cost/hour were calculated to be PKR. 1844, PKR.2407 and 4251 respectively (see fig.18).

Table VI. Performance measures of the queuing system of medical OPD of case hospital 2

Scenarios	1	2	3	4	5
Doctors	3	**4**	5	6	7
Lambda (Arrival Rate)/hour	70	**70**	70	70	70
Mue (Service rate)/server per hour	76.5	**76.5**	76.5	76.5	76.5
Expected Service cost (PKR.)/hour	1843.806	**2458.4074**	3073.009	3687.61	4302.21
Expected Waiting Cost (PKR.)/hour	2407	**740**	603	570	562
Expected total system cost (PKR.)/hour	4250.9	**3198.2**	3675.5	4258.1	4863.9
L'daeff	70	**70**	70	70	70

(Rho) Utilization factor	92%	**69%**	55%	46%	39%
p0	0.0208	**0.0541**	0.0617	0.0636	0.0641
Ls (Number of patients in the system)/hour	11.83	**3.64**	2.96	2.80	2.76
Lq (Number of patients in the queue)hour	9.09	**0.89**	0.22	0.06	0.02
Ws (Waiting time of patients in the system in hours)	0.17	**0.05**	0.04	0.04	0.04
Wq Waiting time of patients in the queue in hours)	0.13	**0.01**	0.00	0.00	0.00

Suppose, four doctors are appointed, the utilization factor will come down from 92% to 69%; which means the doctors will be 23% less busy as compared to scenario 1; furthermore, 0.89 patients will be left awaiting in the queue and 3.64 patients in the system. The expected service cost/hour, expected waiting cost/hour and total system cost/hour were calculated to be PKR.2458, PKR.740 and 3198 respectively (see fig.18). The performance measures calculated for second scenario were considered to be optimum because the expected total system cost was seen to be minimum (see fig.18).

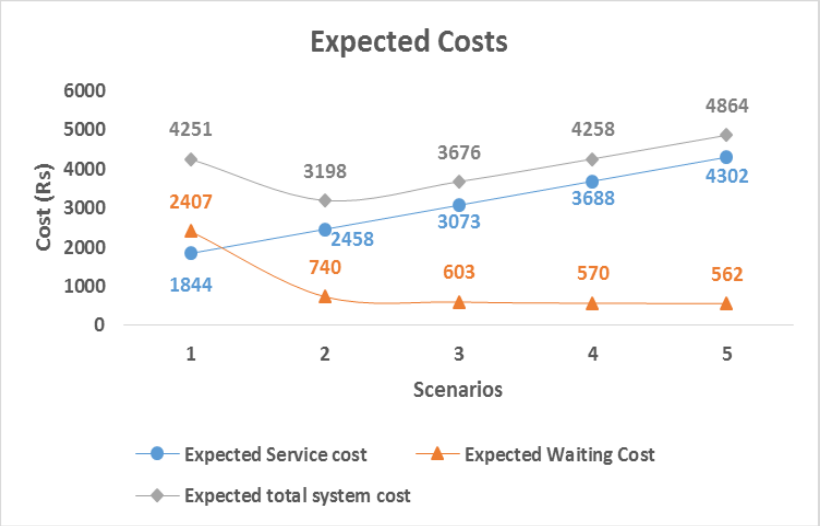


Fig. 18. Calculation of expected costs of the queuing systems of the medical OPD of case hospital 2

In the gravity of the consideration of the calculated results, one doctor at the medical OPD of case hospital 2 was suggested to be increased so that the problem of long waiting line can be simplified.

5. COMPARISON OF THE PERFORMANCE OF OPDs

Comparison of the performance was split into two phases i.e. comparison of the current and optimum performance which are discussed as under in the below given headings:

5.1 Comparison of Current performance

5.1.1 Comparison of the Current Performance of Receptions of the Medical OPDs of Both Hospitals

The comparison of the performance measures of both the receptions is presented in the table III. Mean arrival rate of patients was 101 patients per hour and the receptionist served the patients at the speed of 106 patients per hour at the medical OPD of case hospital 1; whereas, the arrival and service rate of patients at the reception of medical OPD was collected to be 70 and 75.5 patients per hour respectively.

All the performance measures of reception at the medical OPD of case hospital 1 were found to be greater e.g. Utilization factor (95%), Lq (19.25), Ls (20.2), Wq (11.43), Ws (12), Expected service cost (Rs.166.67), Expected waiting cost (Rs.3285); except the probability of the reception to be idle (4.72%) it was less as compared to the reception at the medical OPD case hospital 2 which was 7.29%. From the p0, it is clear that reception at the medical OPD of case hospital 2 was less congested than that of case hospital 1.

Table VII. The table of consisted of performance measures of the current performance of the receptions of both the OPD of both hospitals

Variables	Case Hospital 1	Case Hospital 2
Utilization Factor (Rho)	95%	93%
Probability fo the reception to remain idle (p0)	0.0472	0.0729
Number of the patients in the queue (Lq)	19.25	11.8
Number of the patients in the OPD(Ls)	20.2	12.73
Amount of time the patient spends in queue (Wq) (Minutes)	11.43	10.11
Amount of time the patient spends in the OPD (Ws) (Minutes)	12	10.91
Expected Service Cost	Rs. 166.67	Rs. 155.56
Expected Waiting cost	Rs. 3285	Rs. 2589

5.1.2 Comparison of the Performance Measures of Existing Performance of Medical OPDs of the Both Hospitals

Similarly, comparison of the performance measures of the both OPDs in existing scenario was conducted as shown in the table IV. In this comparison, probability of the OPD of case hospital 1 to remain idle is less (0.271%) than that of the medical OPD of case hospital 2 (0.208%). Interestingly, all other calculated performance measures calculated for the medical OPD of case hospital 1 are greater than the medical OPD of case hospital 2 (see table VIII).

Table VIII. The table of comparison consisted of performance measures of the current performance of both the OPD of both hospitals

Variables	Case Hospital 1	Case Hospital 2
Utilization Factor (Rho)	94%	92%
Probability fo the OPD to remain idle (p0)	0.00271	0.0208
Number of the patients in the queue (Lq)	13.26	9.09
Number of the patients in the OPD (Ls)	17.95	11.83
Amount of time the patient spends in queue (Wq) (Minutes)	7.87	7.79
Amount of time the patient spends in the OPD (Ws) (Minutes)	10.67	10.14
Expected Service Cost	Rs. 5231	Rs. 1843.8
Expected Waiting cost	Rs. 2919	Rs. 2407

5.1.3 The Comparison of the Total Time taken By the Patients in the System

In the existing scenario at case hospital 1, patients were taking 12 minutes in order to get free from the reception and 10.67 minutes in getting served by the doctor; collectively, patients were in the system for 22.67 minutes at the case hospital 1.

Table IX. Total waiting time of patients in the overall OPD in the current scenario

Variables	Time spent at reception (Minutes)	Time spent in the OPD (Minutes)	Total Time (Minutes)
Case Hospital 1	12	10.67	22.67
Case Hospital 2	10.91	10.14	21.05

Whereas, in the scenario of case hospital 2, total reception time of patients was 10.91 minutes and OPD time was 10.14 minutes and total system time was 21.05 minutes as shown in the table IX.

5.2 Comparison of the Optimum Performance of the Receptions and OPDs

5.2.1 Comparison of the Optimum Performance of Receptions of the Medical OPDs of Both Hospitals

In the optimum scenario, the arrival and service rate of patients was assumed to be the same and doctors were supposed to be increased by 1 in each scenario and the comparison of performance measures is presented in the table X. All the optimum performance measures of reception at the medical OPD of case hospital 1 were found to be greater e.g. Utilization factor (48%), Lq (0.28), Ls (1.28), Wq (0.17), Ws (0.73), Expected service cost (Rs.333.34),

Expected waiting cost (Rs.200); except the probability of the reception to be idle (48%) it was less as compared to the reception at the medical OPD case hospital 2 which was 46%. From the p0, it is clear that reception at the medical OPD of case hospital 2 would be less congested than that of case hospital 1 in the optimum scenario if implemented.

Table X. The table of consisted of performance measures of the optimum performance of the receptions of both the OPD of both hospitals

Variables	Case Hospital 1	Case Hospital 2
Utilization Factor (Rho)	48%	46%
Probability of the OPD to remain idle (p0)	0.3546	0.3665
Number of the patients in the queue (Lq)	0.28	0.25
Number of the patients in the OPD(Ls)	1.28	1.18
Amount of time the patient spends in queue (Wq) (Minutes)	0.17	0.22
Amount of time the patient spends in the OPD (Ws) (Minutes)	0.73	1.01
Expected Service Cost	Rs.333.34	Rs.311.12
Expected Waiting cost	Rs. 200	Rs. 240

5.2.2 Comparison of the Optimum Performance of the Medical OPDs of Both Hospitals

Similarly, comparison of the performance measures of the both OPDs in optimum scenario was conducted as shown in the table VII. In this comparison, probability of the OPD of case hospital 1 to remain idle is less (0.704%) than that of the medical OPD of case hospital 2 (5.41%). Interestingly, all other calculated performance measures calculated for the medical OPD of case hospital 1 are greater than the medical OPD of case hospital 2 (see table XI).

Table XI. The table of consisted of performance measures of the optimum performance of both the OPDs of both hospitals

Variables	Case Hospital 1	Case Hospital 2
Utilization Factor (Rho)	78%	69%
Probability of the OPD to remain idle (p0)	0.00704	0.0541
Number of the patients in the queue (Lq)	1.75	0.89
Number of the patients in the OPD (Ls)	6.44	3.64
Amount of time the patient spends in queue (Wq) (Minutes)	1.04	0.76
Amount of time the patient spends in the OPD (Ws) (Minutes)	3.83	3.12
Expected Service Cost	Rs. 6278.167	Rs. 2458.407
Expected Waiting cost	Rs. 1048	Rs. 740

5.2.3 The Comparison of the Total Time taken By the Patients in the System

In the optimum scenario at case hospital 1, patients were taking 0.73 minutes in order to get free from the reception and 3.83 minutes in getting from the doctor; collectively, patients were in the system for 4.56 minutes at the case hospital 1.

Table XII. Total waiting time of patients in the overall OPD in the optimum scenario

Variables	Time spent at reception (Minutes)	Time spent in the OPD (Minutes)	Total Time (Minutes)
Case Hospital 1	0.73	3.83	4.56
Case Hospital 2	1.01	3.12	4.13

Whereas, in the optimum scenario of case hospital 2, total reception time of patients was 1.01 minutes and OPD time was 3.12 minutes and total system time was 4.13 minutes as shown in the table XII.

6. CONCLUSION AND RECOMMENDATIONS

Reception and medical OPD at the case hospital 1 were more congested and absorbing in terms of patients as compared to that of case hospital 2. It was suggested to the hospital administration that they should increase one receptionist and one doctor at both of the OPDs in order to simplify the problem of congestion of patients. After the

increment one receptionist and one doctor, queues of patients at both stages would be minimized and so the total waiting time of patients. Since they will have to wait for less time as the compared to the existing scenario; their waiting cost would certainly be decreased which will be the good point on the side of healthcare providers.

7. LIMITATIONS AND FUTURE WORK

In this study, the time of patients which was spent while coming to the hospital and the time taken by the patient at the pharmacy was not included due to the shortage of time. In the next study, these parameters would be considered in order to produce more accurate results. This work can be extended by conducting the modelling and simulation of queuing systems of both the OPDs.

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