operation, and hence the system is operable, but its performance is inferior since the failure rate has incresed. A pictorial presentation of the two-component series system version undergoing one imperfect repair is shown in Figure 1. This systems consists of seven states, three states (1, 3, 6) constitute the operation states, while the other four states (2, 4, 5, 7) are the failed states.

In this system, in the initial state 1, both components are new (A_0, B_0). Over time, the system deteriorates and any one component could fail with failure rate λ_1 moving to failed state 2, where the failed component is repaired, with repair rate μ_1. Upon repair, the system moves to state 3, where one component has failed once and the second has not failed at all (A_1, B_0 or A_0, B_1). Over time, any one component may fail, and the system moves either to state 4 by the failure of the non-failed component with failure rate λ_1, or to state 5 with the second failure of the once failed component with failure rate with failure rate λ_2. From state 5, and after repairing the failed component with repair rate μ_2, the system moves to state 1 and becomes as good as new.

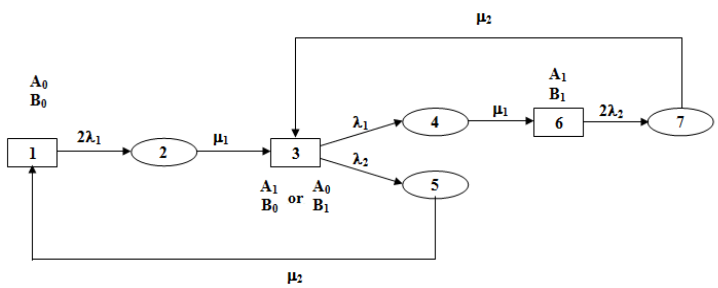

Figure 1: A two-component series system with imperfect repair

Legend:
λ1, λ2 = the first and the second failure rate for each component, μ1, μ2 = the first and the second repair rate for each component. A0, B0 = both components are new, A1, B1 = both components have failed once.

While from state 4, and upon repairing the once failed component, the system moves to state 6. State 6 represents the state where both components have failed once (A_1, B_1). The system could fail and move from state 6 to state 7 upon failure of one of the components for the second time with failure rate λ_2. from state 7, the system moves to state 3 upon repairing the failed component for the second time with repair rate μ_2.

By equating the flow in to the flow out for each node, the steady state transition probabilities of the system are derived, it is as follows:

$$\begin{bmatrix} -2\lambda_1 & 0 & 0 & 0 & \mu_2 & 0 & 0 \\ 2\lambda_1 & -\mu_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mu_1 & -(\lambda_1+\lambda_2) & 0 & 0 & 0 & \mu_2 \\ 0 & 0 & \lambda_1 & -\mu_1 & 0 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 & -\mu_2 & 0 & 0 \\ 0 & 0 & 0 & \mu_1 & 0 & -2\lambda_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\lambda_2 & -\mu_2 \end{bmatrix} \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \\ \pi_5 \\ \pi_6 \\ \pi_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sum_{i=1}^{7} \pi_i = 1$$

Solving the above equations for the system's state probabilities in terms of the probability of being in state 1 (π_1) results in the following:

$$\pi_2 = \frac{2\lambda_1}{\mu_1}\pi_1; \pi_3 = \frac{2\lambda_1}{\lambda_2}\pi_1; \pi_4 = \frac{2\lambda_1^2}{\lambda_2\mu_1}\pi_1;$$
$$\pi_5 = \frac{2\lambda_1}{\mu_2}\pi_1; \pi_6 = \frac{\lambda_1^2}{\lambda_2^2}\pi_1; \pi_7 = \frac{2\lambda_1^2}{\lambda_2\mu_2}\pi_1$$

Substituting the probabilities in (2) into (1) and solving for π_1, we obtain:

$$\pi_1 = \frac{\lambda_2^2\mu_1\mu_2}{(\lambda_1+\lambda_2)\left[2\lambda_1\lambda_2(\mu_1+\mu_2)+\mu_1\mu_2(\lambda_1+\lambda_2)\right]}$$

Adding the expressions for the operational probabilities in (2), i.e., $\pi_1 + \pi_3 + \pi_6$, we obtain the expression for the steady state operational probability of the system (*A*), becomes (Availability):

$$A = \frac{\mu_1\mu_2(\lambda_1+\lambda_2)^2}{\lambda_2^2}$$

Substituting (3) into (4), we obtain:

$$A = \frac{\mu_1\mu_2(\lambda_1+\lambda_2)^2}{(\lambda_1+\lambda_2)\left[2\lambda_1\lambda_2(\mu_1+\mu_2)+\mu_1\mu_2(\lambda_1+\lambda_2)\right]}$$

Dividing the numerator and the denominator of the above expression by $\lambda_1\lambda_2\mu_1\mu_2$ and simplifying, it reduces to:

$$A = \frac{\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right)}{\frac{1}{2}\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right)+\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)}$$

In Similar fashion, for a two-component system with *n* repairs, the long term probability of being in state 1, the expression is as follows:

$$A = \frac{\prod_{i=1}^{n+1}\lambda_{i+1}^2\mu_i}{\prod_{i=1}^{n+1}\mu_i\left[\sum_{i+1}^{n+1}\frac{1}{\lambda_i^2}\prod_{i=1}^{n+1}\lambda_i^2 + 2_i\prod_{i=1}^{n+1}\lambda_i\left(\sum_{i=1}^{n+1}\frac{1}{\lambda_i}\prod_{i=1}^{n+1}\lambda_i\right)\right] + 2_i\prod_{i=1}^{n+1}\lambda_i\left[\sum_{i+1}^{n+1}\frac{1}{\lambda_i}\prod_{i=1}^{n+1}\lambda_i + \left(\sum_{i=1}^{n+1}\frac{1}{\mu_i}\prod_{i=1}^{n+1}\mu_i\right)\right]}$$

Rearranging the above expression and simplifying, it reduces to:

$$A = \frac{\sum_{i=1}^{n+1}\frac{1}{2\lambda_i}}{\sum_{i=1}^{n+1}\frac{1}{2\lambda_i}+\sum_{i=1}^{n+1}\frac{1}{\mu_i}}$$

Case 2: Parallel Components

The parallel system configuration consists of two identical components; the failure of one component does not cause the whole system to fail, but weakens its performance. A pictorial presentation of the system is presented in Figure (2). It consists

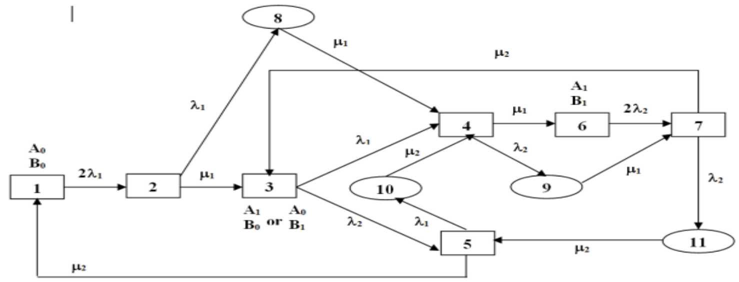

Figure 2: A two-component parallel system with imperfect repai**r**

Legend:
λ_1, λ_2 = the first and the second failure rate for each component, μ_1, μ_2 = the first and the second repair rate for each component. A_0, B_0 = both components are new, A_1, B_1 = both components have failed once.

of 11 states, seven operating states (1 to 7) and four failed states (8 to 11). State one represents the intial state where both components are new (A_0, B_0). With continuous operation, the system deteriorates and any one component could fail with a constant failure rate λ_1, thus moving to state 2. From state 2, the transition is either to state 3 (A_1, B_0 or A_0, B_1) with constant repair rate μ_1, or with the failure of the non failed component, to the failed state 8 with constant failure rate λ_1, and hence the system is down. From state 8 the transition is to state 4 by repairing the first failed component with a constant repair rate μ_1. From the operational state 3, the transition is either to state 4 by the failure of the non-failed component with constant failure rate λ_1, or to the operational state 5 with the second failure of the component who has already failed once with constant failure rate λ_2. The transition from state 4 is either through the repair of the failed component to state 6 (A_1, B_1), with a constant repair rate μ_1, or to another failed state 9 with the failure of the second component with a failure rate of λ_2. The transition from the operating state 5 is either through repair to the initial state, where the system is as good as new with a constant repair rate μ_2,or to the failed state 10 with constant failure rate λ_1. From state 6, the transition is only to state 7 with the failure of either component for the second time with a failure rate of λ_2. The transition from state 7 is either through the repair of the failed component to state 3 with a constant repair rate μ_2, or by the failure of the other component for the second time to state 11 with a constant failure rate of λ_2. The system is pictorially presented in Figure 2, where the rectangular and the oval shapes represent the operational failed states respectively.

By equating the flow in to the flow out for each node, the steady state transition probabilities of the system are derived, it is as follows:

$$\begin{bmatrix} -2\lambda_1 & 0 & 0 & 0 & \mu_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\lambda_1 & -(\lambda_1+\mu_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_2 & 0 \\ 0 & \mu_1 & -(\lambda_1+\lambda_2) & 0 & 0 & 0 & \mu_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_1 & -(\lambda_2+\mu_1) & 0 & 0 & 0 & \mu_1 & 0 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 & -(\lambda_1+\mu_2) & 0 & 0 & 0 & 0 & \mu_1 & 0 \\ 0 & 0 & 0 & \mu_1 & 0 & -2\lambda_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\lambda_2 & -(\lambda_2+\mu_2) & 0 & \mu_1 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & -\mu_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & -\mu_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & -\mu_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & -\mu_2 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \\ P_{10} \\ P_{11} \end{bmatrix} = \underline{0}$$

$$\sum_{i=1}^{11} \pi_i = 1$$

Solving the above equations for the system's state probabilities in terms of the probability of being in state 1 (P_1) results in finding the expression for the probabilities in terms of π_1, invoking the fact that the sum of the probabilities is equal to 1 (8), and solving for P_1, result is as following expression:

$$\pi_1 = \frac{\lambda_2^2 \mu_1 \mu_2^2 (\lambda_1+\mu_1)(\lambda_2+\mu_1)(\lambda_1+\lambda_2+\mu_2)}{\left\{ \begin{array}{l} \mu_1\mu_2 \left[\begin{array}{l} \lambda_1^2(\lambda_1+\mu_2)(\lambda_2+\mu_2)(2\lambda_2+\mu_1)(\lambda_1+\lambda_2+\mu_1) \\ +2\lambda_1\lambda_2(\lambda_1+\mu_2)(\lambda_2+\mu_1)(\lambda_1+\lambda_2+\mu_1)(\lambda_1+\lambda_2+\mu_2) \\ +\lambda_2^2(\lambda_1+\mu_1)(\lambda_2+\mu_1)(2\lambda_1+\mu_2)(\lambda_1+\lambda_2+\mu_2) \end{array} \right] \\ +2\lambda_1^2\lambda_2^2 \left[\begin{array}{l} (\lambda_1+\mu_2)(\lambda_1+\lambda_2+\mu_2)\{\mu_1(\lambda_1+\mu_1)+\mu_2(\lambda_2+\mu_1)\} \\ +(\lambda_2+\mu_1)(\lambda_1+\lambda_2+\mu_1)\{\mu_1(\lambda_1+\mu_2)+\mu_2(\lambda_2+\mu_2)\} \end{array} \right] \end{array} \right\}}$$

Substituting the value of π_1 in equation (10) into the probabilities in (9) and adding the operational probabilities, we obtain the long term operational probability A:

$$A = \frac{\mu_1\mu_2 \left[\begin{array}{l} \lambda_1^2(\lambda_1+\mu_2)(\lambda_2+\mu_2)(2\lambda_2+\mu_1)(\lambda_1+\lambda_2+\mu_1) \\ +2\lambda_1\lambda_2(\lambda_1+\mu_2)(\lambda_2+\mu_1)(\lambda_1+\lambda_2+\mu_1)(\lambda_1+\lambda_2+\mu_2) \\ +\lambda_2^2(\lambda_1+\mu_1)(\lambda_2+\mu_1)(2\lambda_1+\mu_2)(\lambda_1+\lambda_2+\mu_2) \end{array} \right]}{\left\{ \begin{array}{l} \mu_1\mu_2 \left[\begin{array}{l} \lambda_1^2(\lambda_1+\mu_2)(\lambda_2+\mu_2)(2\lambda_2+\mu_1)(\lambda_1+\lambda_2+\mu_1) \\ +2\lambda_1\lambda_2(\lambda_1+\mu_2)(\lambda_2+\mu_1)(\lambda_1+\lambda_2+\mu_1)(\lambda_1+\lambda_2+\mu_2) \\ +\lambda_2^2(\lambda_1+\mu_1)(\lambda_2+\mu_1)(2\lambda_1+\mu_2)(\lambda_1+\lambda_2+\mu_2) \end{array} \right] \\ +2\lambda_1^2\lambda_2^2 \left[\begin{array}{l} (\lambda_1+\mu_2)(\lambda_1+\lambda_2+\mu_2)\{\mu_1(\lambda_1+\mu_1)+\mu_2(\lambda_2+\mu_1)\} \\ +(\lambda_2+\mu_1)(\lambda_1+\lambda_2+\mu_1)\{\mu_1(\lambda_1+\mu_2)+\mu_2(\lambda_2+\mu_2)\} \end{array} \right] \end{array} \right\}}$$

Rearranging and simplifying, the above expression reduces to:

$$\pi_s = \frac{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}}{\left\{ \begin{array}{l} \frac{1}{\lambda_1}+\frac{1}{\lambda_2}+\lambda_1\left[\frac{1}{\mu_1(2\lambda_1+\mu_1)}+\frac{1}{\mu_2(2\lambda_1+\mu_2)}\right] \\ \qquad +\lambda_2\left[\frac{1}{\mu_1(2\lambda_2+\mu_1)}+\frac{1}{\mu_2(2\lambda_2+\mu_2)}\right] \end{array} \right\}}$$

In similar fashion, the long term probability of the system being in the operational state after undergoing n imperfect repairs can be derived. It takes the following form:

$$A = \frac{\sum_{i=1}^{n+1} \frac{1}{\lambda_i}}{\sum_{i=1}^{n+1} \frac{1}{\lambda_i} + \sum_{i=1}^{n+1} \lambda_{i+1} \left[\frac{1}{\mu_i (2\lambda_{i+1} + \mu_i)} + \frac{1}{\mu_i (2\lambda_{i+1} + \mu_i)} \right]}$$

Where $\mu_{n+2} = \mu_1, \lambda_{n+2} = \lambda_1$

Case 3: Two-component Standby System

In this system, the operating and the stand-by components are identical, the number of components in the system is two. When the working (operating) component fails, the cold stand-by component starts operating and the system will continue in the operating mode. The failed component is sent to the repair station for repair and after repair, it becomes a stand-by (spare). There are many examples of stand-by systems in real life, in petroleum refiners where pumps are essential parts of operation, stand-by pumps are used for emergency cases; in nuclear power plants, a stand-by emergency core cooling system (ECCS) is available to take over if the main cooling system fails, and computers have a stand-by generator as a backup for the main supply line.

A pictorial presentation of the transition between states is as shown in Figure 3 for the proposed system.

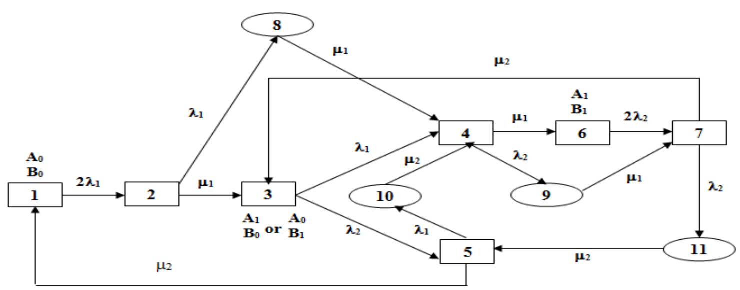

Figure 3: A two-component standby system with imperfect repair.

The system has 12 states, states 1 to 8 (rectangular shape) are the operational states, while states 9-12 (oval) are the failed states. At state 1 (A_0, B_0) the system is as good as new, A_0 is the operational component and B_0 is the standby component over time, and after the failure of the operating component with a constant failure rate λ_1 the standby by component takes over and the system moves to state 2. The transition from the operational state 2 is either to the operational state 3 (A_1, B_0) with constant repair rate μ_1, or to the failed state 9 with constant failure rate λ_1 and hence the system becomes non-operational (down). State 4 is reached either from the operational state 3, with the failure of the non-failed component with constant failure rate λ_1, or from the failed state 9 with a constant repair rate μ_1 due to the repair of the failed component which took place in state 2. Transition from state 4 is either through the repair of the failed component to state 5 (A_1, B_1), with a constant repair rate μ_1, or to the failed state 10 with the failure of the operational component with the constant failure rate λ_2. The transition from the operating state 5 is to state 6 with the failure of the operating component for the second time with a constant failure rate of λ_2. From state 6, the transition is either to state 7 with the second repair of the failed component with a constant repair rate μ_2, or to state 11 with the failure of the operating component for the second time with a constant failure rate of λ_2. State 8 is reached from state 7 with the failure of the operating component

B_1 for the second time with a failure rate of λ_2, or through the repair of the failed component from state 11 with a constant repair rate μ_2. The transition from state 8 is either to the initial state (A_0, $\underline{B_0}$) with a constant repair rate μ_2 where the system becomes as good as new, or by the failure of the operating component to state 12 with a constant failure rate of λ_1. After repair in state 12, the system moves to state 2 with a constant repair rate μ_2.

$$\begin{bmatrix} -\lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_2 & 0 & 0 & 0 & 0 \\ \lambda_1 & -(\lambda_1+\mu_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_2 \\ 0 & \mu_1 & -\lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_1 & -(\lambda_2+\mu_1) & 0 & 0 & 0 & 0 & \mu_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu_1 & -\lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_2 & -(\lambda_2+\mu_2) & 0 & 0 & 0 & \mu_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_2 & -\lambda_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda_2 & -(\lambda_1+\mu_2) & 0 & 0 & \mu_2 & 0 \\ 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & 0 & -\mu_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & -\mu_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & -\mu_2 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \\ P_{10} \\ P_{11} \\ P_{12} \end{bmatrix} = \underline{0}$$

By equating the flow in to the flow out for each node, the steady state transition probabilities of the system are derived, it is as follows:

$$\sum_{i=1}^{12} \pi_i = 1$$

Solving the above equations for the system's state probabilities in terms of the probability of being in state 1 (π_1), results in the expression for all probability expressions in terms of π_1. Solving the expreesion for π_1 is found as follows:

$$\pi_1 = \frac{\lambda_2 \mu_1 \mu_2^2}{\left\{ \begin{array}{l} (\lambda_1^2 + \lambda_1\mu_2 + \mu_2^2)\lambda_2\mu_1 + (\lambda_1 + \mu_2)\left[\dfrac{(\lambda_1^2 + \lambda_1\mu_1 + \mu_1^2)\lambda_2\mu_2}{(\lambda_1 + \mu_1)} \right] \\ +(\lambda_1 + \mu_2)\left[\dfrac{(\lambda_2^2 + \lambda_2\mu_1 + \mu_1^2)\lambda_1\mu_2}{(\lambda_2 + \mu_1)} + \dfrac{(\lambda_2^2 + \lambda_2\mu_2 + \mu_2^2)\lambda_1\mu_1}{(\lambda_2 + \mu_2)} \right] \end{array} \right\}}$$

In order to find the long run probability of the system being operational, we have to add the probability of the operational states, i.e. $\pi_1 + \pi_2 + \ldots + \pi_7 + \pi_8$. Therefore, the expression for the long run probability of being operational P_s becomes:

$$A = \frac{2\mu_1\mu_2(\lambda_1+\lambda_2)}{\left\{\begin{array}{l}(\lambda_1^2+\lambda_1\mu_2+\mu_2^2)\lambda_2\mu_1+(\lambda_1+\mu_2)\left[\dfrac{(\lambda_1^2+\lambda_1\mu_1+\mu_1^2)\lambda_2\mu_2}{(\lambda_1+\mu_1)}\right] \\ +(\lambda_1+\mu_2)\left[\dfrac{(\lambda_2^2+\lambda_2\mu_1+\mu_1^2)\lambda_1\mu_2}{(\lambda_2+\mu_1)}+\dfrac{(\lambda_2^2+\lambda_2\mu_2+\mu_2^2)\lambda_1\mu_1}{(\lambda_2+\mu_2)}\right]\end{array}\right\}}$$

Dividing the numerator and the denominator of the above expression by $2\lambda_1\lambda_2\mu_1\mu_2(\lambda_1+\mu_2)$, rearranging, and simplifying, the above expression reduces to:

$$P_s = \frac{\dfrac{1}{\lambda_1}+\dfrac{1}{\lambda_2}}{\dfrac{1}{\lambda_1}+\dfrac{1}{\lambda_2}+\dfrac{1}{2}\lambda_1\left[\dfrac{1}{\mu_1(\lambda_1+\mu_1)}+\dfrac{1}{\mu_2(\lambda_1+\mu_2)}\right]+\dfrac{1}{2}\lambda_2\left[\dfrac{1}{\mu_1(\lambda_2+\mu_1)}+\dfrac{1}{\mu_2(\lambda_2+\mu_2)}\right]}$$

In general for a two component system undergoing *n* imperfect repairs, the probability of being in state 1 has the following expression:

$$\pi_1 = \frac{\mu_{n+1}^2\prod_{i=1}^{n+1}\lambda_{i+1}\,\mu_i}{\sum_{i=1}^{n+1}\sum_{j=1}^{n+1}\left[\dfrac{(\lambda_i+\lambda_i\mu_j+\mu_j^2)\prod_{\substack{k=1,k\neq i\\ l=1,l\neq j}}^{n+1}\lambda_k\mu_l}{(\lambda_i+\mu_j)}\right]}$$

In similar fashion, the steady state probability of the system being in operational state after *n* imperfect repairs has the following general formula:

$$A = \frac{2\prod_{i=1}^{n+1}\lambda_i\mu_i\left[\sum_{i=1}^{n+1}\dfrac{1}{\lambda_i}\right]}{\sum_{i=1}^{n+1}\sum_{j=1}^{n+1}\left[\dfrac{(\lambda_i+\lambda_i\mu_j+\mu_j^2)\prod_{\substack{k=1,k\neq i\\ l=1,l\neq j}}^{n+1}\lambda_k\mu_l}{(\lambda_i+\mu_j)}\right]}$$

Simplifying and rearranging expression (20), it is reduced to the following:

$$A = \frac{\sum_{i=1}^{n+1}\dfrac{1}{\lambda_i}}{\sum_{i=1}^{n+1}\dfrac{1}{\lambda_i}+\dfrac{1}{2}\sum_{i=1}^{n+1}\lambda_{i+1}\left[\dfrac{1}{\mu_i(\lambda_{i+1}+\mu_i)}+\dfrac{1}{\mu_i(\lambda_{i+1}+\mu_i)}\right]}$$

Where $\mu_{n+2}=\mu_1,\ \lambda_{n+2}=\lambda_1$

3. Example

In this example, the long term operational probability for the various configurations is calculated with five sets of failure and repair rate rates (5 runs). The values of the repair (λ_i) and replacement rates (μ_i) are given in Table 1.

Using the values of the failure rates and repair rates for the different runs, the long term operational probabilities for the various configurations are calculated in Table 1 by utilizing the expressions in equations (5), (12), and (18), respectively. Table 2 shows the operational probabilities for different number of repairs.

Table 1. The failure and repair rates for the different runs

Failure & Repair Rates	Run				
	1	2	3	4	5
λ_1	0.009	0.800	1.100	3.090	7.330
λ_2	0.087	0.210	2.500	7.270	13.45
λ_3	0.430	0.540	5.700	11.50	19.15
λ_4	2.200	1.200	8.700	19.21	26.09
μ_1	2.700	3.700	10.20	12.87	10.57
μ_2	1.090	1.900	7.300	9.890	6.220
μ_3	0.530	0.800	3.800	3.120	2.090
μ_4	0.097	0.300	1.290	1.770	0.082

It can be observed from Table 2 below that the standby configuration has the highest long term Operational probability in all cases and the series system has the lowest. In addition, the long term operational probability decreases as the number of repairs increases. Moreover, it is not significantly different for the various configurations when the system is new (0 repair), and after it becomes very old. The lower the failure rate, and the higher the repair rate the better the system's performance. In fact, as we examine the operational probability of the different runs, it can be deduced that the operational probability is positively correlated with the ratio of the repair rate to the failure rate (μ_i/λ_i).

Table 2. The long term operational probability for the various configurations vs. the number of imperfect repairs allowed before replacement

Run	No. of Repairs	Configuration		
		Series	Parallel	Standby
1	0	0.993	0.998	0.999
	1	0.979	0.993	0.995
	2	0.952	0.980	0.985
	3	0.823	0.885	0.899
2	0	0.958	0.976	0.985
	1	0.915	0.993	0.995
	2	0.824	0.924	0.947
	3	0.649	0.771	0.815
3	0	0.822	0.917	0.957
	1	0.736	0.858	0.898
	2	0.598	0.731	0.787
	3	0.386	0.469	0.518
4	0	0.675	0.684	0.746
	1	0.563	0.858	0.898
	2	0.423	0.521	0.580
	3	0.242	0.285	0.315
5	0	0.419	0.497	0.664
	1	0.292	0.345	0.387
	2	0.152	0.169	0.184
	3	0.012	0.013	0.014

4. Conclusions and Future Research

As it has been shown, the closed form expressions for the steady state operational probability becomes more complicated as the number of repairs increases. In addition, the analytical derivation of the operational probability is easier for the series system than the standby system, while it is very tedious for the parallel configuration. The derived expressions for the steady state operational probability are generic in nature; they could be used for the perfect repair case under certain assumptions. Moreover, if other than the exponential distribution is assumed for the failure and repair rates, measuring the performance analytically becomes extremely difficult and tedious. Future works should examine systems with more than two-component with non-identical components, more than one repair channel, more complex configurations, and non exponential distributions.

References

Chiang, Cheng-Hsiuing, & Liang-Hsuan Chen, Availability allocation and multi-objective optimization for parallel-series systems. *European Journal of Operational Research,* vol. 180, pp. 1231-1244,2007.

Chung, Who Kee., Reliability of imperfect switching of cold standby systems with multiple non-critical errors. *Microelectronics and Reliability,* vol. 35, no.12, pp. 1479-1482, 1995.

Dohi, Tadashi, Akira Ashioka, Naoto Kaio & Shunji Osaki, The optimal repair-time limit replacement policy with imperfect repair: Lorenz transform approach, *Mathematical and computer Modeling*, vol. 38, no. 11-13, pp. 1169-1176, 2003.

Doyen, Laurent & Olivier Gaudoin, Classes of imperfect repair models based on reduction of failure intensity or virtual age, *Reliability Engineering & System Safety*, vol. 84, no. 1, pp. 45-56, 2003.

Guo, L., Wang, Y., and Li, K. (). Availability Modeling of Two-Component Repairable Systems Subject to Switch-Off, IEEE, *DOI.10.1109/ACCESS.2018.2806386,* 2018.

Ke, Jau-Chuan, & Yunn-Kuang Chu, Comparative analysis of availability of redundant repairable system. *Applied Mathematics and Computation,* vol.188, pp. 323-338, 2007.

Ke, Jau-Chuan, Ssu-Lang Lee, & Ying-Lin Hsu, On a repairable system with detection, imperfect coverage and reboot: Bayesian approach, *Simulation Modeling Practice and Theory,* vol. 16, no. 3, pp. 353-367, 2008.

Oliveira, E. A., Alvim, A.C.M. & Frutuoso e Melo, P.F, Unavailability analysis of safety systems under gaining by supplementary variables with imperfect repair, *Annals of Nuclear Energy,* vol. 31, no. 2, pp. 241-252, 2005

Sheu, Shey-Huei, Yuh-Bin Lin, and Gwo-Liang Liao. Optimum polices for a system with general imperfect maintenance, *Reliability Engineering & System Safety,* vol. 91, no. 3, 362-369, 2006.

Zhang, J., Ma, X., and Zhao, Y., Reliability demonstration for long-life products based on hardened testing method and gamma process," *IEEE Access*, vol. 5, no., pp. 19322-19332, 2017.

Zhang, N., Fouladirad M., and Barros, A., Optimal imperfect maintenance cost analysis of a two-component system with failure interactions, *Reliability Engineering & System Safety*, Vol. 177, September, pp. 24-34, 2018.

Mohammed Hajeeh is a research scientist in the Techno-Economics Division (TED). He Holds a Bachelor of Science (BS) degree in Nuclear Engineering, Master of Science (MS) in Mechanical/ Nuclear Engineering and a Ph.D. in Operations Research. During his employment at KISR, he has participated in and contributed to several research projects. He has published several papers in various topics in different international referred journals and conference proceedings. He has conducted courses inside and outside KISR in the areas of Statistics, Linear Programming, Operations Management, Design and Analysis of Experiments, Project Management, and Mathematical Analysis.

Shaima Al-Shamali is a senior research associate at the Techno-Economics Division (TED). She joined KISR in October 1997. She is a graduate of Kuwait University (B.Sc.) in Computer Science as a major and minor is in Political Science,1997. She has participated in many projects undertaken by TED in different fields such as: banking, labor, agriculture, trade, databank and Information Technology (IT). Her fields of interests are computer programming, database development, and learning new topics. She has taken numerous training courses in her fields of interest

Wafa Awadh is a graduate of Kuwait University (B.Sc.) in Economics with minor in accounting, July 1990, and The George Washington University, Washington, DC, USA (MA) in economics, May 1997. She joined KISR/TED in 1991. She has participated in many research projects as task leader and team member. She has contributed to several KISR research reports and scientific papers as a joint author. Her areas of research interest are: macroeconomics, international economics, econometrics modeling, labor economics and regional economics.