

The Complete Solution of the Production, Remanufacture and Waste Disposal Model with Lost Sales

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Abstract

Inventory management in reverse logistics has been receiving increasing attention in recent years. Our modelling approach generalizes a whole class of various models that draw attention to different aspects of production, inventory, and recovery. A complete solution in the form of a theorem for that general model class is provided. Furthermore, the paper illustrates how that theorem can be applied to one of the mentioned models from the literature, Jaber and Saadany, 2009. The paper Jaber and Saadany, 2009 extends along this line of research and assumes that demand for manufactured items is different from that for remanufactured (repaired) ones. This assumption results in lost sales situations where there are stock-out periods for Manufactured and remanufactured items. However, in their paper the authors provide solution procedure in the form of algorithm and did not provide the complete explicit solution to this complex problem. We provide the solution in this paper.

Keywords

Reverse Logistics, Remanufacturing, Lot Sizing

1. Introduction

In recent years, reverse logistics has received increasing attention from both academia and industry. There is increasing recognition that careful management can bring both environmental protection and lower costs: environmental and economic considerations have led to manufacturers taking their products back at the end of their lifetimes. As a result, the reverse logistics process is now considered as a basis for generating real economic value as well as supporting environmental concerns.

Rogers and Tibben-Lembke, 2009 defined reverse logistics as the process of planning, implementing, and controlling the efficient, cost-effective flow of raw materials, in-process inventory, finished goods, and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal. The integration of forward and reverse supply chains resulted in the origination of the concept of a closed-loop supply chain. The whole chain can be designed in such a way that it can service both forward and reverse processes efficiently.

A latest actual survey of mathematical inventory models for reverse logistics can be found in Bazan et al., 2016. An extensive survey of research related to quantitative modeling for inventory and production planning in a closed-loop supply chain was provided by Akcalı and Cetinkaya, 2011. According to Akcalı and Cetinkaya, 2011 inventory models are divided into two main categories: deterministic and stochastic, according to the modelling of demand and return processes.

The subject of this paper is deterministic inventory models with constant demand and return. The economic order quantity model (EOQ model), which was derived by Ford W. Harris in 1913, became the basis for many reverse logistics models because of its simplicity and intelligibility. Shady, 1967 was the first to apply the EOQ model to reverse logistics processes. He introduced an EOQ model with instantaneous production and repair rates. A closed-form solution was developed. In his work an efficient policy $P(m, I)$ was established, which means that within each remanufacturing cycle a number m of remanufacturing batches of equal size are followed by exactly one manufacturing batch. This work was extended by Nahmias and Rivera, 1979 and Mabini et al., 1992 extended

Shrady's model to the multi-item case. Koh et al., 2002 analyzed a model similar to that of Shrady, 1967, but with some differences. They considered two types of policies, $P(m, I)$ and $P(I, n)$, under a limited repair capacity, where n is the number of manufacturing batches. They examined the cases of a smaller and a larger recovery rate compared to the demand rate.

Teunter, 2001 generalized the results of Schrady by examining different structures of the remanufacturing cycle. He considered different types of policies by placing the n manufacturing batches and m recovery batches in different orders.

He concluded that the policy $P(m, n), m > I, n > I$ will never be optimal if the above-mentioned m and n are simultaneously larger than one, and that only the two policies $P(I, n)$ and $P(m, I)$ are relevant.

Choi et al., 2007 generalized the $P(m, n)$ policy of Teunter by considering the ordered sequence of manufacturing and remanufacturing batches within the cycle as decision variables. Through sensitivity analysis they found that only 0.2% out of the 8,100,000 tested instances of the model have an optimal solution with both m and n greater than one. Liu et al., 2009 generated and solved 60,000 instances and found that only 0.19% of them have an optimal solution in $P(m, n)$ with both m and n greater than one. Konstantaras and Papachristos, 2008 extended Teunter's approach and found the exact solutions for the optimal numbers m and n .

Richter was the author of a series of papers where he considered an EOQ model with respect to the waste disposal problem. Richter, 1996 proposed an EOQ model that differed from that of Shrady, who assumed a continuous flow of used products to the manufacturer. Richter, 1996 assumed a system of two shops: the first shop provided a product used by a second shop; the first shop manufactures new products and repairs (in contemporary terms--remanufactures) products already used by the second shop and collected there according to some rate; other products are disposed of according to a disposal rate. At the end of a certain time interval the collected items are brought back to the first shop. Richter, 1997 examined the optimal inventory holding policy if the waste disposal (return) rate is a decision variable. The result of this study was that the optimal policy has an extremal property: either reuse all items without disposal or dispose of all items and produce new products; that is, the policy of the type $P(m, n)$ with $m > I$ and $n > I$ is never optimal. He also derived a closed-form for the optimal policy parameters. This analysis of the repair and waste disposal model was continued in the papers by Richter and Dobos, 1999 and Dobos and Richter, 2000.

Dobos and Richter, 2003 and Dobos and Richter, 2004 studied a production/recycling system with constant demand that is satisfied by non-instantaneous production and recycling. They concluded that it is optimal either to produce or to recycle all items that are brought back. Dobos and Richter, 2006 extended their previous work by considering the quality of the returned items.

Saadany and Jaber, 2010 argued that such a pure policy of no waste disposal is technologically infeasible and suggested the introduction of a demand function that depends on two decision variables: purchasing price and acceptance quality level.

Saadany et al., 2012 regarded the assumption that an item can be recovered indefinitely as unrealistic: material degrades in the process of recycling and loses some of its mass and quality, thereby making the option of 'multiple recovery' somewhat infeasible. Saadany et al., 2012 developed a model where an item can be recovered only a finite number of times.

Some authors extended the above-mentioned models to take account of various assumptions. One option is to allow for backorders, where some customers are compensated for having to wait for their delayed orders by either a reduction in price or some other form of discount, which is a cost incurred by the supplying firm. This results in a backorder cost. Konstantaras and Papachristos, 2006 extended the work of Richter, 1996 by allowing for backorders in remanufacturing and production while keeping the other assumptions the same.

In the study of Konstantaras et al., 2010, which extended the work of Koh et al., 2002, a combined inspection and sorting process is introduced with a fixed setup cost and unit variable costs. This study assumes that remanufactured and newly purchased products are sold in a primary market whereas refurbished units are sold in a secondary market.

Konstantaras and Scouri, 2010 considered two models: one with no shortages and the other with shortages. Both models are considered for the case of variable setup numbers of equal sized batches for the production and remanufacturing processes. For these two models, sufficient conditions for the optimal type of policy, referring to the parameters of the models, are proposed.

Saadany and Jaber, 2009 extended the work of Richter, 1996 by assuming that demand for manufactured items is different from that for remanufactured (repaired) items. This assumption results in lost sales situations where there are stock-out periods for manufactured and remanufactured items; that is, demand for newly manufactured items is lost during remanufacturing cycles and vice versa.

Hasanov et al., 2012 extended the work of Jaber and Saadany, 2009 for the full-backorder and partial-backorder cases, where recovered items (remanufactured or repaired) are perceived by customers to be of lower quality; that is, not as good as new items.

More recent papers which have considered inventory and production planning models with constant demand and return on the base of EOQ Guo and Ya, 2015, Bazan et al., 2015.

Guo and Ya, 2015 analyzed the model of the recycled products which are considered with the minimum quality level in manufacturing/remanufacturing system. In this model, a constant demand is satisfied by manufacturing raw materials and remanufacturing recycled products which are up to the quality level.

Bazan et al., 2015 consider energy used for manufacturing and remanufacturing as well as greenhouse gas emissions from manufacturing, remanufacturing and transportation activities with emissions penalty tax as per The European Union Emissions Trading System. The objective of the model is to develop a total cost function that is minimized by determining the following: the manufacturing batch size per cycle, the number of manufacturing batches per cycle, the number of remanufacturing batches per cycle, and the number of times an item may be remanufactured.

However, for some of the above-mentioned models, so far no complete solutions have been presented. In the paper of Saadany and Jaber, 2008 the extended EOQ production, repair and waste disposal model of Richter, 1996 was modified to show that ignoring the first time interval results in an unnecessary residual inventory and consequently an over estimation of the holding costs. They also introduced switching costs in order to take into account production losses, deterioration in quality or additional labour. When shifting from producing (performing) one product (job) to another in the same facility, the facility may incur additional costs, referred to as switching costs, when alternating between production and repair runs. The special case of even numbers m and n was studied and

conditions were provided to decide which of two policies $P(m,n)$ and $P(\frac{m}{2}, \frac{n}{2})$ is preferable, but a general optimal policy for the problem was not presented.

Our modeling approach generalizes a whole class of various other models that draw attention to different aspects of production, inventory, and recovery. A complete solution in the form of a theorem for that general model class is provided. Furthermore, the paper illustrates how that theorem can be applied to one of the mentioned models from the literature.

Our paper is organized in the following way: in the second section, the solution of the general model is presented; in the third section the production, remanufacture and waste disposal model with lost sales (Jaber and Saadany, 2009) and additionally switching costs is solved using general approach and the forth section contains our conclusions.

2. Solution of the general model

The general production and recovery model was considered in the working paper Kozlovskaya et al., 2017. Consider the following two-dimensional nonlinear integer optimization problem:

$$\min_{(x_1, x_2, \dots, x_n)} K(x_1, x_2, \dots, x_n) = \min_{(x_1, x_2, \dots, x_n)} (b_0 + \sum_{i=1}^i b_i x_i) \cdot (a_0 + \sum_{i=1}^n \frac{a_i}{x_i}), \quad (1)$$

$$x_i \in \{1, 2, \dots\}, i = 1, 2, \dots, n.$$

First, let us consider the following continuous auxiliary problem:

$$\min_{(x_1, x_2, \dots, x_n)} K(x_1, x_2, \dots, x_n) = \min_{(x_1, x_2, \dots, x_n)} (b_0 + \sum_{i=1}^i b_i x_i) \cdot (a_0 + \sum_{i=1}^n \frac{a_i}{x_i}), \quad (2)$$

$$x_i \geq 1, i = 1, 2, \dots, n.$$

By analysing the first partial derivatives, we can prove the following lemma:

Lemma. *If $x_i > 0, i = 1, 2, \dots, n$, there are n curves of local minima (2) with respect to x_j :*

$$X_j(x_1, x_2, \dots, x_{j-1}, x_{j+1}, \dots, x_n) = \sqrt{\frac{a_j(b_0 + \sum_{i=1, i \neq j}^n b_i x_i)}{b_j(a_0 + \sum_{i=1, i \neq j}^n \frac{a_i}{x_i})}}$$

and the point of the local minimum

$$x_j^* = \sqrt{\frac{a_j b_0}{a_0 b_j}}, \quad i = 1, 2, \dots, n. \quad (3)$$

Let us denote the radicands of the expressions (3) by

$$A_i = \frac{a_i b_0}{a_0 b_i}, \quad i = 1, 2, \dots, n. \quad (4)$$

Without loss of generality, it is supposed that $A_1 \leq A_2 \leq \dots \leq A_n$.

We denote:

$$B_i(j) = \frac{a_i(b_0 + \sum_{k=1}^j b_k)}{b_i(a_0 + \sum_{k=1}^j a_k)}, \quad i = 1, 2, \dots, n. \quad (5)$$

Then the optimal solution for the continuous problem (1) is provided by the following theorem.

Theorem. *The optimal solution to the problem (1) has the following structure depending on the value of the parameters $A_i, B_i(j)$:*

If $A_i \geq 1, i = 1, 2, \dots, n$, then $x_i = \sqrt{A_i}, i = 1, 2, \dots, n$.

If $A_1 < 1$, then consider $B_2(1), B_3(2), \dots, B_j(j-1), \dots, B_n(n-1)$; if $B_j(j-1) < 1$ and $B_{j+1}(j) \geq 1$ then $x_i = 1, i = 1, \dots, j, x_i = \sqrt{B_i(j)}, i = j+1, \dots, n$.

If $B_n(n-1) < 1$, then $x_i = 1, i = 1, \dots, n$.

In the paper Kozlovskaya et al., 2017, the problem of Saadany and Jaber, 2008 was solved by using this approach. The theorem can be used for the solution of more complicated models, with more activities and more stock points. More detailed analysis can be found in working paper Kozlovskaya et al., 2015. In the next section, the application of the theorem will be demonstrated for the model Jaber and Saadany, 2009.

3. Solution of the Production, Remanufacture and Waste Disposal Model with Lost Sales

In this section, the above discussed methodology is applied to the model Jaber and Saadany, 2009.

3.1. Assumptions

1. A single product case with two different qualities.
2. Production and recovery rates are instantaneous.
3. Demand rates for produced and remanufactured items are known, constant and of different values.
4. Collection rates for previously produced and remanufactured items are known, constant and of different values.
5. Lead time is assumed to be zero.
6. Inventory stock-out occurs with unsatisfied demand (newly produced items or used/repaired items) lost.
7. Unlimited storage capacity is available.
8. Planning horizon is infinite.

3.2. Notations

Decision variables:

n - number of production cycles in an interval of length T .

m - number of remanufacturing cycles in an interval of length T .

γ_p - collection percentage of available returns of newly produced items ($0 < \gamma_p < 1$).

γ_r - collection percentage of available returns of previously remanufactured items ($0 < \gamma_r < 1$).

Input parameters:

D_p - demand rate for newly produced items (units/ unit of time).

D_r - demand rate for remanufactured items (units/ unit of time), where D_r is not necessarily equal to D_p .

P - total switching cost for a production and a remanufacturing cycle (\$).

S_p - setup cost for a production cycle (\$).

S_r - setup cost for a remanufacturing cycle (\$).

h_p - holding cost per unit per unit of time of a produced item (\$/unit/unit of time).

h_r - holding cost per unit per unit of time of a remanufactured item (\$/unit/unit of time).

h_u - holding cost per unit per unit of time of a used item (\$/unit/unit of time).

c_p - cost per unit of a lost demand for a produced item (\$/unit).

c_r - cost per unit of a lost demand for a remanufactured item (\$/unit).

β_p - percentage of available returns from the primary market for produced items.

β_r - percentage of available returns from the secondary market for remanufactured items ($0 < \beta_r \leq \beta_p < 1$). Note that $1 - \beta_r$ and $1 - \beta_p$ are the waste disposal rates.

Decision variables dependent parameters

x_2 - lot size quantity (in units) to be remanufactured/ repaired in an interval of length T .

x_1 - lot size quantity (in units) to be produced in an interval of length T .

T_p - length of a production interval (units of time), where $T_p = x_1 / D_p$.

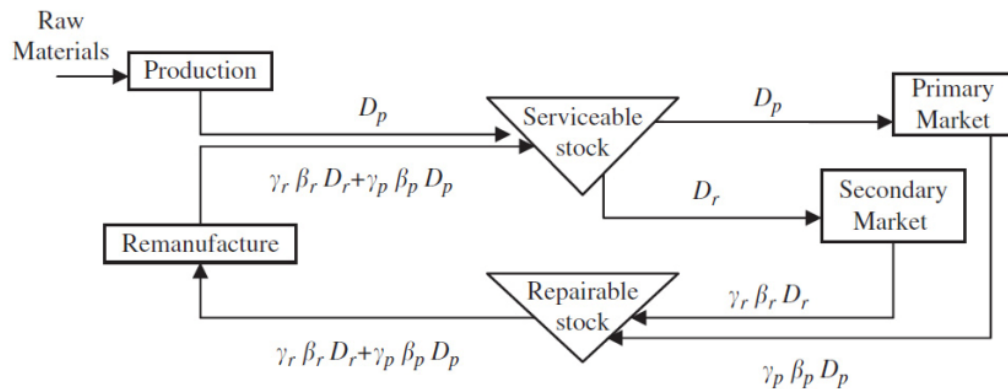


Figure 1. Material flow for a production and remanufacture system.

3.4. Problem statement

The paper Jaber and Saadany, 2009 considers the Richter's model (Richter, 1996) except for the assumption that remanufactured (repaired) items are not perceived by customers to be of the same quality as newly produced items. There are therefore a primary market and a secondary market for produced and remanufactured items respectively. In an interval of length T , there are two sub-intervals representing remanufacturing and production of lengths T_r and T_p , respectively. Each repair interval consists of m remanufactured batches of size x_2 / m each, while a production interval consists of n production batches of size x_1 / n each. The total remanufactured and production quantities per interval T are determined respectively as $x_2 = D_r T_r$ and $x_1 = D_p T_p$. Used items are collected at rates $\gamma_r \beta_r$ over T_r and $\gamma_p \beta_p$ over T_p accumulating $x_2 = D_r T_r = \gamma_r \beta_r D_r T_r + \gamma_p \beta_p D_p T_p$ units. Accordingly, $x_1 / x_2 = (1 - \gamma_r \beta_r) / (\gamma_p \beta_p)$, where x_1 / x_2 is the ratio of produced to remanufactured units. Material flow for a

production and remanufacture system is illustrated in Fig.1. The paper Jaber and Saadany, 2009 considers two cases for lost sales. The first case assumes that demand for newly manufactured (produced) items are lost over T_r and that demand for remanufactured items are lost over T_p . This case is referred to as the total lost sales case. The second case assumes that it may be possible to entice some customers to settle for a remanufactured (manufactured) item at a cost, this case is referred to as the partial lost sales case. Two mathematical models are developed accordingly. In the paper Jaber and Saadany, 2009 the algorithm for solution was proposed for solution of this model, but strict formulas weren't derived. Consider the first model – total lost sales case. The behavior of inventory for remanufactured produced and collected used items over interval T is represented in Fig.2. The mathematical programming problem was derived Jaber and Saadany, 2009:

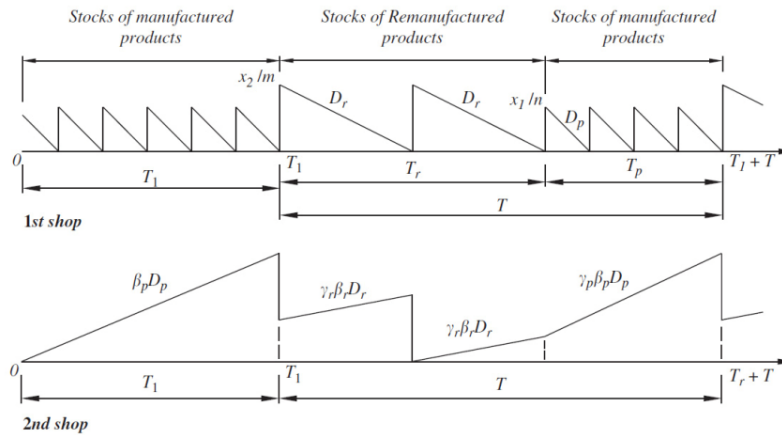


Figure 2. The behavior of inventory for remanufactured produced and collected used items over interval T .

$$\min \psi(n, m) = A(2\sqrt{(mS_r + nS_p)H(n, m)} + C_{pr}), \quad (6)$$

$$m, n \geq 1$$

where

$$A = \frac{1}{\frac{1 - \gamma_r \beta_r}{\gamma_p \beta_p D_p} + \frac{1}{D_r}}$$

$$H(n, m) = \frac{h_r}{2mD_r} + \frac{h_p}{2nD_p} \left(\frac{1 - \gamma_r \beta_r}{\gamma_p \beta_p} \right)^2 +$$

$$+ h_u \left[\frac{m(\gamma_r \beta_r - 1) + 1}{2mD_r} + \frac{\gamma_p \beta_p}{2D_p} \left(\frac{1 - \gamma_r \beta_r}{\gamma_p \beta_p} \right)^2 + \left(\frac{\gamma_r \beta_r}{mD_p} + \frac{\gamma_p \beta_p (m-1)}{mD_r} \right) \left(\frac{1 - \gamma_r \beta_r}{\gamma_p \beta_p} \right) \right] \quad (7)$$

$$C_{pr} = \frac{c_r D_r}{D_p} \left(\frac{1 - \gamma_r \beta_r}{\gamma_p \beta_p} \right) + \frac{c_p D_p}{D_r}$$

3.5. Problem solution

The minimum value of the function (6) could be found using Theorem. We obtain the coefficients of the model in the following form:

$$\begin{aligned}
 b_2 &= S_p, b_1 = S_r, b_0 = 0, \\
 a_0 &= \frac{1}{2}h_u(1-\gamma_r\beta_r)\left(\frac{1}{D_p} + \frac{1-\gamma_r\beta_r}{\gamma_p\beta_p} + \frac{1}{D_r}\right) \\
 a_1 &= \frac{h_r-h_u}{2D_r} + h_u\gamma_r\beta_r\left(\frac{1}{D_r} + \frac{1-\gamma_r\beta_r}{\gamma_p\beta_p} + \frac{1}{D_p}\right) \\
 a_2 &= \frac{h_p}{2D_p}\left(\frac{1-\gamma_r\beta_r}{\gamma_p\beta_p}\right)^2
 \end{aligned} \tag{8}$$

We have $A_1 = A_2 = 0$. From the theorem follows that the solution has the form: $(m, 1)$, if $C = \frac{a_1(b_0 + b_2)}{b_1(a_0 + a_2)} \geq 1$ or

$(n, 1)$, if $D = \frac{a_2(b_0 + b_1)}{b_2(a_0 + a_1)} \geq 1$, or $(1, 1)$, if both $C < 1$ and $D < 1$, where $m = \lceil \sqrt{C} \rceil$, $n = \lceil \sqrt{D} \rceil$.

In this section, we consider more general cases of policies, taking into account switching costs. When shifting from producing (performing) one product (job) to another in the same facility, the facility may incur additional costs, referred to as switching costs, when alternating between production and repair runs. Suppose that switching costs are positive: $P > 0$. Then the mathematical programming problem (6) has the following form:

$$\begin{aligned}
 \min \psi(n, m) &= A(2\sqrt{(P + mS_r + nS_p)H(n, m)} + C_{pr}), \\
 m, n &\geq 1
 \end{aligned}$$

with $b_0 = P$ and $A_1 \neq 0, A_2 \neq 0$.

Without loss of generality let $0 < A_1 < A_2$. In this case the solution can have different structure. If $1 < A_1 < A_2$, the solution has the form (m, n) , where $m = \lceil \sqrt{A_1} \rceil$, $n = \lceil \sqrt{A_2} \rceil$. If $A_1 \leq 1 < A_2$, then consider $B_2(1) = \frac{a_2(b_0 + b_1)}{b_2(a_0 + a_1)}$. If

$B_2(1) > 1$, the solution has the form $(1, n)$, where $n = \lceil \sqrt{B_2(1)} \rceil$. If $B_2(1) \leq 1$, the solution has the form $(1, 1)$. The case $0 < A_2 < A_1$ can be solved by the same way.

The impact of the switching cost becomes apparent for sufficiently high values. In this case the optimal numbers (m, n) can both be greater than one. It is illustrated in the next section.

3.6. Numerical analysis

The input parameters for numerical analysis are represented in the Table 1. Each of the model parameters has been set to vary in a range, which are represented in the Table 1.

Table 1. The input parameters for the numerical analysis.

	D_p	D_r	S_p	S_r	h_p	h_r	h_u	c_p	c_r	β_p	β_r	γ_p	γ_r	P
Max	1000	1000	500	500	100	100	100	100	100	0,8	0,8	0,99	0,99	500
Min	100	100	5	5	1	1	1	10	10	0,1	0,1	0,01	0,01	5

The sets of parameters $(D_p, D_r, S_p, S_r, h_p, h_r, h_u, c_p, c_r, \beta_p, \beta_r, \gamma_p, \gamma_r, P)$ for 10000 examples were randomly generated. When generating the h_p, h_r, h_u , the constraint $h_p > h_r > h_u$ was respected. When generating the c_p and c_r , the constraint $c_p > c_r$ was respected. The results confirmed that the $P(1, n)$ is optimal over 3307 examples (50,8%), $P(1, 1)$ over 3128(48,1%), $P(m, 1)$ over 43 (0,7%), $P(m, n)$ over 30(0,5%); some more results are displayed in Table 2.

Table 2. Results of the numerical analysis.

Switching Cost	$P(n, m)$	$P(1, m)$	$P(n, 1)$	$P(1, 1)$	Total
$P = 0$	0	339	7298	2363	10000
$P > 0$	2373	262	6672	693	10000

4. Conclusion

Faster generation of waste leads to rise of collecting and remanufacturing cores to extend their useable lives and thus reduce waste. Economical incentives enticed and later governmental legislations compelled companies to initiate product recovery programs. European waste legislation currently gives a global framework for the implementation of extended producer responsibility in Europe. Inventory and production planning focuses on the effective utilization of existing inventories and production resources, therefore managing inventory in reverse logistics represents an important problem. The focus of this paper is on the mathematical modeling of inventory with return flows that were developed on the base EOQ. The general objective of the inventory management models is to control product orders, inventory levels, and recovery processes to minimize total costs.

Our modeling approach generalizes a whole class of various models that draw attention to different aspects of production, inventory, and recovery. A complete solution in the form of a theorem for that general model class is provided. Furthermore, the paper illustrates how that theorem can be applied to one of the mentioned models from the literature, Jaber and Saadany, 2009. In their paper the authors provide solution procedure in the form of algorithm and did not provide the complete explicit solution to this complex problem. We provide the solution in this paper. We found the optimal policy $P(m; n)$ for the problem, it can have a different structure depending on the value of the parameters. The impact of the switching cost becomes apparent for sufficiently high values. In this case the optimal numbers (m, n) can both be greater than one. This was illustrated by the numerical analysis.

The consideration of the government regulations related to extend producer responsibility instruments such as minimum recovery target and disposal fee is the future direction of this research. A legislative environment will be considered, where the government requires the producers to collect and recover at least a certain percentage of their sales and imposes a disposal fee on each product not recovered up to this quota

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