Dynamic Asset Allocation using Elastic Asset Allocation

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Abstract

When Markowitz proclaimed in his research paper, there were many crises in economic system. The majority of economist and quant had suggested their own model of portfolio and theories. Our research suggested a new portfolio which is named 'T&M' using elastic asset allocation, diversified investment, and real momentum. Using Kelly Criterion, T&M portfolio's efficiency and stability were verified. Market indexes used in T&M portfolio are US 10 Year T-Note Futures, US 30 Year T-Bond Futures, S&P 500, Canadian S&P 500,S&P/TSX Composite, DAX, Hang Seng, KOSPI, USD/KRW(Forex), and Gold Future. The data was used from Jan 1st, 1988 to Sep 1st, 2016. The period verified was computed from 1989, monthly. Microsoft Excel 2013's inner Function was used as a main analysis program. Finally, T&M Portfolio increased by about 15.503%(CAGR), and MDD decreased by about 6.3%. With this, this study verified he high stability and efficiency of elastic asset allocation.

Both of two students got the data for completion of study. Taewan Kim had conducted for comparing, sorting data, Back-Test, writing about elastic asset allocation and Kelly Criterion, and Mihyeop Lee had conducted for writing the stock market, momentum, standard deviation, theory of portfolio, introduction and conclusion.

Keywords
Elastic Asset Allocation, Diversified Investment, Kelly Criterion

1. Introduction

Since Makowitz(1952) presented the theory of portfolio and diversified investment, many researches have been studied. In the financial market, the value of theory of portfolio and diversified investment takes up a large proportion. In this paper, two researchers designed T&M portfolio that applied strategy of Elastic Asset Allocation and momentum. Moreover, Kelly Criterion Ratio of T&M portfolio was calculated because the value should be verified. The T&M portfolio uses the low correlation coefficient, monthly rebalancing, and momentum. The data includes the Hang Seng(HSI), Nifty 50(NSEI), DAX(GDAXI), S&P500(SPX), S&P/TSX Composite (GSPTSE), US 10 Year T-

Note Futures, US 30 Year T-Bond Futures, and KOPSI(KS11). Market indexes used in T&M portfolio are US 10 Year T-Note Futures, US 30 Year T-Bond Futures, S&P 500, Canadian S&P 500, S&P/TSX Composite, DAX, Hang Seng, KOSPI, USD/KRW(Forex) and Gold Future. The data from Jan 1, 1998 to Sep 1, 2016 was used for the research. Also, allocated investment ratio was in proportion to generalized momentum score. The correlation coefficient was calculated by covariance, and the data used in the study was provided at www.investing.com.

2. Body

2.1 Kelly Criterion

We assume there is stock when capital is 1. We will get two-point profit or loss and probability is 0.5. For example, when the man who invested his money to the stock predicted exactly future's price, the expected profit is priced by how much the man invested. In other words, an investor will invest his total capital at the stock because there are not any risks if he exactly knows the price. In this case, after times were invested, capital got 2^N times profit. In addition, his compound profit will be 100% up at each trade. G of the [Equation 1] represents investor's geometric profit. V_N stands for capital after N times were invested, and base of the log function is 2. If investor's prediction is wrong, new method should be considered. The probability stands for p, and opposite probability stands for q. This case is related to investment of the total capital at [Equation 2]. However, as frequency of N increases, the probability of bankruptcy will get higher. If N faces to infinity, the probability of bankruptcy is 1. When an investor invests part l of his capital, the formula is used at [Equation 3]. This symbol W stands for a frequency of profit, and L stands for a frequency of loss. [Equation 4] is based on [Equation 1], [Equation 2], and [Equation 3].

$$\mathrm{G} = \lim_{N\to\infty} \frac{1}{N} \cdot \log_2 \frac{V_N}{V_0}$$
[Equation 1]

$$V_N = (2q)^N V_0$$
[Equation 2]

$$V_N = (1+l)^W (1-l)^L V_0$$
[Equation 3]

$$\mathrm{G} = \lim_{N\to\infty} \left[\frac{W}{N}\log(1+l) + \frac{W}{N}\log(1-l)\right]$$
$$= q\log(1+l) + p\log(1-l)$$
[Equation 4]

At this moment, if using l which maximizes G, the investor is far superior to any other investors keeping in this pace. Gaul Sin (2015) defined signs on her paper.
$\mathrm{p(s)}$: Probability s^{th} sign that transmitted.
$\mathrm{p}(r/s)$: Suppose that s^{th} sign transmitted, conditional probability that transmitted r^{th} sign.
$\mathrm{p(s,r)}$: Probability of sth sign transmitted, and r^{th} sign received.
$\mathrm{q(r)}$: Probability of sign transmitted.
$\mathrm{q(s/r)}$: Suppose that the sign transmitted, conditional probability that transmitted sign.
a_s : Profit per 1
$\mathrm{a(s/r)}$: Invested amount at s^{th} sign transmitted.
A stock market has a lot of signs and complicated conditions must be considered deductive amount. It cannot suppose that $\sum_s a(s/r) = 1$, and capital is checked at [Equation 5] when r^{th} is received. In order to maximize G of [Equation 6], r should be maximized independently as [Equation 7] shown. *G* of [Equation 8] is an exponential growth rate. In this study, when maximizing G, a capital growth rate gets more maximized than any other invest ratios.

$$b_r = 1 - \sum_s a(s/r)$$
[Equation 5]

$$\mathrm{G} = \sum_r \sum_s p(s,r) \log\left[b_r + a_s a\left(\frac{s}{r}\right)\right] \ (b_r + \sum_s a(s/r) = 1)$$

[Equation 6]

$$G_r = \mathrm{q}(r) \sum_r \sum_s q(s,r) \log[b_r + a_s a(s/r)]$$

[Equation 7]

$$\mathrm{G} = \sum_s p(s) \log[b + a_s a(s)]$$

[Equation 8]

2.2 Portfolio Theory

2.2.1 Markowitz Portfolio

When there is no accurate information to gain profit, Markowitz Portfolio do diversified investment at several indexes. The reason why a purchaser invests at index A and index B is to prepare that one index's fall and one index's rise. Making a profit by diversified investment at 2 or more index is a basic definition of Markowitz Portfolio. Quant Investment constitutes portfolio which ranks 20-30 indexes by company's value, supply and technical indexes.

2.2.2 Elastic Asset Allocation

Elastic Asset Allocation is a strategy for allocating assets based on the correlation, volatility, and return rate of each asset, and allocating the asset proportionally. In this case, the investor can construct the optimal portfolio by considering the weight on the correlation, the volatility, and the yield. The following is a strategy formula for Elastic Asset Allocation.

$$w_i \sim z_i = \frac{(r_i)^{\omega R} \cdot (1 - c_i)^{\omega C}}{(v_i)^{\omega V}} \ (i, j = 1 \ldots N)$$

[Equation 9]

If the value of r_i is less than 0, the value of w_i and z_i are 0. The symbol '∼' indicates that it is proportional. The following is a description of each symbol.

z_i : The generalized momentum score of asset i, based on the EAA formula.

w_i : The investment amount of the asset i proportional to score z_i $\left(w_i \geq 0, \sum w_j = 1\right)$

r_i : Return of assets i.

v_i : Volatility of assets i.

c_i : Correlation coefficient between equally weighted portfolio and assets i.

$\omega\mathrm{R}, \omega\mathrm{C}, \omega\mathrm{V}$: Weight value

The modified Elastic Asset Allocation strategy presented in this paper is as follows.

$$w_i \sim z_i = \frac{(m_i)^{\omega M} \cdot (1 - c_i)^{\omega C}}{(v_i)^{\omega V}} \ (i, j = 1 \ldots N)$$

[Equation 10]

If the value of r_i is less than 0, the value of w_i and z_i are 0. The symbol '∼' indicates that it is proportional. The following is a description of each symbol.

m_i : Momentum score of assets i.

$\omega\mathrm{M}, \omega\mathrm{C}, \omega\mathrm{V}$: Weight value

We replaced the existing r_i by m_i, m_i represents the average momentum score from 1 month to 12 months. Assuming that the annual return of cash assets is 3%, we calculate the average by recognizing relatively higher momentum scores than cash assets. The score of the asset with a momentum score was lower than the cash asset's momentum score which is calculated as zero. In this study, we compensated the shortcoming of the trend following strategy by removing the market noise using m_i. In a basic portfolio, we set all weighting variables to 1 and calculated the momentum values and investment ratios that are generalized under the same conditions for all variables.

2.3 T&M Portfolio

The portfolio consists of nine representative indices and cash assets assuming a risk-free rate of return of 3%. Nine indices are Hang Seng(HSI), Nifty 50(NSEI), DAX(GDAXI), S&P500(SPX), S&P/TSX Composite (GSPTSE), US 10 Year T-Note Futures, US 30 Year T-Bond Futures, and KOPSI(KS11). Market indexes used in T&M portfolio are US 10 Year T-Note Futures, US 30 Year T-Bond Futures, S&P 500, Canadian S&P 500, S&P/TSX Composite, DAX, Hang Seng, KOSPI, USD/KRW(Forex) and Gold Future. The data from Jan 1, 1998 to Sep 1, 2016 was used for the research. Testing period was started from 1989, and we calculated momentum score monthly. Inner function on the Microsoft Excel 2016 was used for the analysis. The CAGR and the maximum drawdown were used to evaluate the portfolio.

3. Results

3.1 Comparison of Cash Ratio

3.1.1 Investment ratio of cash asset 0%, leverage x1

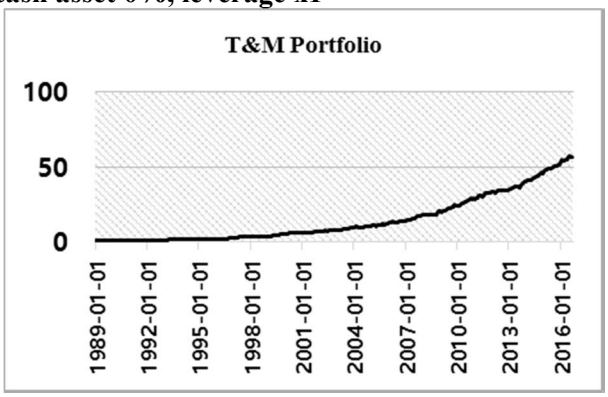

Figure 1. Cash asset 0%, leverage x1
CAGR: about 15.503% change and MDD: about −6.385% change.

3.1.2 Investment ratio of cash asset 25%, leverage x1

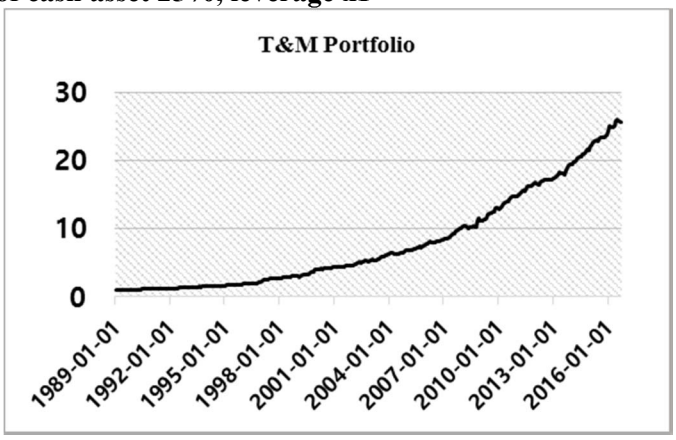

Figure 2. Cash asset 25%, leverage x1
CAGR: about 12.293% change and MDD: about −4.727% change.

The maximum loss was recorded in August 1997. Although the financial crisis occurred in Southeast Asia in 1997, the portfolio presented in this study showed a small loss even in this crisis. The average of the generalized momentum scores of government bonds in this period was 16.58 as the highest for the 10 Year Bond and 14.86 as the second

highest for the 30 Year Bond, the second highest. The relatively safe ratio of government bonds has increased and has not been significantly affected by the financial crisis in Southeast Asia. When the results of back-testing in a comprehensive manner were conducted, we can confirm that the portfolio has a low volatility and stable profitability.

3.2 Comparison of Portfolio Leverage

3.2.2 Investment ratio of cash assets 0%, leverage x2

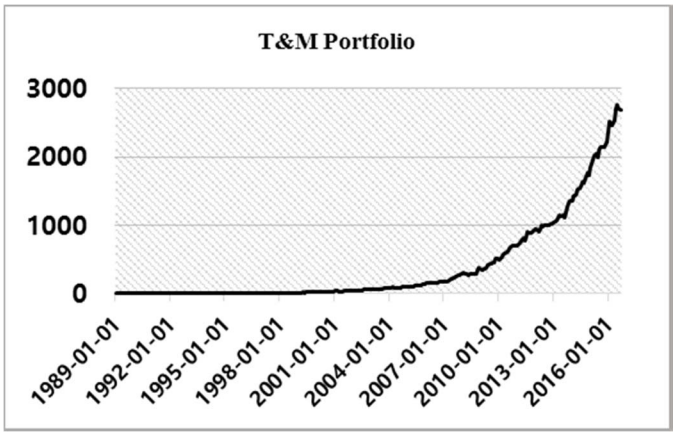

Figure 3. Cash assets 0%, leverage x2
CAGR: about 32.557% change and MDD: about −12.771% change.

3.2.3 Investment ratio of cash assets 25%, leverage x2

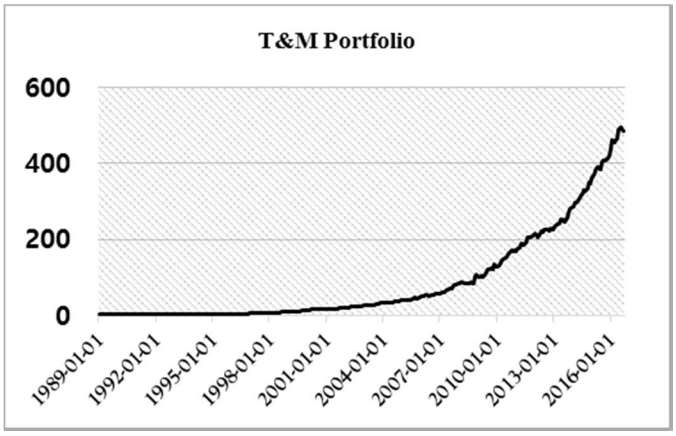

Figure 4. Cash assets 25%, leverage x2
CAGR: about 24.722% change and MDD: about −9.516% change.

3.2.4 Simple Momentum Portfolio Back-Testing Results

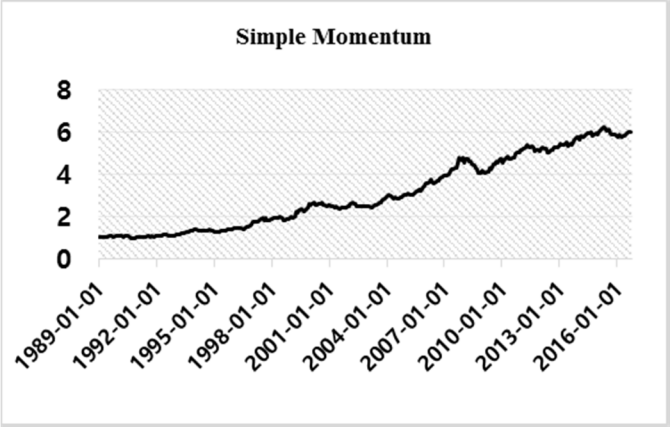

Figure 5. Simple Momentum
CAGR: about 6.739% change and MDD: about −15.645% change.

The following is a summary of the back test results of the portfolio presented above.

3.2.5 Kelly Criterion Ratio

Kelly Criterion Ratio was calculated based on investment ratio of cash assets 0%, leverage x1. The value was measured to be about 15.21. The following is a graph showing profits by investment ratio.

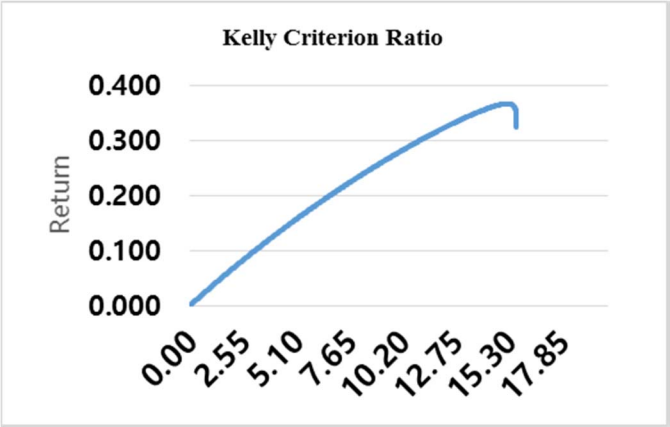

Figure 6. Kelly Criterion ratio

The following is the cumulative profit graph of the cash 0% portfolio with leverage x15.21 according to the Kelly Criterion Ratio.

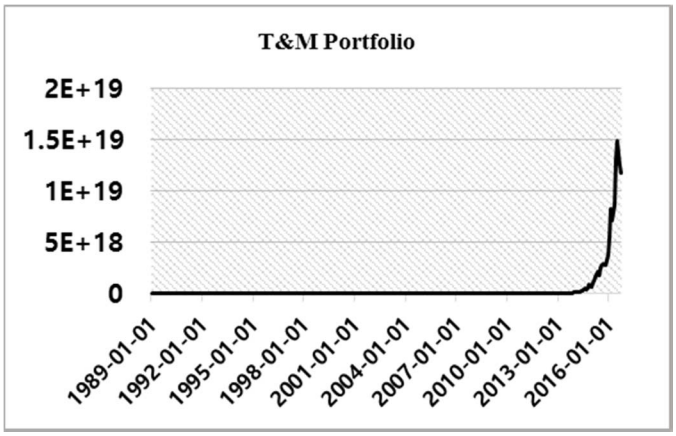

Figure 7. Cash 0% portfolio with leverage x15.21
CAGR and MDD were measured to be about 379.861%, and -97.12%.

The following is the cumulative profit graph of the cash 0% portfolio with leverage x7.605, half of Kelly Criterion ratio.

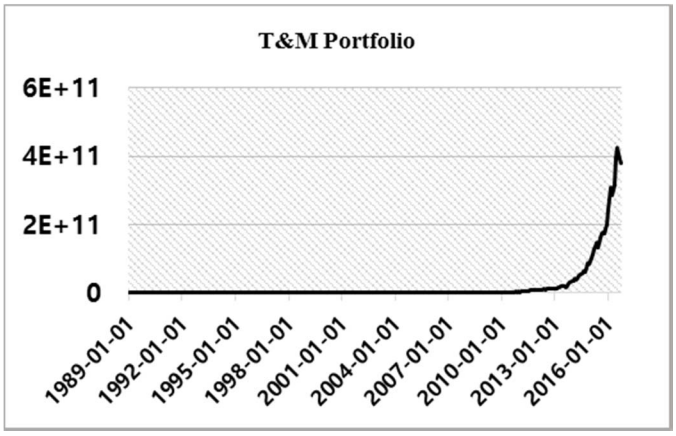

Figure 8. Cash 0% portfolio with leverage x7.605

3.2.6 Portfolio that changes the weight of $\omega M, \omega C, \omega V$

The following is the cumulative profit graph of the cash 0% portfolio with $\omega M = 2$.

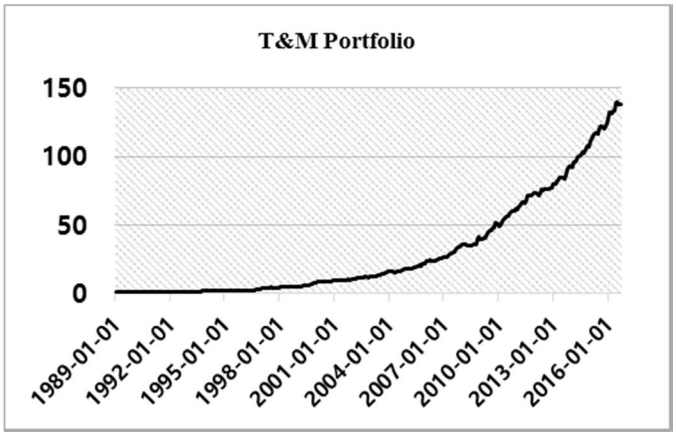

Figure 9. Set $\omega M = 2$

CAGR: about 19.243% change and MDD: about −15.645% change. The following is the cumulative profit graph of the cash 0% portfolio with $\omega C = 2$.

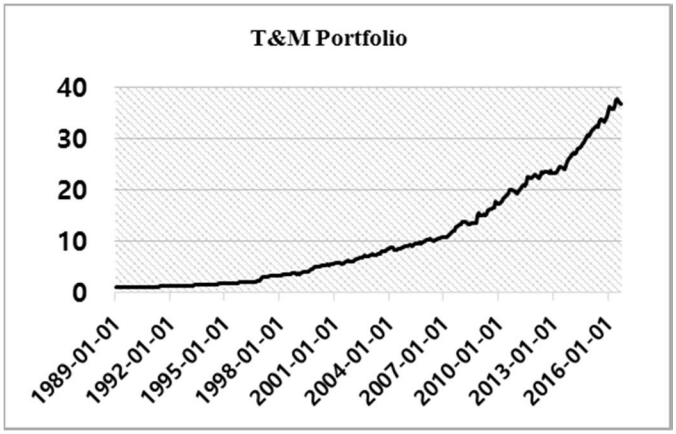

Figure 10. Set $\omega C = 2$

CAGR: about 13.746% change and MDD: about −6.654% change. The following is the cumulative profit graph of the cash 0% portfolio with $\omega V = 2$.

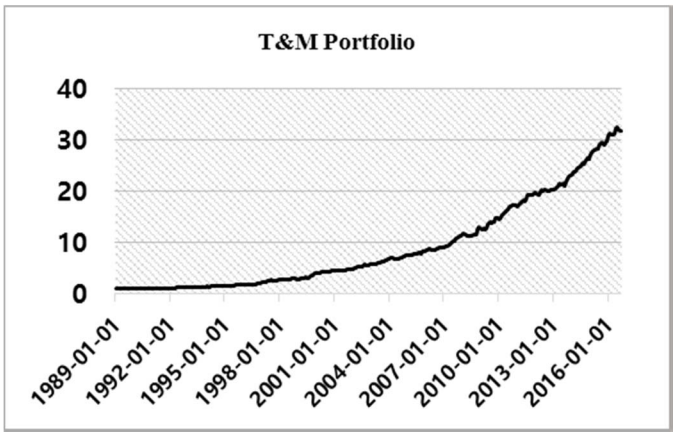

Figure 11. Set $\omega V = 2$

CAGR: about 13.155% change and MDD: about −4.975% change. The following table summarizes the backtesting results of the portfolio presented above.

Table 1. Summary by setting value

Types of portfolios	CAGR(Compound Annual Growth Rate)	MDD(Maximum Drawdown)
Cash assets 0%, Leverage x1 portfolio: P1	15.503%	−6.385%
Cash assets 25%, Leverage x1 portfolio: P2	12.293%	−4.727%
Cash assets 0%, Leverage x2 portfolio: P3	32.557%	−12.771%
Cash assets 25%, Leverage x2 portfolio: P4	24.722%	−9.516%
Simple Momentum Portfolio: P5	6.739%	−15.645%

Cash assets 0%, Leverage x15.21 portfolio: P6	379.861%	−97.12%
Cash assets 0%, Leverage x7.605 portfolio: P7	159.123%	−48.56%
$\omega M = 2$ Portfolio: P8	19.243%	−7.19%
$\omega C = 2$ Portfolio: P9	13.746%	−6.654%
$\omega V = 2$ Portfolio: P10	13.155%	−4.975%

4. Conclusion

The newly developed T&M portfolio using volatility, correlation coefficient, and momentum strategy is a portfolio that focuses on profitability while minimizing the proportion of cash. The core of the T&M portfolio proposed in this study rebalances at the beginning of each month. Rebalancing is to balance the portion of highly valued assets with those of undervalued assets and takes into account the profits and losses of the group of assets that generated the losses. The rebalancing has been incorporated into the T&M portfolio.

The T&M portfolio uses the low correlation coefficient, monthly rebalancing, and momentum. The data includes the Hang Seng(HSI), Nifty 50(NSEI), DAX(GDAXI), S&P500(SPX), S&P/TSX Composite (GSPTSE), US 10 Year T-Note Futures, US 30 Year T-Bond Futures, and KOPSI(KS11). Market indexes used in T&M portfolio are US 10 Year T-Note Futures, US 30 Year T-Bond Futures, S&P 500, Canadian S&P 500, S&P/TSX Composite, DAX, Hang Seng, KOSPI, USD/KRW(Forex) and Gold Future. The data from Jan 1, 1998 to Sep 1, 2016 was used for the research. According to the results, T&M Portfolio rebalances ratio at every single month. Using the momentum score, it was calculated through comparison of non-risk cash asset(CAGR 3%) to get a proper ratio, and T&M Portfolio showed less loss and stable profit. For example, simple momentum portfolio had got a lot of loss during 2008 financial depression, but T&M Portfolio recovered the loss by bond and gold asset. When T&M Portfolio set the ratio of cash asset high, both CAGR and MDD got a loss. When another way set the ratio of cash asset low, CAGR and MDD got higher. Also, multiplying the leverage can get more profits at a short term, but MDD got higher to increase of volatility. The conclusion drawn from this study is the ratio of multiplying leverage and cash asset which must increase at the same time. Up to now we have tested the profit depending on ωM, ωC, and ωV. In conclusion: the research framework and research methods were as follows: (1) when ωM increases, both CAGR and MDD increase. (2) when ωV increases, both CAGR and MDD decrease. (3) It is the worst result that makes ωC increase. The biggest limitation of this study is the deduction. Each stock firm in countries shows different deduction.

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Biography.

Taewan Kim is a high school student at Paul Math school. The researcher is interested in financial engineering, asset allocation, Kelly criterion, future market and data science field. He had researched about 'Application to Kelly Criterion in KOSPI 200 Futures Market' and 'Technical Analysis Indicators Prognosis Earning Rate Analysis in the KOSPI Market'. He presented the poster 'Technical analysis indicators and candlestick chart prognosis earning rate analysis in the KOSPI market' at The Korean Society of Mathematical Education.

Mihyeop Lee is a high school student at Paul Math school. The researcher is interested in mathematics, finance, industrial engineering, risk management, and Kelly criterion field. He had researched about 'Application to Kelly Criterion in KOSPI 200 Futures Market', 'Technical Analysis Indicators Prognosis Earning Rate Analysis in the KOSPI Market' at The Korean Society of Mathematical Education. He had presented the poster 'Technical Analysis Indicators Prognosis Earning Rate Analysis in the KOSPI Market'.

Daeryong Seo is a mathematics teacher who is charge of mathematics researcher class at Paul Math school. The researcher is interested in mathematics, computer science and educational technology. His undergraduate major is mathematics education, and his master degree is math & computer science. Moreover, his doctoral degree is education al technology specializing in curriculum, instruction, and media technology.