Analysis of Natural Convection Heat Transfer from Slender Body in a Channel

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Abstract

For the purpose of heating and cooling, natural/free convection is one of the cheapest methods. When fluid flows inside a channel and if there remains a temperature difference between fluid and solid surface, heat transfer takes place by convection heat transfer mechanism. After taking heat from the heated object, fluid density decreases and it lifts up due to buoyancy force. Thus a vacancy is created which is filled up by relatively cooler fluid with relatively larger density. In our analysis, the flow is considered two dimensional, steady-state, incompressible, laminar and all physical properties are assumed to be constant through the channel with a heated slender body. Effects of Rayleigh number, Prandtl number, locations of slender body and aspects ratios have been observed in the paper. Maximum temperature occurs at the vicinity of slender body. The average Nusselt number slowly decreases with the increase of Rayleigh Number but the average Nusselt number remains constant for the range of Prandtl Number investigated. Slender body average Nusselt number decreases initially but becomes minimum at the middle portion of the channel and then increases when the slender body is moved from location 1.0 to 6.0.

Keywords
Natural Convection, Rayleigh and Prandtl Number, Slender Body, Aspects Ratio, Nusselt Number

1. Introduction

Free convective heat transfer in flow through a channel is of great interest recent days because of its frequent practical applications. Although the value of the coefficient of free convective heat transfer is relatively low, many devices depend largely on this type of cooling. Heat is trapped in thermal devices for various reasons. Due to lack of efficient cooling of heat entrapped in the thermal devices, they can under- perform or even breakdown under certain condition. The ever increasing miniaturization, packaging density, quality demands and reduction in life cost will put ever increasing pressure on the solution of these problems. The solution of such heat transfer depends on successfully solving Navier-Stokes equation along with the energy equation. The specific problem dealt with here is dependent on a number of parameters, an optimization of which is necessary to yield reasonably accurate result.

Cebeci et al. (1974) studied free convection heat transfer from slender cylinders which was subjected to uniform wall heat flux. Heat transfer from such surfaces was investigated by the solution of a set of partial differential equations with different boundary conditions and Prandtl number. For such flows, heat transfer largely depends on the slenderness of the cylinder or transverse curvature parameter. Na et al. (1980) studied the difficulties associated with laminar free convection flow over a slender frustrum of a cone with constant wall heat flux. Numerical solutions were obtained by finite-difference method for a variety of prandtl number (0.1 to 100) and transverse curvature parameter. It was found that transverse curvature parameter has great significance in such flows. Turki et al. (2003) performed a numerical investigation for analyzing the unsteady flow field and heat transfer characteristics in a horizontal channel with a built-in heated square cylinder. Hydrodynamic behavior and results of heat transfer were obtained by solving the complete Navier–Stokes and energy equations utilizing a control volume finite element method (CVFEM) which was adapted to the staggered grid.

Alami et al. (2005) studied the numerical solution of free convection from a two dimensional horizontal channel with rectangular heated blocks and investigated its effect on electrical component cooling. Block spacing and mass flow rate were varied. It was found that flow structure and heat transfer depends significantly on control parameters. Bakkas et al. (2006) studied two-dimensional laminar free convection in a horizontal channel. The upper wall was maintained cold at a fixed temperature and the lower wall contained rectangular heated blocks. The study was performed by using air as working fluid (Pr=0.72) and spacing between blocks was maintained constant (C=0.5). The relative height of the blocks and Rayleigh number were varied. Different flow structures were obtained based upon the length of the computational domain and other governing parameters. Kader et al. (2013) studied fluid flow and heat transfer behavior in the very large enclosure with heated object at the bottom. The two dimensional Continuity, Navier-Stokes equation and Energy equation were solved by the finite difference method. It was observed that the average heat transfer remains constant for higher values of Rayleigh number and heating efficiency varies with Rayleigh number up to the value of Ra=35 and beyond this value heating efficiency remains constant.

The purpose of this paper is to study the fluid flow and heat transfer characteristics at different Rayleigh and Prandtl numbers. Fluid flow and heat transfer characteristics are analyzed for different aspect ratios. Effect of locations of slender body on fluid flow and heat transfer characteristics in a channel are also discussed in this paper.

2. Mathematical Formulation

2.1. Approximations

The flow is considered two dimensional, steady state, incompressible, laminar and all physical properties are assumed to be constant through the channel with a heated slender body. The body forces are neglected and viscous dissipation are also neglected.

2.2. Governing Equations

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Navier-Stokes equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+v\left(\frac{\partial^2\text{-}u}{\partial x^2}+\frac{\partial^2\text{-}u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+v\left(\frac{\partial^2\text{-}v}{\partial x^2}+\frac{\partial^2\text{-}v}{\partial y^2}\right)+g\beta(T-T_\propto)$$

Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}+\left(\frac{\partial^2\text{-}T}{\partial x^2}+\frac{\partial^2\text{-}T}{\partial y^2}\right)$$

2.3. Boundary Conditions

Boundary conditions at top wall, bottom wall and heated slender body: The velocity components (u and v) are zero at the wall and heated slender body. We know that the velocity components are –

$$u=\frac{\partial\psi}{\partial y};v=-\frac{\partial\psi}{\partial x}$$

Applying no slip conditions, $u=0\ and\ v=0$
So, stream function, ψ = constant

The vorticity at the top wall, bottom wall and heated slender body are prescribed by the following equation-

$$\omega=-\frac{C_f Re}{2}$$

Where, $C_f=0.05$

The top and bottom walls are adiabatic. So, at the top and bottom wall,

$$\frac{\partial \theta}{\partial y} = 0$$

The slender body is assumed to be heated. So, the temperature field associated with the slender body is: $\theta = 1.0$

Boundary conditions for both left and right boundaries:

$$\frac{\partial u}{\partial x} = 0; \frac{\partial v}{\partial x} = 0; \frac{\partial \psi}{\partial x} = 0; \frac{\partial \omega}{\partial x} = 0; \frac{\partial \theta}{\partial x} = 0$$

2.4. Grid and Co-Ordinate System

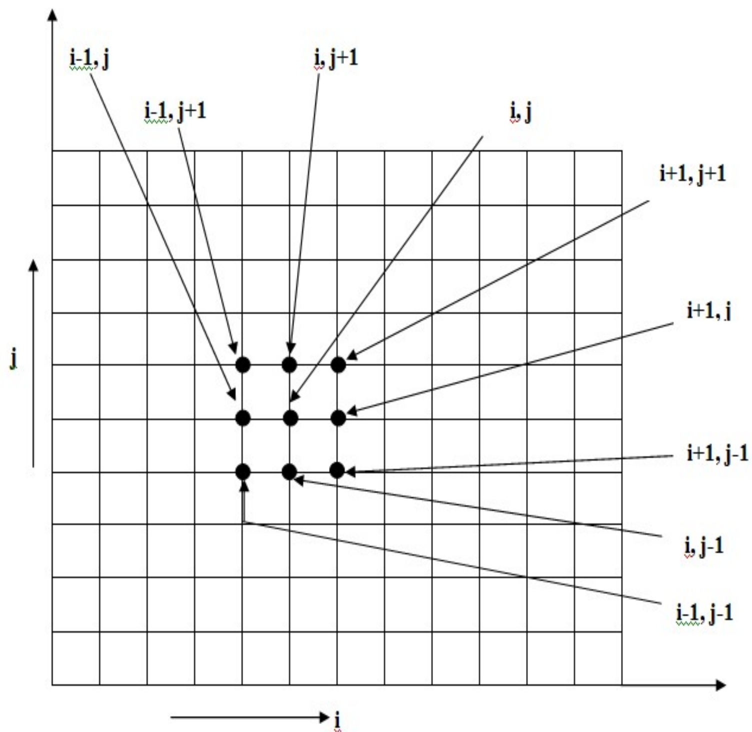

Figure 1. Grid and Co-ordinate system in the Computational Domain

3. Result and Discussion

Here, the governing four non-dimensional partial differential equations with the boundary conditions are solved to get stream function, vorticity, velocity components and temperatures at every internal grid system of 480x240 points in the computational domain which provides a mesh of 115200 nodes. Forward and central difference methods are utilized for discretizing the differential equations. The discretised equations in the first order finite difference form are used. Successive Under Relaxation method (SUR) is employed. The length of the channel is chosen 8 and the height of the channel is chosen 4.

The governing physical parameters in the problem considered are Prandtl number and Rayleigh number, where Rayleigh number is the product of Grashof number and Prandtl number. The flow domain is solved for Rayleigh numbers in the range $50.0 \leq Ra \leq 10000$. Slender body of five different aspect ratios 9.98, 4.99, 3.33, 2.49, 2.0 are also considered. For investigating the effect of locations of slender body, the heated slender body is placed at six different horizontal locations 1.0 to 6.0. The distribution of local Nusselt number, average Nusselt number, velocity profile and temperature profile for different Rayleigh numbers, locations and different aspect ratios have been calculated here. For effective heating, average Nusselt number at heated object surfaces should be higher and average fluid temperature should be lower.

3.1. The Effect of Rayleigh Number

At low Rayleigh numbers, weak air flow is induced by the low thermal driving force. The main flow is symmetric from hot surface zone of object to upper zone near to top wall. At low Rayleigh numbers, with the increase of Rayleigh number, the average Nusselt numbers remains nearly constant. It also means the slowly increase of heat transfer. At higher Rayleigh numbers up to 25000, similar occurrence is observed. At near about Rayleigh number 50, the effective heat transfer is found. The flow from the openings of the channel passes over the heated slender body. The flow is laminar and there is no external force except buoyancy force acts on fluid which drives the fluids getting heated from the heated slender body passes over the slender body through the channel.

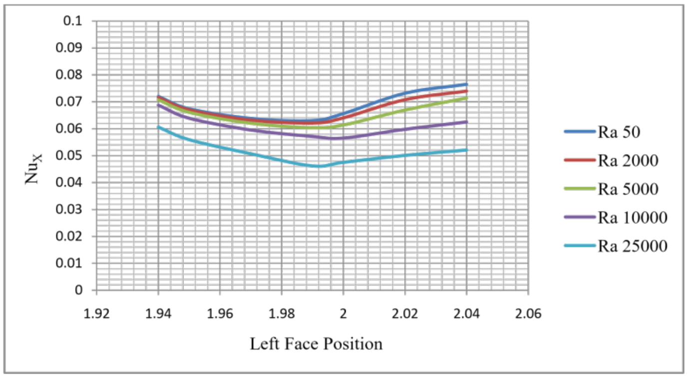

Figure 2. Distribution of Local Nusselt Number at Left face at different Rayleigh Number

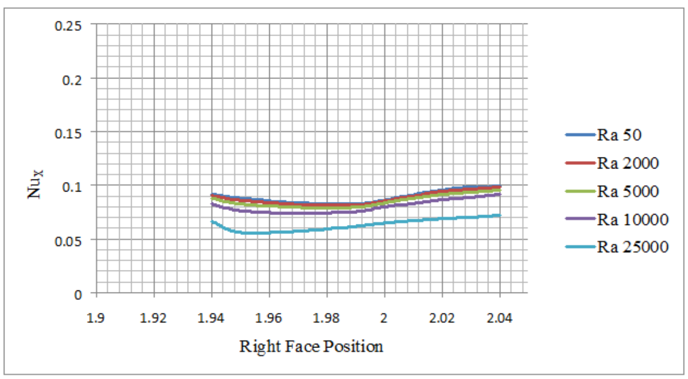

Figure 3. Distribution of Local Nusselt Number at Right face at different Rayleigh Number

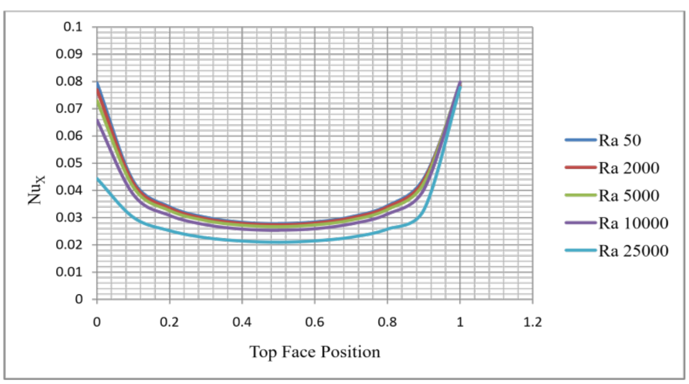

Figure 4. Distribution of Local Nusselt Number at Top face at different Rayleigh Number

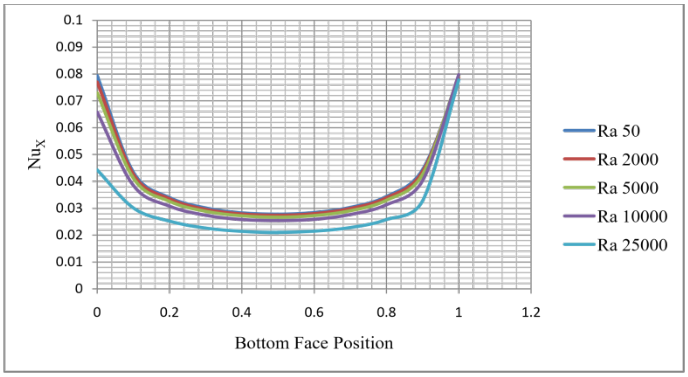

Figure 5. Distribution of Local Nusselt Number at Bottom face at different Rayleigh Number

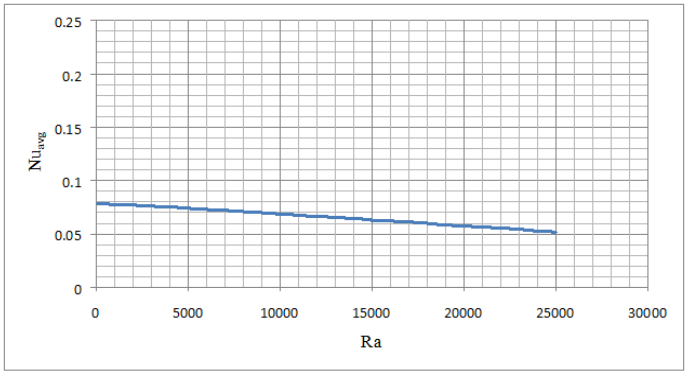

Figure 6. Variation of Average Nusselt Number at Left face with Rayleigh Number

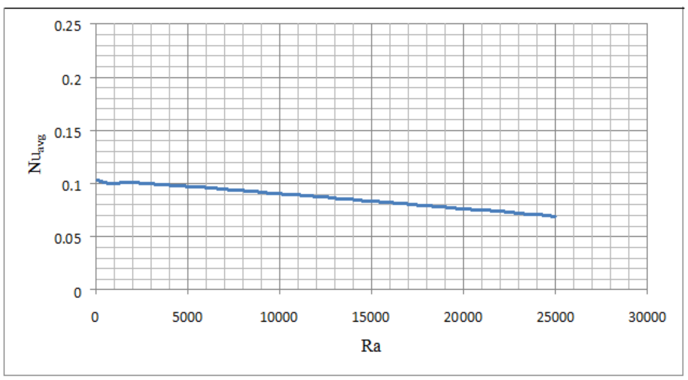

Figure 7. Variation of Average Nusselt Number at Right face with Rayleigh Number

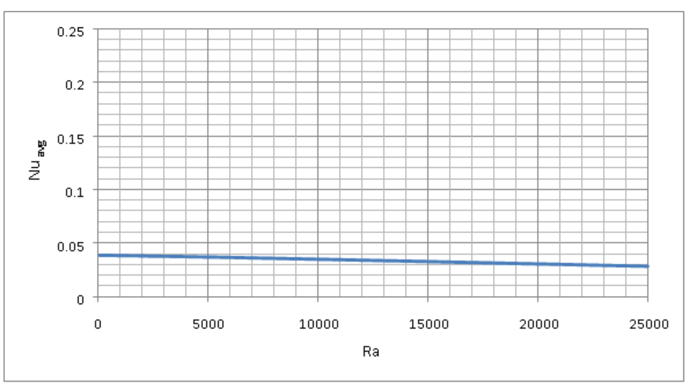

Figure 8. Variation of Average Nusselt Number at Top face with Rayleigh Number

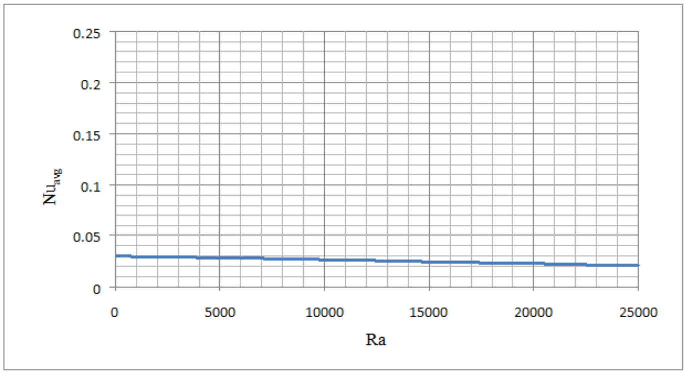

Figure 9. Variation of Average Nusselt Number at Bottom face with Rayleigh Number

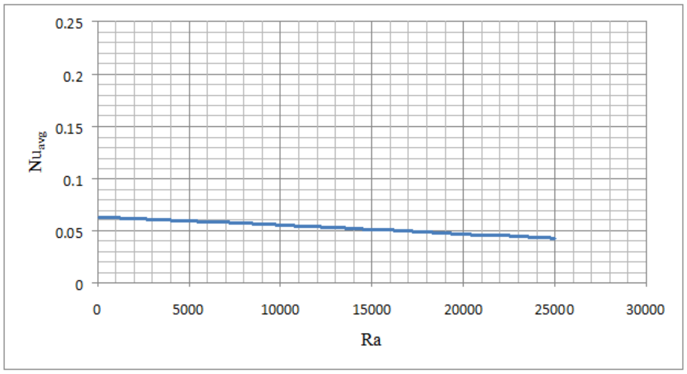

Figure 10. Variation of Slender Body Average Nusselt Number with Rayleigh Number

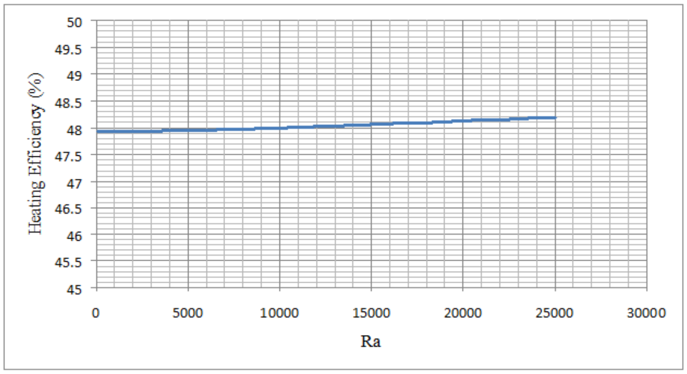

Figure 11. Variation of Efficiency with Rayleigh Number

3.2. The Effect of Prandtl Number

The average Nusselt number remains around constant for the range of Prandtl number investigated. The overall heating efficiency also remains almost constant for the range of Prandtl number considered. Average Nusselt number at all the four faces of the slender body remains nearly constant for the range of Prandtl number considered.

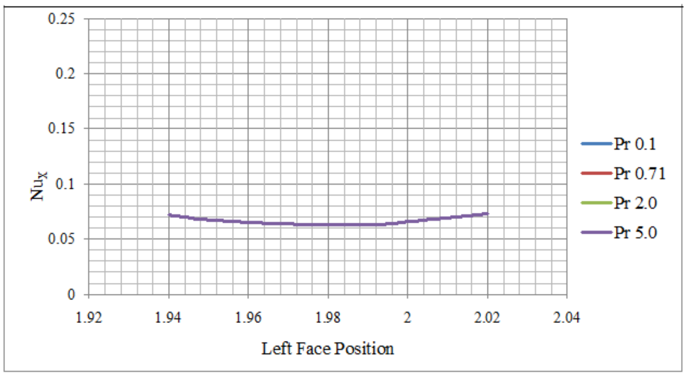

Figure 12. Distribution of Local Nusselt Number at Left face at different Prandtl Number

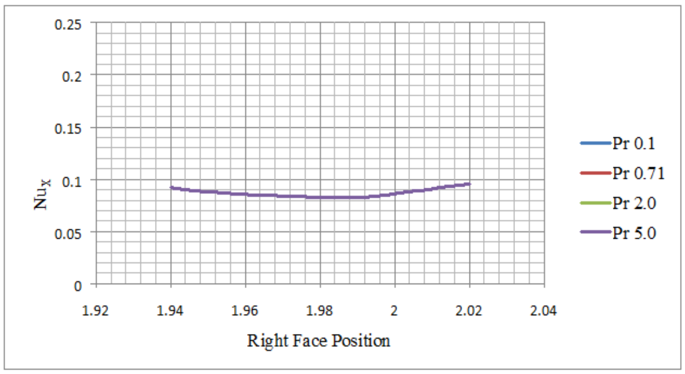

Figure 13. Distribution of Local Nusselt Number at Right face at different Prandtl Number

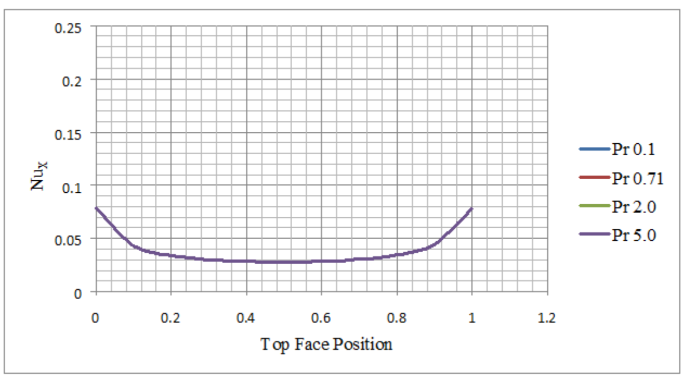

Figure 14. Distribution of Local Nusselt Number at Top face at different Prandtl Number

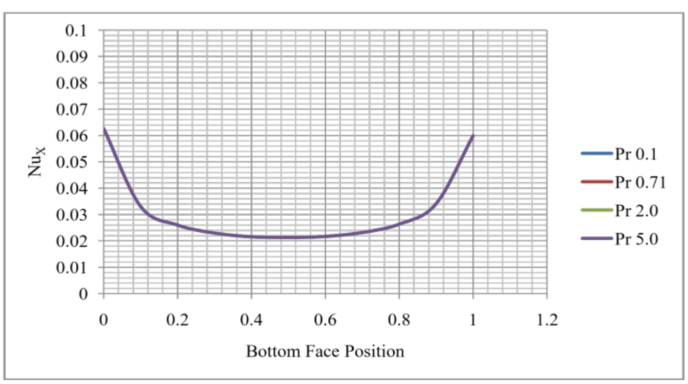

Figure 15. Distribution of Local Nusselt Number at Bottom face at different Prandtl Number

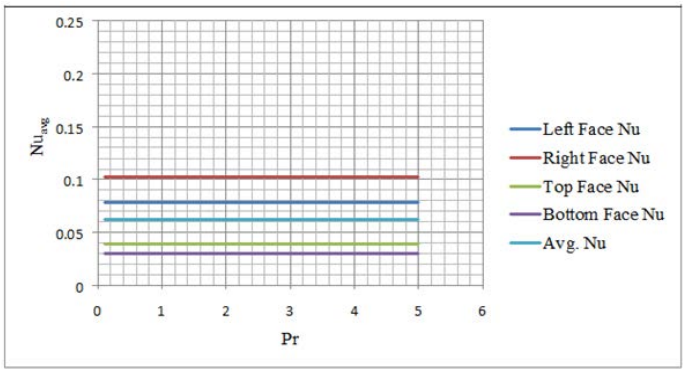

Figure 16. Variation of Average Nusselt Number at various faces with Prandtl Number

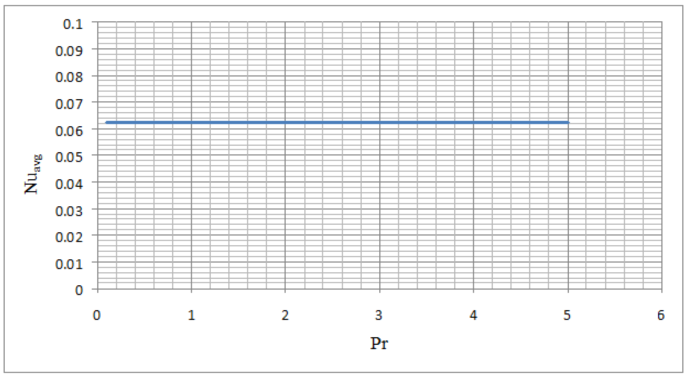

Figure 17. Variation of Slender Body Average Nusselt Number with Prandtl Number

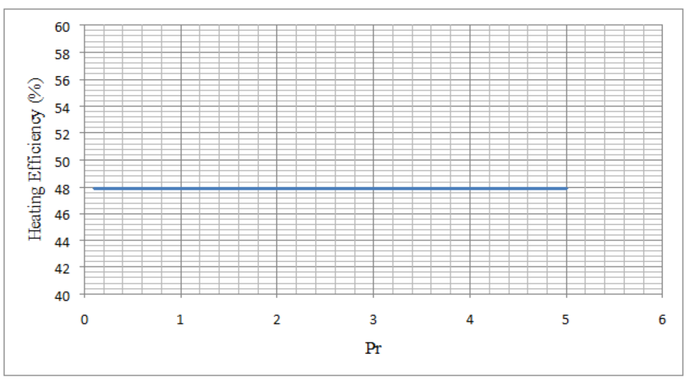

Figure 18. Variation of Efficiency with Prandtl Number

3.3. The Locations of Slender Body

Average Nusselt number at all the four faces of the slender body decreases initially, becomes minimum at the middle portion of the channel and then increases when the slender body is moved from locations from 1.0 to 6.0. Slender body average Nusselt number also shows similar phenomena. The overall heating efficiency increases initially, becomes maximum at the middle, then decreases when the slender body is moved from the locations 1.0 to 6.0.

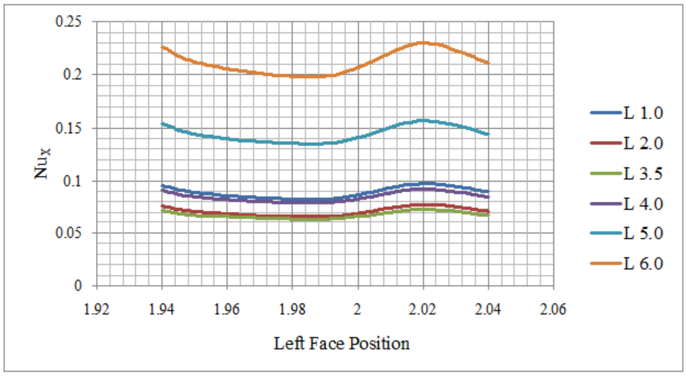

Figure 19. Distribution of Local Nusselt Number at Left face at different Locations

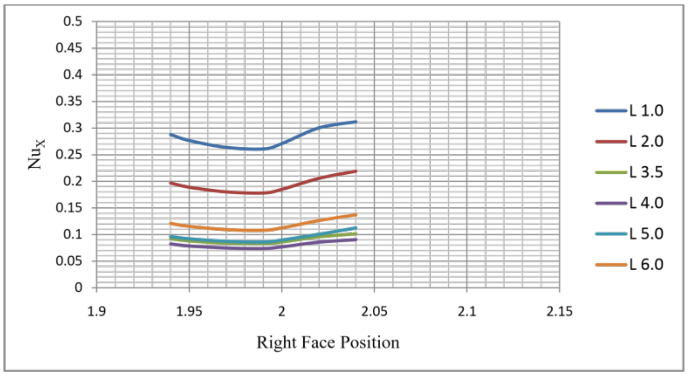

Figure 20. Distribution of Local Nusselt Number at Right face at different Locations

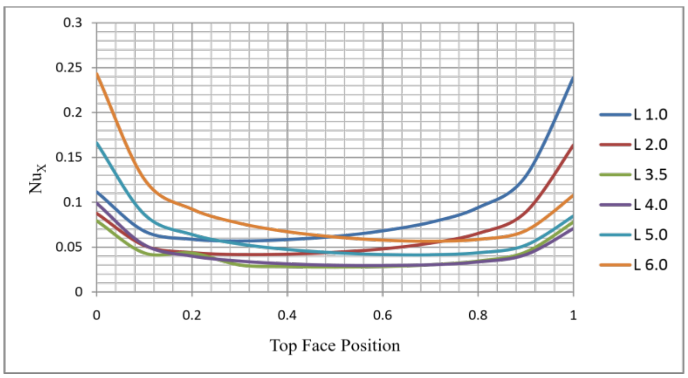

Figure 21. Distribution of Local Nusselt Number at Top face at different Locations

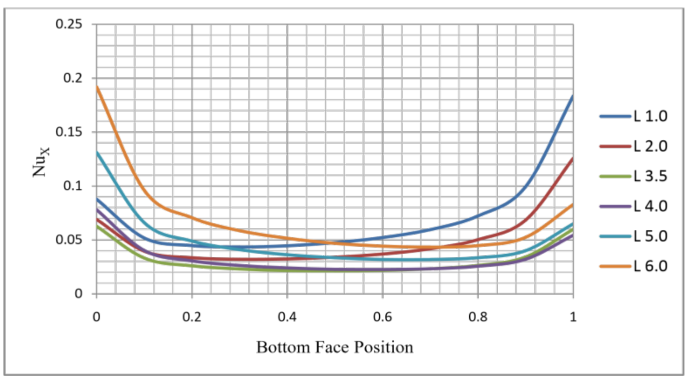

Figure 22. Distribution of Local Nusselt Number at Bottom face at different Locations

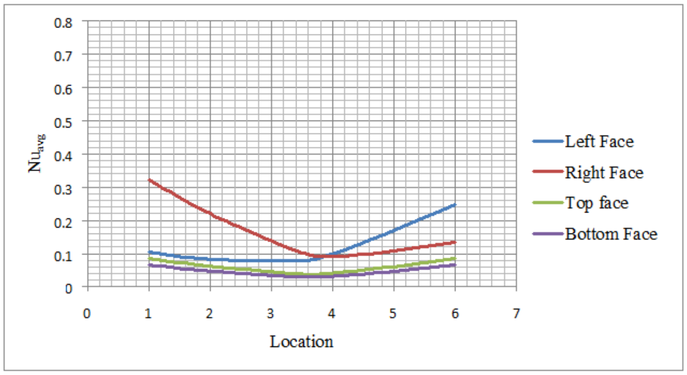

Figure 23. Variation of Aveage Nusselt Number at faces with Location

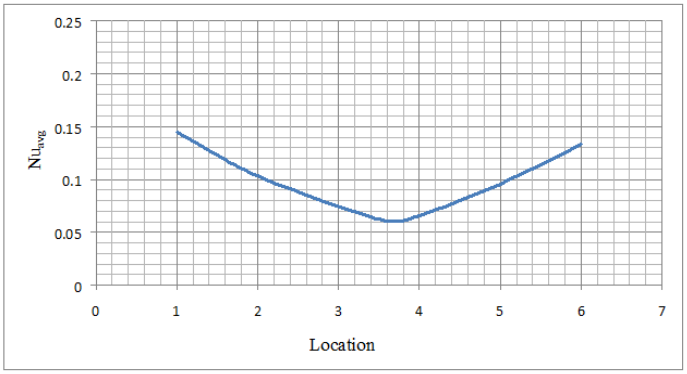

Figure 24. Variation of Slender Body Average Nusselt Number with Location

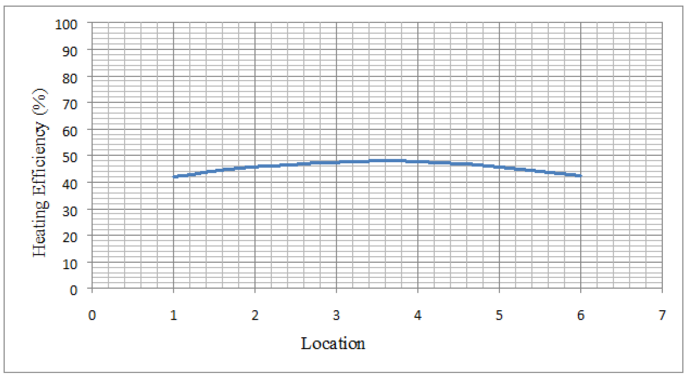

Figure 25. Variation of Efficiency with Slender Body Locations

3.4. The Effect of Aspect Ratios

The overall heating efficiency of the fluid gradually increases with the increase of slender body width or with the decrease of aspect ratio. The average Nusselt number on the left face, right face, top face, bottom face decreases with the increment of slender body width or with the decrement of aspect ratio. So, the average Nusselt number of the slender body decreases with the increment of the slender body width or with the decrement of aspect ratio.

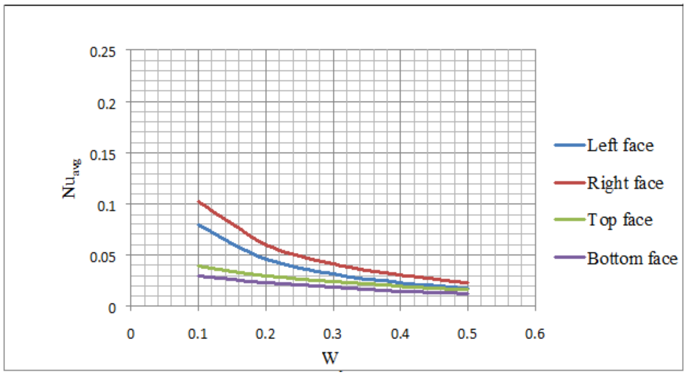

Figure 26. Variation of Average Nusselt Number at different faces with Slender Body Width

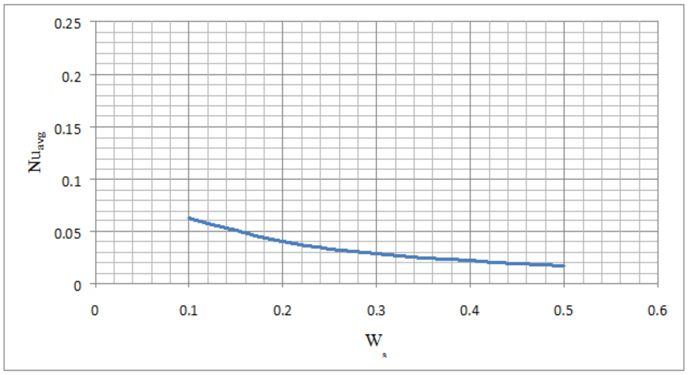

Figure 27. Variation of Slender Body Average Nusselt Number with Slender Body Width

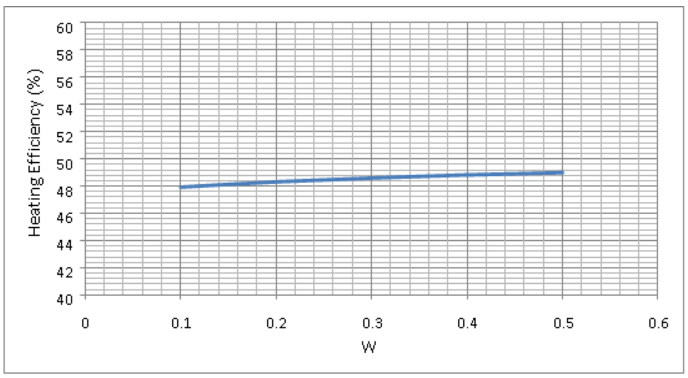

Figure 28. Variation of Efficiency With Slender Body Width

Conclusion

In this paper, finite difference numerical method was developed for laminar free convection flow in a channel with adiabatic side walls and heated slender body. The study is taken at the range of Rayleigh Number from 50 to 25000 and Prandtl Number range from 0.1 to 5.0. Study was also carried out for different locations of slender body through the channel and aspect ratios of 9.98, 4.99, 3.33, 2.49, and 2.0. The main findings of this analysis can be summarized as follows:

(i) Maximum temperature occurs at the vicinity of slender body.

(ii) The average Nusselt Number slowly decreases with the increase of Rayleigh Number.

(iii) The average Nusselt Number remains constant for the range of Prandtl Number investigated.

(iv) Slender body average Nusselt Number decreases initially, becomes minimum at the middle portion of the channel and then increases when the slender body is moved from location 1.0 to 6.0.

(v) Slender body average Nusselt Number decreases with the increase of slender body width or with the decrease of aspect ratio.

(vi) The heating Efficiency increases with the increase of Rayleigh Number and remains constant for the range of Prandtl Number considered.

(vii) The heating efficiency increases initially, becomes maximum at the middle portion of the channel and then decreases when the slender body is moved from location 1.0 to 6.0.

(viii) The overall heating efficiency of the fluid gradually increases with the increase of slender body width or with the decrease of aspect ratio.

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Biographies

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