A Mathematical Model to Investigate the Frequent Impact of Global Warming on Coastal Lives

Sajib Mandal
Department of Mathematics
Bangabandhu Sheikh Mujibur Rahman Science and Technology University
Gopalganj-8100, Bangladesh
sajibmandal1997@gmail.com

Md. Sirajul Islam, Sonia Akter
Department of Mathematics
Bangabandhu Sheikh Mujibur Rahman Science and Technology University
Gopalganj-8100, Bangladesh
sirajulku@gmail.com, soniaaa053@gmail.com

Md. Haider Ali Biswas
Mathematics Discipline
Science Engineering and Technology School
Khulna University, Khulna-9208, Bangladesh.
mhabiswas@yahoo.com

Abstract

Global warming is the main concern of environmental scientists at present. The rapid emissions of greenhouse gases (GHGs) from different sources promote global warming and introduce several natural calamities. According to scientists, about 83% of greenhouse gases in the atmosphere are produced by human activities, and about 17% GHGs in the atmosphere are produced by wild animals. For global warming, the ice is melting and the lower lands in the world such as Bangladesh are going underwater. In a report, the annual increasing rate of seawater label is approximately 0.13 inches. In this study, such a situation has been investigated by developing a mathematical model in terms of four dynamical species. The model has been solved both analytically and numerically. The particular focus of this study aims to illustrate the impact of rapidly emitting GHGs on coastal lives. This study also discloses a mathematical relationship between increasing GHGs and global warming.

Keywords
Mathematical model, Greenhouse gases, Global warming, Coastal lives, Natural calamities.

1. Introduction
We live in the age of the modern world and different types of engines, aircraft, or vehicles are invented every day. It is enough to say that most of these produce greenhouse gas. Greenhouse gases (GHGs) mean a combination of such types of gases that are responsible for global warming. Besides, they change the climate by warming the atmosphere by forming an invisible layer around the world that restricts to omit the temperature of the earth. According to scientists, about 83% of greenhouse gases in the atmosphere is produced by human activities. Vehicles, factories, brickfields, industries, fossil burning, man-made forest fires, and cut down the amount of forest are the main human destructive activities to increase the amount of GHGs. As a result, the climate is being changed with the rising temperature. Due to climate change, nature restricts the lifestyle both of the human community and forest ecosystems near the coastal areas introducing various natural disasters such as hurricanes, cyclones, tornados, tsunami, floods.

Mandal et al. (2020a) developed a mathematical model both analytically and numerically to describe the potential impact of climate change on coastal ecosystems. Biswas et al. (2016) discussed the effect of climate change and described the corresponding effect in Bangladesh through mathematical modeling. Paul and Biswas (2019) specially described the bad effect of carbon dioxide on the Sundarbans. Sekerci and Petrovskii (2015) described the effect of climatic change on the marine ecosystem especially the plankton through modeling. Hader et al. (2017) statistically discussed climate change due to solar radiation and the effects of solar UV-radiation on the aquatic ecosystem. Khan and Alom (2015) discussed literally that greenhouse gases are the main causes of global warming. Mandal et al. (2021) investigated the increasing global warming on the planktonic ecosystem in oceans analytically and numerically. Devi and Mishra (2020) recently developed and analyzed a mathematical model to describe the effect of increasing atmospheric temperature due to rapid emissions of GHGs. Montzka et al. (2011) disclosed that the anthropogenic non-CO_2 GHGs are the most responsible for global warming as well as climate change. Besides, Samimi et al. (2013) claimed that rapid emissions of GHGs are responsible for climate change and introduced some controlling techniques briefly, whereas Ladley et al. (1999) briefly described some factors of GHGs for which the atmospheric temperature becomes hotter.

1.1 Objectives

The objectives of the paper are

- to describe the effect of greenhouse gases on global warming and living beings in the coastal area.
- to represent the interrelationship among these dynamical populations.
- to perform the analysis both analytically and numerically for illustrating the dynamical behaviors of the considered dynamics.

2. Model Formulation

To describe the effect of greenhouse gases on global warming and living beings on the earth near the coastal area, we consider a dynamic system consisting of four dynamic variables as the density of greenhouse gases, the rising of atmospheric temperature in the earth, the human community in the coastal area and the forest ecosystem near the coastal area. This study aims to estimate the approximate change in global warming and lives on the earth because of the rapid emissions of greenhouse gases for different activities of the human population. The interrelationship among the species can be represented in the following schematic diagram which is shown in Figure 1 as

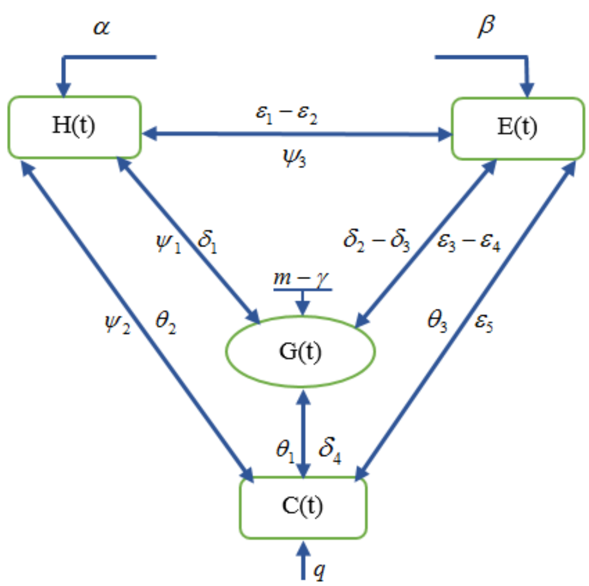

Figure 1. Effect of GHGs on climate change, human community**,** and forest ecosystems near coastal areas

According to the diagram (Figure 1), the modified mathematical model can be rewritten as the following form:

$$\frac{dG}{dt} = (m - \gamma) + \delta_1 HG + (\delta_2 - \delta_3)EG + \delta_4 C \tag{1}$$

$$\frac{dC}{dt} = q + \theta_1 GC + \theta_2 HC - \theta_3 EC \tag{2}$$

$$\frac{dH}{dt} = \alpha - \psi_1 GH - \psi_2 CH + \psi_3 EH \tag{3}$$

$$\frac{dE}{dt} = \beta + (\varepsilon_1 - \varepsilon_2) HE + (\varepsilon_3 - \varepsilon_4) GE - \varepsilon_5 CE \tag{4}$$

with initial conditions $G(0) > 0,\ C(0) > 0,\ H(0) \ge 0,\ E(0) \ge 0$

Here, G(t), C(t), H(t), and E(t) are considered as the density of GHGs, the rising of atmospheric temperature in the earth, the human community in the coastal area, and the forest ecosystem near the coastal area.

A brief description of the parameters in the model are as follows:

m	:	the natural increasing rate of GHGs in the environment
γ	:	depleting rate of GHGs due to the ultraviolet radiations
δ_1	:	producing rate of GHGs by the human population
δ_2	:	producing rate of GHGs by animals in the forest ecosystem near the coastal area
δ_3	:	absorbing rate of GHGs by trees in the forest ecosystem
δ_4	:	producing rate of GHGs after natural disasters
q	:	natural climate change in the absence of lives
θ_1	:	changing rate of global warming by the GHGs
θ_2	:	changing rate of global warming by the activity of the human population
θ_3	:	absorbing rate of temperature by the forest ecosystem
α	:	the normal growth rate of the human population at any instant
ψ_1	:	hampering rate of human due to GHGs
ψ_2	:	hampering rate of human due to climate change
ψ_3	:	the growth rate of the human by the forest ecosystem
β	:	the natural growth rate of forest ecosystem
ε_1	:	planting rate of trees in the forest ecosystem by the human population
ε_2	:	deforesting rate in the forest ecosystem by the human population
ε_3	:	the growth rate of forest ecosystems with the help of GHGs especially CO_2
ε_4	:	hampering rate of forest ecosystems because of GHGs especially acid rain
ε_5	:	hampering rate of forest ecosystem especially wildfire because of global warming

3. Analytical Analysis

In this section, we test the positivity of the dynamical variables of the system, stability analysis at equilibrium points, and numerical simulation [Biswas et al. (2017), Biswas et al. (2020), Kabir et al. (2020), Mandal et al. (2020b), Sardar et al. (2016)].

3.1 Boundedness of the System

Now, we will establish that the system of nonlinear ordinary differential equations (1)-(4) is bounded. By the following lemma, we start to prove.

Lemma 1. The set $\Phi = \left\{ (G(t), C(t), H(t), E(t)) \in \Re_4^+ : 0 \le G(t) + C(t) + E(t) \le \frac{\upsilon}{\eta}, H(t) \le \alpha \right\}$ is a region of attraction for each solution and initially, all the variables are positive, where η is a constant and $\upsilon = m - \gamma + q + \beta$.

Proof: Consider a function from the system (1)-(4) as

$$g(t) = G(t) + C(t) + E(t)$$

$$\Rightarrow \frac{dg}{dt} + \eta g = \frac{dG}{dt} + \frac{dC}{dt} + \frac{dE}{dt} + \eta G + \eta C + \eta E$$

$$\therefore \frac{dg}{dt} + \eta g = (m-\gamma) + \eta G + q + \beta + \{(\delta_2 - \delta_3) - (\varepsilon_3 - \varepsilon_4)\} EG + (\delta_4 + \eta) C + \theta_1 GC + \{\delta_1 G + \theta_2 C + (\varepsilon_1 - \varepsilon_2) E\} H - (\varepsilon_5 + \theta_3) EC + \eta E$$

Since the density of the human community is absent in this section, let put $H = 0$. then we get from the above equation as

$$\frac{dg}{dt} + \eta g = (m-\gamma) + \eta G + q + \beta + \{(\delta_2 - \delta_3) - (\varepsilon_3 - \varepsilon_4)\} EG + (\delta_4 + \eta) C + \theta_1 GC - (\varepsilon_5 + \theta_3) CE + \eta G + \eta E$$

$$\therefore \frac{dg}{dt} + \eta g \le m - \gamma + q + \beta$$

Since $\{(\delta_2 - \delta_3) - (\varepsilon_3 - \varepsilon_4)\} EG + (\delta_4 + \eta) C + \theta_1 GC - \varepsilon_5 CE + \eta G + \eta E \le (m - \gamma + q + \beta)$

$$\therefore \frac{dg}{dt} + \eta g \le \upsilon, \quad \text{where } \upsilon = m - \gamma + q + \beta$$

Applying the differential inequalities, we have $0 \le g(t) \le \frac{\upsilon}{\eta}(1 - e^{-\eta t})$,

Taking limit as $t \to \infty$, we get $0 \le g(t) \le \frac{\upsilon}{\eta}$ i.e. $0 \le G(t) + C(t) + E(t) \le \frac{\upsilon}{\eta}$

Then $\frac{dH}{dt} + hH = \alpha$, where $h = \psi_1 G + \psi_2 C - \psi_3 E$

Taking limit as $t \to \infty$, we have $H(t) \le \frac{\alpha}{h}$ i.e. $0 \le H(t) \le \frac{\alpha}{h}$

Hence the solution of the system is bounded in Φ .

3.2 Equilibrium Points

We obtain equilibrium points of the system by setting $\frac{d\overline{G}}{dt} = \frac{d\overline{C}}{dt} = \frac{d\overline{H}}{dt} = \frac{d\overline{E}}{dt} = 0$. The system (1)-(4) produces three equilibrium points $E_i(\overline{T}, \overline{P}, \overline{Z}, \overline{D})$. Then we get

(i) $E_L(\overline{G}, \overline{C}, 0, 0) = E_L\left(\frac{q(m-\gamma)}{\theta_1 \delta_4}, \frac{\gamma - m}{\delta_4}, 0, 0\right)$

(ii) $E_H(\overline{G}, \overline{C}, 0, \overline{E}) = E_H\left(\frac{q\delta_4}{(m-\gamma)(\theta_1 - \theta_3 A)}, \frac{\beta}{\varepsilon_5 A} + \frac{q\delta_4(\varepsilon_3 - \varepsilon_4)}{\varepsilon_5 (m-\gamma)(\theta_1 - \theta_3 A)}, 0, \frac{q\delta_4 A}{(m-\gamma)(\theta_1 - \theta_3 A)}\right)$

(iii) $E_E(\overline{G}, \overline{C}, \overline{H}, \overline{E}) = E_H\left(a + \frac{\theta_3}{\theta_1} \frac{\beta q \psi_1 \delta_4}{\alpha\beta\delta_1\theta_3 - b}, \frac{\alpha\delta_1}{\psi_1 \delta_4}, \frac{\alpha\theta_1(m-\gamma)}{q\psi_1\delta_4}, \frac{\beta q \psi_1 \delta_4}{\alpha\beta\delta_1\theta_3 - b}\right)$

3.3 Stability Analysis

The system of equations (1)-(4) can be represented in the Jacobian matrix as

$$J = \begin{bmatrix} \delta_1 H + (\delta_2 - \delta_3) E & \delta_4 & \delta_1 G & (\delta_2 - \delta_3) G \\ \theta_1 C & \theta_1 G + \theta_2 H - \theta_3 E & \theta_2 C & -\theta_3 C \\ -\psi_1 H & -\psi_2 H & -\psi_1 G - \psi_2 C + \psi_3 E & \psi_3 H \\ (\varepsilon_3 - \varepsilon_4) E & -\varepsilon_5 E & (\varepsilon_1 - \varepsilon_2) E & (\varepsilon_1 - \varepsilon_2) H + (\varepsilon_3 - \varepsilon_4) G - \varepsilon_5 C \end{bmatrix} \quad (5)$$

Theorem 1. The system (1)-(4) is a saddle point at the life's free equilibrium point E_L .

Proof: The Jacobian matrix (5) becomes at the lives free equilibrium point E_L

$$J_{E_L}=\begin{bmatrix} 0 & \delta_4 & \delta_1\dfrac{q(m-\gamma)}{\theta_1\delta_4} & (\delta_2-\delta_3)\dfrac{q(m-\gamma)}{\theta_1\delta_4} \\ \theta_1\dfrac{\gamma-m}{\delta_4} & \theta_1\dfrac{q(m-\gamma)}{\theta_1\delta_4} & \theta_2\dfrac{\gamma-m}{\delta_4} & -\theta_3\dfrac{\gamma-m}{\delta_4} \\ 0 & 0 & -\psi_1\dfrac{q(m-\gamma)}{\theta_1\delta_4}-\psi_2\dfrac{\gamma-m}{\delta_4} & 0 \\ 0 & 0 & 0 & (\varepsilon_3-\varepsilon_4)\dfrac{q(m-\gamma)}{\theta_1\delta_4}-\varepsilon_5\dfrac{\gamma-m}{\delta_4} \end{bmatrix}$$

The characteristic equation of $\left|J_{E_L}-\lambda I\right|=0$ takes the form as

$$J_{E_L}=\begin{bmatrix} -\theta_1-\lambda_1 & q & -\theta_2 & \theta_3 \\ 0 & \delta_4-\lambda_2 & \delta_1\dfrac{q(m-\gamma)}{\theta_1\delta_4} & (\delta_2-\delta_3)\dfrac{q(m-\gamma)}{\theta_1\delta_4} \\ 0 & 0 & \dfrac{q\psi_1}{\theta_1}-\psi_2-\lambda_3 & 0 \\ 0 & 0 & 0 & -\dfrac{q(\varepsilon_3-\varepsilon_4)}{\theta_1}-\varepsilon_5-\lambda_4 \end{bmatrix}$$

Hence the eigenvalues are

$$\lambda_1=-\theta_1,\ \lambda_2=\delta_4,\ \lambda_3=\frac{q\psi_1}{\theta_1}-\psi_2,\ \lambda_4=-\left(\frac{q(\varepsilon_3-\varepsilon_4)}{\theta_1}+\varepsilon_5\right)$$

Here λ_1 and λ_4 are negative, λ_2 and λ_3 are positive ($\because \psi_1>\psi_2$). Therefore, the lives' free equilibrium point is asymptotically stable.

Theorem 2. The system (1)-(4) is a stable point at the human community free equilibrium E_H.

Proof: The Jacobian matrix (5) becomes at the human community free equilibrium E_H

$$J_{E_H}=\begin{bmatrix} (\delta_2-\delta_3)N & \delta_4 & \delta_1K & (\delta_2-\delta_3)K \\ \theta_1M & \theta_1K-\theta_3N & \theta_2M & -\theta_3M \\ 0 & 0 & -\psi_1K-\psi_2M+\psi_3N & 0 \\ (\varepsilon_3-\varepsilon_4)N & -\varepsilon_5N & (\varepsilon_1-\varepsilon_2)N & (\varepsilon_3-\varepsilon_4)K-\varepsilon_5M \end{bmatrix}$$

$$=\begin{bmatrix} -N & \dfrac{-\delta_4}{(\delta_2-\delta_3)} & \dfrac{-\delta_1K}{(\delta_2-\delta_3)} & -K \\ 0 & \theta_1K-\theta_3N-\dfrac{\theta_1\delta_4M}{(\delta_2-\delta_3)N} & \theta_2M-\dfrac{\theta_1\delta_1KM}{(\delta_2-\delta_3)N} & -\theta_3M-\dfrac{\theta_1KM}{N} \\ 0 & 0 & -\psi_1K-\psi_2M+\psi_3N & 0 \\ 0 & 0 & 0 & -\varepsilon_5M+d_1\left\{\theta_3M-\dfrac{\theta_1KM}{N}\right\} \end{bmatrix} \qquad (6)$$

where $K=\dfrac{q\delta_4}{(m-\gamma)(\theta_1-\theta_3A)}$, $N=\dfrac{q\delta_4A}{(m-\gamma)(\theta_1-\theta_3A)}$, $M=\dfrac{\beta}{\varepsilon_5A}+\dfrac{q\delta_4(\varepsilon_3-\varepsilon_4)}{\varepsilon_5(m-\gamma)(\theta_1-\theta_3A)}$

Hence the eigenvalues from the characteristic equation of equation (6) are

$$\lambda_1=-N=-\frac{q\delta_4A}{(m-\gamma)(\theta_1-\theta_3A)}\quad,\quad \lambda_2=-\left(\theta_3N+\frac{\theta_1\delta_4M}{(\delta_2-\delta_3)N}-\theta_1K\right)\quad,\quad \lambda_3=-\left(\psi_1K+\psi_2M-\psi_3N\right)\quad,$$

$$\lambda_3=-\left\{\varepsilon_5M+d_1\left\{\theta_3M-\frac{\theta_1KM}{N}\right\}\right\}$$

Here all of the eigenvalues are negative. Hence the human community free equilibrium point is stable.

Theorem 3. The system (1)-(4) is a stable point or saddle point at the global equilibrium point E_G .

Proof: The Jacobian matrix (5) becomes at the equilibrium point E_G

$$J_{E_G} = \begin{bmatrix} \delta_1 R + (\delta_2 - \delta_3)S & \delta_4 & \delta_1 P & (\delta_2 - \delta_3)P \\ \theta_1 Q & \theta_1 P + \theta_2 R - \theta_3 S & \theta_2 Q & -\theta_3 Q \\ -\psi_1 R & -\psi_2 R & -\psi_1 P - \psi_2 Q + \psi_3 S & \psi_3 R \\ (\varepsilon_3 - \varepsilon_4)S & -\varepsilon_5 S & (\varepsilon_1 - \varepsilon_2)S & \begin{pmatrix} (\varepsilon_1 - \varepsilon_2)R + \\ (\varepsilon_3 - \varepsilon_4)P - \varepsilon_5 Q \end{pmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} -\psi_1 R & -\psi_2 R & -\psi_1 P - \psi_2 Q + \psi_3 S & \psi_3 R \\ (\varepsilon_3 - \varepsilon_4)S & -\varepsilon_5 S & (\varepsilon_1 - \varepsilon_2)S & \begin{pmatrix} (\varepsilon_1 - \varepsilon_2)R + \\ (\varepsilon_3 - \varepsilon_4)P - \varepsilon_5 Q \end{pmatrix} \\ \delta_1 R + (\delta_2 - \delta_3)S & \delta_4 & \delta_1 P & (\delta_2 - \delta_3)P \\ \theta_1 Q & \theta_1 P + \theta_2 R - \theta_3 S & \theta_2 Q & -\theta_3 Q \end{bmatrix} \begin{matrix} [R_1 \Leftrightarrow R_3] \\ [R_2 \Leftrightarrow R_4] \end{matrix}$$

$$\therefore J_{E_E} = \begin{bmatrix} -\psi_1 R & -\psi_2 R & -\psi_1 P - \psi_2 Q + \psi_3 S & \psi_3 R \\ 0 & -\left[\varepsilon_5 + \frac{\psi_2(\varepsilon_3 - \varepsilon_4)}{\psi_1}\right]S & (\varepsilon_1 - \varepsilon_2)S - \frac{(\varepsilon_3 - \varepsilon_4)PS}{R} - \frac{\psi_2(\varepsilon_3 - \varepsilon_4)SQ}{\psi_1} & a_{24} \\ 0 & 0 & b_{33} & b_{34} \\ 0 & 0 & 0 & b_{44} - \frac{b_{43}b_{34}}{b_{33}} \end{bmatrix} \tag{7}$$

where $P = a + \frac{\theta_3}{\theta_1}\frac{\beta q \psi_1 \delta_4}{\alpha\beta\delta_1\theta_3 - b}$, $Q = \frac{\alpha\delta_1}{\psi_1\delta_4}$, $R = \frac{\alpha\theta_1(m-\gamma)}{q\psi_1\delta_4}$, $S = \frac{\beta q \psi_1 \delta_4}{\alpha\beta\delta_1\theta_3 - b}$, $a_{44} = -\theta_3 Q + \frac{\theta_1\psi_3 QS}{\psi_1 R}$

$a_{24} = (\varepsilon_1 - \varepsilon_2)R - (\varepsilon_3 - \varepsilon_4) - \varepsilon_5 Q + \frac{\psi_3(\varepsilon_3 - \varepsilon_4)S}{\psi_1}$, $a_{32} = \delta_4 - \frac{\delta_1 R + (\delta_2 - \delta_3)S}{\psi_1}\psi_2$, $b_{33} = a_{33} - \frac{a_{32}a_{23}}{a_{22}}$

$a_{33} = \delta_1 P - \frac{\delta_1 R + (\delta_2 - \delta_3)S}{\psi_1 R}(\psi_1 P + \psi_2 Q)$, $a_{34} = (\delta_2 - \delta_3)P + \frac{\psi_1\delta_1 R + \psi_3(\delta_2 - \delta_3)S}{\psi_1}$, $b_{34} = a_{34} - \frac{a_{32}a_{24}}{a_{22}}$

$a_{42} = \theta_1 P + \theta_2 R - \theta_3 S - \frac{\theta_1\psi_2 Q}{\psi_1}$, $a_{43} = \theta_3 Q - \frac{\theta_1 QP}{R} + \frac{\theta_1\psi_3 QS}{\psi_1 R}$, $b_{43} = a_{43} - \frac{a_{42}a_{23}}{a_{22}}$, $b_{44} = a_{44} - \frac{a_{42}a_{24}}{a_{22}}$.

Hence the eigenvalues of the characteristic equation of (7) are

$$\lambda_1 = -\psi_1 R = -\frac{\alpha\theta_1(m-\gamma)}{q\delta_4},\ \lambda_2 = -\left[\varepsilon_5 + \frac{\psi_2(\varepsilon_3 - \varepsilon_4)}{\psi_1}\right]\frac{\beta q \psi_1 \delta_4}{\alpha\beta\delta_1\theta_3 - b},\ \lambda_3 = b_{33} = a_{33} - \frac{a_{32}a_{23}}{a_{22}},\ \lambda_4 = b_{44} - \frac{b_{43}b_{34}}{b_{33}}$$

It is clear that λ_1 and λ_2 contain negative values. There exist three cases as

Case-1: If λ_3 and λ_4 contain negative values, the extreme equilibrium will be stable.

Case-2: If λ_3 and λ_4 contain positive values, the extreme equilibrium will be asymptotically stable.

Case-3: If λ_3 and λ_4 contain the opposite value (i.e. one contains a positive value and another contains a negative value), the extreme equilibrium will be unstable.

4. Results and Discussion

Numerical simulation is the most useful task to represent the interactions among the dynamical variables. Here we use Maple coding to check the feasibility of our analysis concerning stability axioms. Numerical computations have been driven by using a set of parametric values given below:

Table 1. Parametric values of parameters used in the model.

Symbol	Values	Symbol	Values	Symbol	Values	Symbol	Values
m	$0.01\, gm\, day^{-1} km^{-2}$	δ_4	$0.0001\, gm\, km^{-2}$	α	$0.05\ year^{-1} Thusand^{-1}$	ε_1	$0.001\ km^{-2}\ year^{-1}$
γ	$0.003\ gm\, day^{-1} km^{-2}$	q	$0.001\ {}^oC\, year^{-1}$	ψ_1	$0.048\ year^{-1} Thusand^{-1}$	ε_2	$0.098\ km^{-2} year^{-1}$
δ_1	$0.011\ gm\, day^{-1} km^{-2}$	θ_1	$0.0020\ {}^oC\, year^{-1}$	ψ_2	$0.096\, year^{-1} Thusand^{-1}$	ε_3	$0.015\ km^{-2} year^{-1}$
δ_2	$0.029\ gm\, day^{-1} km^{-2}$	θ_2	$0.00016\ {}^oC\, day^{-1}$	ψ_3	$0.00001\ year^{-1} Thusand^{-1}$	ε_4	$0.095\ km^{-2}\ year^{-1}$
δ_3	$0.050\ gm\, day^{-1} km^{-2}$	θ_3	$0.002\ {}^oC\, day^{-1}$	β	$0.0001\ km^{-2} year^{-1}$	ε_5	$0.0988\ {}^oC^{-1} km^{-2}$

Figures 2-5 represent the competition among these dynamical variables with the change of time. Since the human population is rising at a constant rate, as a result, they increase the amount of GHGs for various purposes which are shown in Figure 2. Because of the increasing GHGs, it raises the temperature of the earth's atmosphere which carries a bad change in the global climate. Therefore, an excess increase of GHGs is responsible for global warming which is shown in Figure 3. On the other hand, because of the increase of GHGs and climate change, the growth rate of the human population will be negative after a while and the total human population will be started to decline during this century shown in Figure 4. The forest ecosystem will be declined due to the excess GHGs and global climate change because of frequent acid rains, global warming of the earth's atmosphere, frequent natural disasters, etc. Besides, depopulating the forest and acting unconsciously to prevent the depopulation, the human community contributes to creating the forest ecosystem imbalanced. The change of forest ecosystem with the interaction among other populations concerning time is represented in Figure 5.

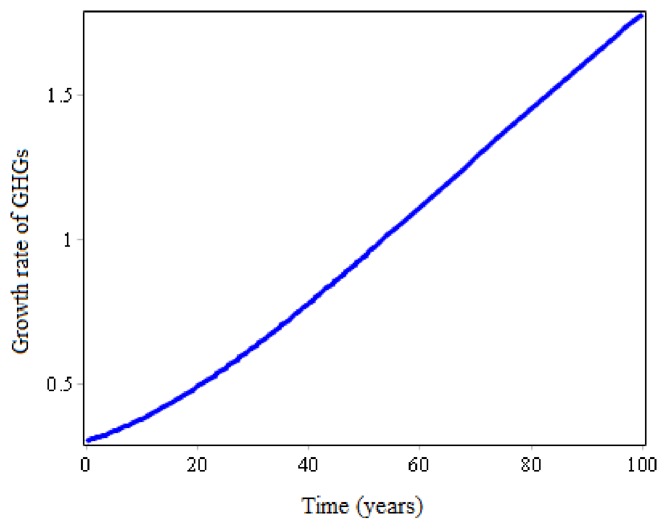

Figure 2. The growth rate of environmental GHGs per year.

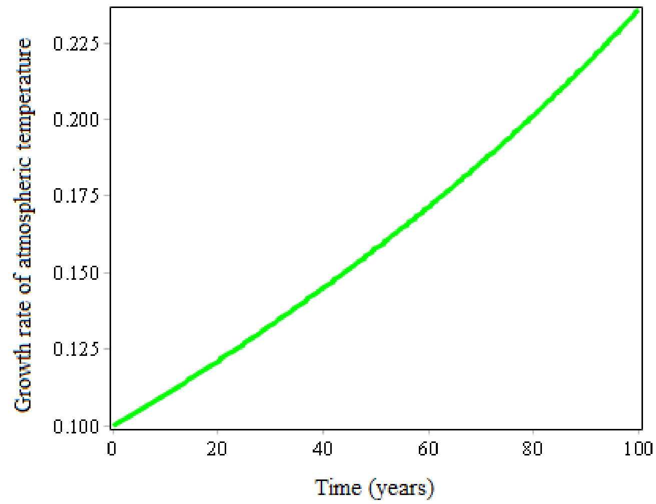

Figure 3. The growth rate of atmospheric temperature per year.

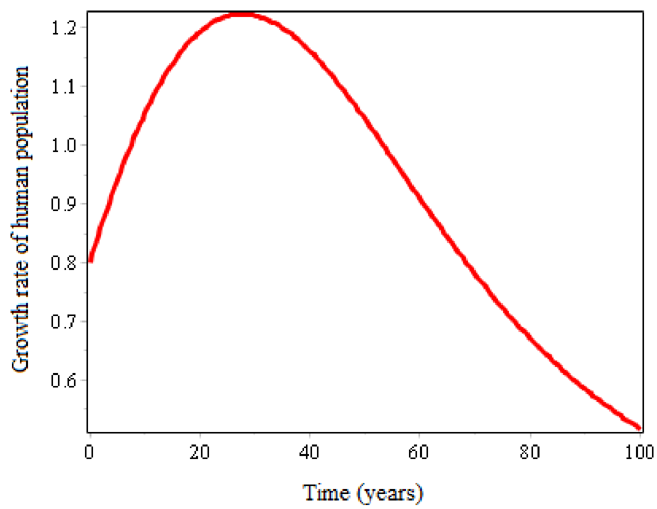

Figure 4. The growth rate of the human population near the coastal area per year.

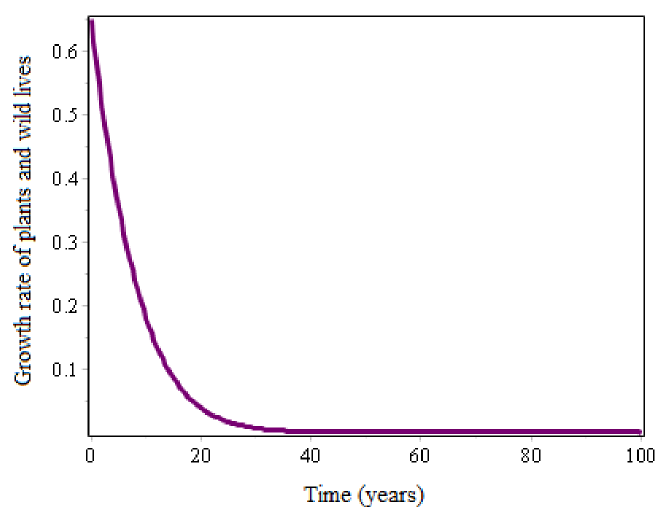

Figure 5. The growth rate of the plants and wildlife near the coastal area per year.

When the GHGs emitting activities of the human population (δ_1) increase, it proportionally raises the amount as well as the emissions of GHGs in the environment. Figure 6 represents the growth rate of GHGs in the environment when the GHGs emitting activities of the human population increase. When the emissions of GHGs increase due to the human population, the atmospheric temperature proportionally increases shown in Figure 7. The increasing GHGs contribute to heavy acid rain and deficiency of fresh oxygen for breath, and the rapid global warming contributes to different natural destructive phenomena. As a result, the growth rate of the human population and the density of the forest ecosystem decrease with the increase of emissions of GHGs in the environment which are respectively shown in Figure 8 and Figure 9.

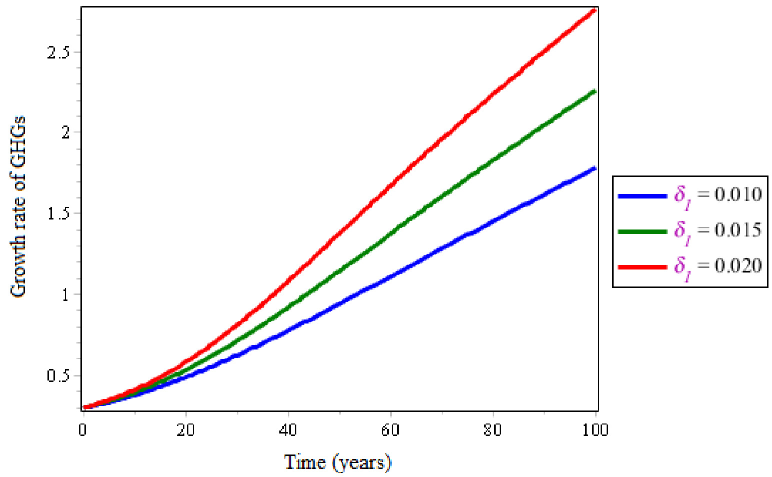

Figure 6. The growth rate of environmental GHGs for different values of δ_1.

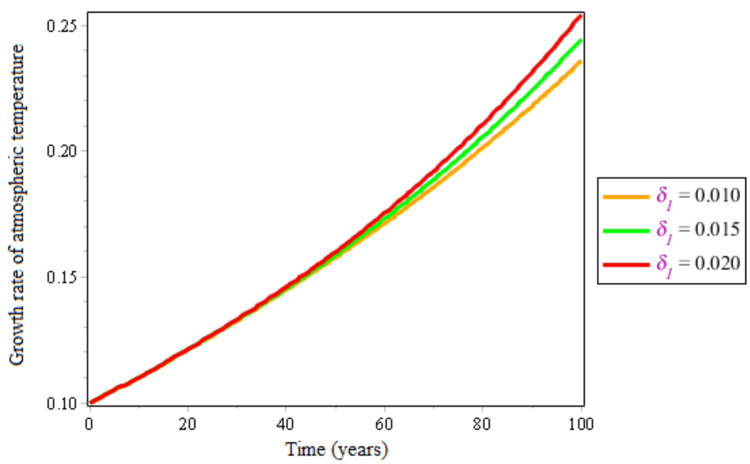

Figure 7. The growth rate of atmospheric temperature for different values of δ_1.

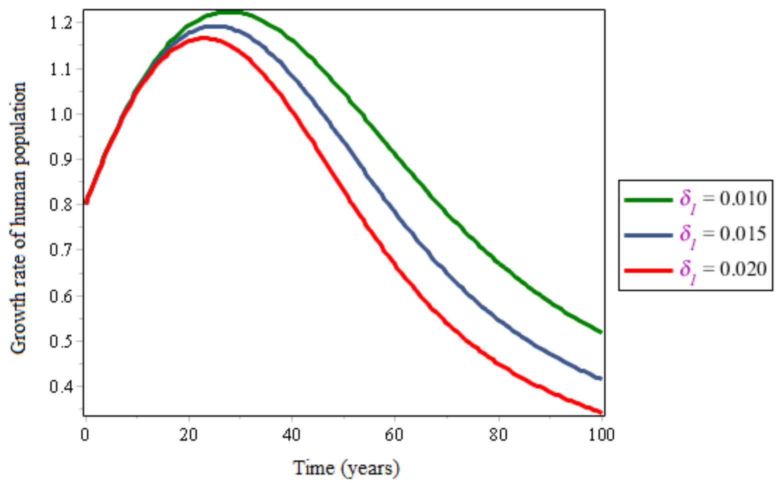

Figure 8. The growth rate of the human population near coastal areas for different values of δ_1.

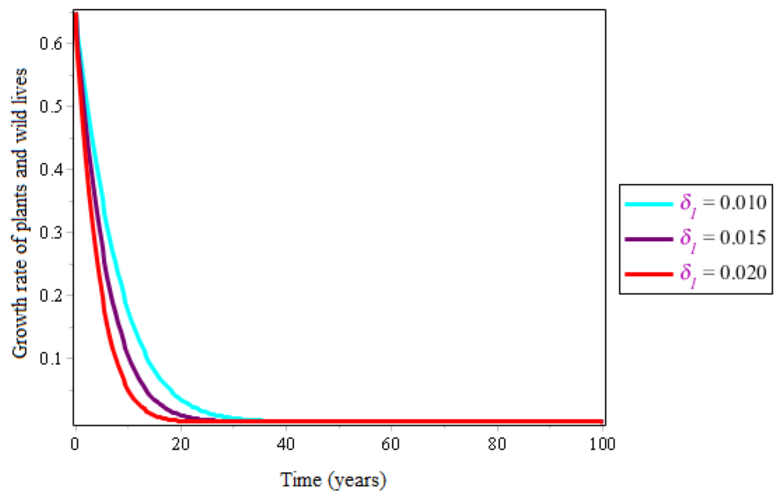

Figure 9. The growth rate of the forest area in the coastal area for different values of δ_1.

5. Conclusion

In this study, a nonlinear mathematical model has been newly formulated to describe the harmful effect of increasing GHGs on global warming and the living beings in the coastal areas. The study analytically analyzed the model to ensure the existence of the considered species and to obtain the nature of the system at each equilibrium point. To describe the dynamical behavior of the species under different concentrations of environmental GHGs, we also carried the study numerically. The study presents that the human population is responsible for increasing GHGs and global warming. And inversely, the human population along with the forest ecosystem are the main victim of the harmful effect of excess emissions of environmental GHGs and rapid global warming. The study also shows that the environmental GHGs and global warming can be controlled by enriching the coastal forestation. On the other hand, if the situation will be continuous, the human population may lose its survival capacity to live and about one-third of the total forest ecosystems may be lost their existence at the end of this century. Therefore, the study concerns with climate change and presents a relationship to environmental management along with mathematical modeling.

Acknowledgments

The work is a part of the M.Sc. thesis of **Sajib Mandal** and supported by NST fellowship, bearing ID: MSc-19201325, Serial: 479, Merit: 601, No: 39.00.0000.012.002.04.19-06, Session: 2019-2020 which is provided by the Ministry of Science and Technology, Government of the People's Republic of Bangladesh.

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Biographies

Sajib Mandal is currently an M.Sc. research (thesis) student in Applied Mathematics at the Department of Mathematics, Bangabandhu Sheikh Mujibur Rahman Science and Technology University, Gopalganj-8100, Bangladesh. He has completed his B.Sc. (Honors) degree in Mathematics at the end of 2018 from the same University. He has already published **Seven** research papers in the peer-reviewed journal (2019, 2020) and **One** book chapter in Springer (2021). He already attended **Six** International conferences and **One** International Workshop at home and abroad during 2019-2020. His research interest consists of Mathematical Modelling, Numerical Analysis, Ordinary Differential Equations (ODEs), Numerical Simulations, Dynamical Systems, Optimal Control, Biomathematics, Climate Change, Global Warming, Marine Ecosystem, and Environmental Management. Recently, **S. Mandal** has become an ordinary member of the Bangladesh Society for Mathematical Biology (BSMB) since the year 2021.

Md. Sirajul Islam is currently affiliated with Bangabandhu Sheikh Mujibur Rahman Science and Technology University, Gopalganj-8100, Bangladesh as an Assistant Professor, Department of Mathematics. Mr. Islam obtained his B.Sc. (Honors) in Mathematics in 2008 and M.Sc. in Applied Mathematics in 2011 from the University of Khulna, Bangladesh. He has more than 9 years of teaching and research experience at the undergraduate and postgraduate levels at different public universities in Bangladesh. He published **15** research papers in peer-reviewed journals and has also attended ICRAMA-2019 (International Conference on Recent Advances in Mathematics and Its Applications organized by the Department of Mathematics, Tripura University, India, during July 16-18, 2019). He has worked on several R & D projects at home and abroad as a PI and/or Researcher. His present research interests include Fluid Mechanics, Computational Fluid Dynamics, Mathematical Modelling, Optimal Control, Numerical Simulation, and Numerical Analysis.

Sonia Akter has completed her M.Sc. in Applied Mathematics in 2018 and B.Sc. in Mathematics in 2016 from Bangabandhu Sheikh Mujibur Rahman Science and Technology University. Now I am a lecturer at the department of EEE at Gono University, Bangladesh. Now I am presently working on Mathematical Modeling along with Optimal Control strategy. My research focuses both on theory and application of optimal control problems with particular attention to real-life problems in Biology and Biomedical Engineering Strengthen on the mathematical epidemiology of Emerging Infectious Diseases (EID).

Dr. Md. Haider Ali Biswas is currently affiliated with Khulna University, Bangladesh as a Professor of Mathematics under Science Engineering and Technology School and he served as the **Head of Mathematics** Discipline from 2015 to 2018. Prof. Biswas obtained his B Sc (Honors) in Mathematics and M Sc in Applied Mathematics in the year 1993 and 1994 respectively from the University of Chittagong, Bangladesh, M Phil in Mathematics in the year 2008 from the University of Rajshahi, Bangladesh, and PhD in Electrical and Computer Engineering from the University of Porto, Portugal in 2013. He has more than 20 years of teaching and research experience at the graduate and post-graduate levels at different public universities in Bangladesh. He published **Three Books**, **Five Book Chapters,** and more than 150 research papers in peer-reviewed journals and international conferences. Prof. Biswas has worked at several R & D projects at home and abroad as PI and/or Researcher, particularly he is conducting different research projects funded by the Ministry of Science and Technology, Bangladesh, University Grants Commission of Bangladesh, and The World Academy of Science (TWAS), Trieste, Italy. His present research interests include Optimal Control with Constraints, Nonsmooth Analysis, ODEs, and Dynamical Systems, Mathematical Modeling, Mathematical Ecology, Environmental modeling, and Climate change, Mathematical Biology and Biomedicine, Epidemiology of Infectious Diseases. For the last ten years, Prof. Biswas has been working on the applications of mathematical models for designing and implementing those to real-life problems, especially for the sustainable/optimal management under the changing environment due to global warming. He is the life/general members of several professional societies and/or research organizations like Bangladesh Mathematical Society (BMS), Asiatic Society of Bangladesh (ASB), Institute of Mathematics and its Applications (IMA), UK, European Mathematical Society (EMS) and Society for Mathematical Biology (SMB). Dr. Biswas is the founder member of Mathematical Forum Khulna and served as the General Secretary of the Forum in 2013-2015. Dr. Biswas organized several national and international seminars/workshops/conferences at home and abroad and he has been working as Editor/Member of editorial boards of several international peer-reviewed journals. Professor Biswas contributed as Keynote/Invited/Plenary/Panel speaker at several international conferences/seminars/workshops at home and abroad. Professor Biswas has been nominated as a Member of the Council of Asian Science Editors (CASE) for 2017-2020 and the Associate Member of the Organization for Women in Science for the Developing World (OWSD) since 2017. Recently, Professor Biswas has been elected as a Member of the Executive Committee of Bangladesh Mathematical Society (BMS) for the year 2020-2021, and also nominated as the Associate Editor of the international journal *GANIT*-Journal of Bangladesh Mathematical Society (BMS) for the year 2020-2021. Dr. Biswas has been nominated as a Member of the Executive Committee of the IEOM Society, Bangladesh Chapter, and also serving as the Treasurer of the IEOM Society, Bangladesh Chapter. He is also serving as the Faculty Advisor of the IEOM Society Khulna University Chapter. Professor Biswas is presently serving as the President of the Bangladesh Society for Mathematical Biology (BSMB) for the year 2020-2022.