Mathematical Assessment for the Dynamical Model of Sexual Violence of Women in Bangladesh

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Abstract

Sexual violence is increasing by significance rate now-a-days. Girls aged 3 to 60 all are the victims of sexual harassment directly or indirectly. According to World Health Organization (WHO), globally one-third of the world's women are being subjected to sexual violence. Sexual violence is also increasing in Bangladesh on a daily basis. About 84% of Bangladeshi women are constantly being subjected to sexual harassment. Risk factors, related to sexual violence include lower levels of education, harmful use of alcohol, antisocial mental disorder, witnessing family violence, gender inequality, etc. Governments of each and every country are taking action against it, but the rate of sexual violence is increasing day by day. To investigate the underlying causes of sexual violence, we propose to formulate a five compartmental model to describe the dynamics of the system of social crime. For the analytical validation of the model, we perform equilibrium analysis, stability of the system at equilibrium points, and basic reproduction number. Using the parameter estimation of the real data of sexual violence, we investigate the numerical simulations for various parameter combinations. To visualize our findings, we illustrate the numerically simulated solutions with analytical results for better prediction of reducing the violence.

Keywords: Sexual violence, Mathematical model, Basic reproduction number, Stability analysis, Numerical simulation.

1. Introduction

Among all the violence against women, sexual violence is an alarming problem today. It happens globally. Sexual violence is any sexual act or attempt to obtain a sexual act through violence, trafficking in an individual or acts against a person's sexuality, irrespective of the relationship of the victim (WHO Report, 2020). The main victims of sexual violence are women. It has an influence on physical and mental health. In addition to causing physical injury, it is associated with an increased risk of a range of sexual and reproductive health issues, with both immediate and long-term implications. Its impact on mental health may be as serious as its physical impact. Sexual abuse includes sexual assault, sexual abuse of children, sexual harassment, rape, attempted rape, gang rape, sexual violence, slavery, prostitution, forced pregnancy, abortion, forced marriage, etc (WHO Report, 2015). Global estimates show that in their lifetime, about 1 in 3 (35%) of women worldwide have experienced either physical and/or sexual intimate partner violence or non-partner sexual violence, and up to 38% of women's killings are committed by a male intimate partner (WHO 2017). In 2019, there were 1080, 149 and 189 incidences of rape, attempted rape and sexual harassment in Bangladesh, respectively (Odhiker, 2019). According to the Bangladesh Mahila Parishadad, Bangladesh saw 1093 incidences of rape from January to November 2020. It can lead to fatal outcomes, such as homicide or suicide, injuries, unintentional pregnancies (abortions, gynecological problems, and sexually transmitted infections, including HIV), health problems (headaches, back pain, abdominal pain, gastrointestinal disorders), etc (WHO Report, 2017).

Mathematical modelling plays an incredible role in every sector nowadays. It has already contributed to a better understanding of the mechanisms of different critical phenomena and has received increasing attention because rapid, cost-effective, and illuminating evaluation is possible through modeling and simulation. Many mathematical models have been studied as well. Sebil and Otoo (2014) constructed the model to describe the domestic violence epidemic model. The model discusses the simple continuous model for the spread of domestic violence. Delgadillo-Aleman *et al*. (2019) formulated a mathematical model based on differential equations for the intimate partner violence that described the dynamics of intimate partner violence in which the man perpetrated violence against the women. It also focused on different key factors reported in literature as causal or motivational factors to perpetrate intimate partner violence that include the man's need to control the women, social pressure on the women to remain married, failures in self-regulation, and empowerment programs. In 2014, Biswas et al. investigate a SEIR model for control of infectious diseases with constraints. We also studied some theoretical and statistical model. Krantz and Garcia-Moreno (2005) described theoretically about violence against women and discussed difference types of violence against women. Adjah and Agbemafle (2016) presented a statistical analysis that investigates the factors whose are increasing the likelihood of an event of domestic violence as reported by ever married Ghanaian women. It also indicates that place of residence, alcohol use by husband and family, history of violence do increase a women's risk of domestic violence. Also, higher than secondary education acted as a protective buffer against domestic violence. Livingston, M. (2010) discussed that whether alcohol outlet density is related to domestic violence and whether this relationship is due to alcohol availabil-ity or to co-occurring economic disadvantage and social disorganisation.

Sexual violence against women becomes a major prbolem worlwide. It has became a cure for our country. So it is important to reduce the violence. As per our view, there is no work that highlight on mathematical model of sexual violence. This paper deals with six compartmental model: potentially violent, violent, recovered violent, susceptible victim, victim and recovered victim. The purpose of this study is to develop a mathematical model of sexual violence against women in terms of a set of nonlinear ordinary differential equations showing that this violence can be reduced.

2. Compartmental Model of Sexual Violence

In order to study the dynamics of sexual violence in two interacting populations of violent individuals and victims, we formulate a model that divides the population of violent population into: potentially violent $S_1(t)$, violent $V_1(t)$, and recovered violent $R_1(t)$, while the victim population is divided into: susceptible victim $S_2(t)$, victim $V_2(t)$, and recovered victim $R_2(t)$. We consider b_1 and b_2 as the recruitment rate of potentially violent population and victims' population, α_1 and α_2 as the rate of potentially violent becomes violent and susceptible victims becomes victims, r_1 and r_2 as the recovery rate of violent population by punishment and victims, γ_1 and γ_2 as the rate of potentially violent becomes recovered violent by counseling and recovered victim becomes susceptible victim, ω as the contact rate between violent individuals and susceptible victim, θ as the violence includes death rate and ρ as the natural death rate for this model.

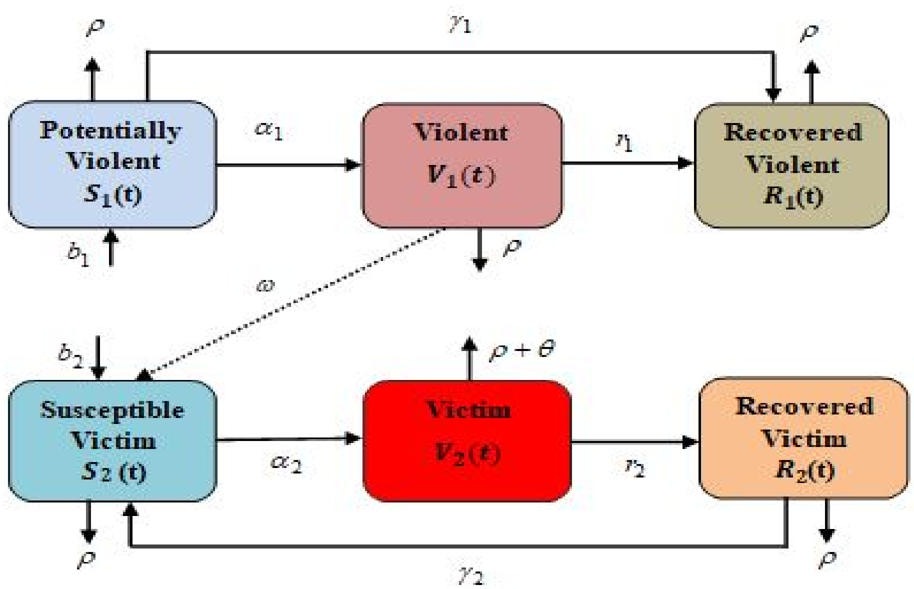

Figure 1.Flowchart of the compartmental model of Sexual violence.

The above model can be formulated by the following system of nonlinear ordinary differential equations:

$$\begin{aligned}
\frac{dS_1}{dt} &= b_1 - \alpha_1 S_1 V_1 - (\rho + \gamma_1) S_1 \\
\frac{dV_1}{dt} &= \alpha_1 S_1 V_1 - (\rho + r_1) V_1 \\
\frac{dR_1}{dt} &= r_1 V_1 + \gamma_1 S_1 - \rho R_1 \\
\frac{dS_2}{dt} &= b_2 - \omega \alpha_2 S_2 V_1 - \rho S_2 + \gamma_2 R_2 \\
\frac{dV_2}{dt} &= \alpha_2 S_2 - (\rho + \theta) V_2 - r_2 V_2 \\
\frac{dR_2}{dt} &= r_2 V_2 - (\gamma_2 + \rho) R_2
\end{aligned} \tag{1}$$

with $S_1(0) = S_{1_0}$, $V_1(0) = V_{1_0}$, $R_1(0) = R_{1_0}$ $S_2(0) = S_{2_0}$, $V_2(0) = V_{2_0}$, $R_2(0) = R_{2_0}$.

3. Analysis of the Model

We investigate the positivity of the proposed model in order to show the validation and well-positioning of the developed model. In addition, different equilibria (violence free and endemic equilibrium points) are calculated, the basic reproduction number is also determined, and the stability analysis is performed at two different equilibria.

3.1 Positivity Analysis

Lemma 1: If $S_1(t) > 0$, $V_1(t) > 0$, $R_1(t) > 0$, $S_2(t) > 0$, $V_2(t) > 0$ and $R_2(t) > 0$ then the solutions $S_1(t)$, $V_1(t)$, $R_1(t)$, $S_2(t)$, $V_2(t)$ and $R_2(t)$ of the model (1) are non-negative.

Proof: We use the equations of the model (1) to prove Lemma 1.
From model (1), we have

$$\begin{aligned}
&\frac{dS_1(t)}{dt} \geq b_1 - (\rho + \gamma_1) S_1 \\
&\Rightarrow \frac{dS_1(t)}{dt} + (\rho + \gamma_1) S_1 \geq b_1 \\
&\therefore \text{I.F} = e^{\int (\rho+\gamma_1) dt} = e^{(\rho+\gamma_1)t}
\end{aligned} \tag{2}$$

Multiplying both sides of (2) by $e^{(\rho+\gamma_1)t}$ we have,

$$\begin{aligned}
e^{(\rho+\gamma_1)t} \frac{dS_1(t)}{dt} + e^{(\rho+\gamma_1)t} (\rho + \gamma_1) S_1 \geq b_1 e^{(\rho+\gamma_1)t} &\Rightarrow \frac{d}{dt}(S_1 e^{(\rho+\gamma_1)t}) \geq b_1 e^{(\rho+\gamma_1)t} \\
&\Rightarrow d(S_1 e^{(\rho+\gamma_1)t}) \geq b_1 e^{(\rho+\gamma_1)t} dt
\end{aligned} \tag{3}$$

Now integrating (3), we have

$$S_1 e^{(\rho+\gamma_1)t} \geq \frac{b_1 e^{(\rho+\gamma_1)t}}{(\rho+\gamma_1)} + c \tag{4}$$

where c is a constant. Applying the initial condition at $t = 0,\ S_1(\text{t}) \geq S_1(0)$.
Hence from (4), we get

$$S_1(0) \geq \frac{b_1}{(\rho+\gamma_1)} + c \Rightarrow S_1(0) - \frac{b_1}{(\rho+\gamma_1)} \geq c$$

Putting the value of c into (4), we get

$$S_1 e^{(\rho+\gamma_1)t} \geq \frac{b_1 e^{(\rho+\gamma_1)t}}{(\rho+\gamma_1)} + \left(S_1(0) - \frac{b_1}{(\rho+\gamma_1)} \right) \Rightarrow S_1(t) \geq \frac{b_1}{(\rho+\gamma_1)} + e^{-(\rho+\gamma_1)t} \left(S_1(0) - \frac{b_1}{(\rho+\gamma_1)} \right) \tag{5}$$

Hence, $S_1(t) > 0$ at $t = 0$ and $t \to \infty$. Similarly we can find the positivity of $V_1(t)$, $R_1(t)$, $S_2(t)$, $V_2(t)$ and $R_2(t)$ under the initial conditions.

Therefore, it is proved that ($S_1(t) > 0$ $V_1(t) > 0$, $R_1(t) > 0$, $S_2(t) > 0$, $V_2(t) > 0$, $R_2(t) > 0$ $\forall t \geq 0$).

3.2 Equilibrium Points

3.2.1 Violence Free Equilibrium Points

Let $E_{vfe}(S_1^*, V_1^*, R_1^*, S_2^*, V_2^*, R_2^*)$ be the violence free equilibrium point of the model (1). We need to solve $\frac{dS_1^*}{dt} = \frac{dV_1^*}{dt} = \frac{dR_1^*}{dt} = \frac{dS_2^*}{dt} = \frac{dV_2^*}{dt} = \frac{dR_2^*}{dt} = 0$ of the model (1) in order to find the violence-free equilibrium point.

At the violence free equilibrium point $E_{vfe}(S_1^*, V_1^*, R_1^*, S_2^*, V_2^*, R_2^*)$, the model (1) can be expressed as:

$$
\begin{aligned}
& b_1 - \alpha_1 S_1^* V_1^* - (\rho + \gamma_1) S_1^* = 0 \\
& \alpha_1 S_1^* V_1^* - (\rho + r_1) V_1^* = 0 \\
& r_1 V_1^* + \gamma_1 S_1^* - \rho R_1^* = 0 \\
& b_2 - \omega \alpha_2 S_2^* V_1^* - \rho S_2^* + \gamma_2 R_2^* = 0 \\
& \alpha_2 S_2^* - (\rho + \theta + r_2) V_2^* = 0 \\
& r_2 V_2^* - (\gamma_2 + \rho) R_2^* = 0
\end{aligned}
\tag{6}
$$

Since there is no violence at the violence free equilibrium point (i.e. $V_1^* = V_2^* = 0$), we obtain from (6),

$$S_1^* = \frac{b_1}{(\rho + \gamma_1)},\ R_1^* = \frac{\gamma_1 b_1}{\rho(\rho + \gamma_1)},\ S_2^* = \frac{b_2}{\rho},\ R_2^* = 0$$

Then, the violence free equilibrium point is $E_{vfe}(S_1^*, V_1^*, R_1^*, S_2^*, V_2^*, R_2^*) = \left(\frac{b_1}{(\rho + \gamma_1)}, 0, \frac{\gamma_1 b_1}{\rho(\rho + \gamma_1)}, \frac{b_2}{\rho}, 0, 0 \right)$.

3.1.2 Endemic Equilibrium Points

For endemic equilibrium points, sexual violence exists everywhere. So here $V_1^* \neq V_2^* \neq 0$.

Now we have to solve the system of equations (6) for finding the endemic equilibrium points $E_{ee}(S_1^*, V_1^*, R_1^*, S_2^*, V_2^*, R_2^*)$.

Where

$$S_1^* = \frac{b_1}{\gamma_1 + \rho}$$

$$V_1^* = 0$$

$$R_1^* = \frac{b_1 \gamma_1}{\rho(\gamma_1 + \rho)}$$

$$S_2^* = \frac{b_2(\gamma_2 + \rho)(r_2 + \rho + \theta)}{\gamma_2 \rho^2 + r_2 \rho^2 + \rho^2 \theta + \rho^3 - \alpha_2 \gamma_2 r_2 + \gamma_2 r_2 \rho + \gamma_2 \rho \theta}$$

$$V_2^* = \frac{\alpha_2 b_2(\gamma_2 + \rho)}{\gamma_2 \rho^2 + r_2 \rho^2 + \rho^2 \theta + \rho^3 - \alpha_2 \gamma_2 r_2 + \gamma_2 r_2 \rho + \gamma_2 \rho \theta}$$

$$R_2^* = \frac{\alpha_2 b_2 r_2}{\gamma_2 \rho^2 + r_2 \rho^2 + \rho^2 \theta + \rho^3 - \alpha_2 \gamma_2 r_2 + \gamma_2 r_2 \rho + \gamma_2 \rho \theta}$$

3.3 Basic Reproduction Number

In this section, the Basic Reproduction Number is a measure of potentiality of the spread of sexual violence in a given population. We will use the next generation matrix method in order to determine the basic reproduction number of the model (1), by considering the violence and victims compartments of the system.

Let F_i be the rate of sexual violence appearance in the compartment *i* and V_i^* be the rate at which people are transferred from, given the sexual violence free equilibrium, then largest eigen value of the next generation matrix donated by:

$$G = F(V^*)^{-1}$$

where

$$F = \begin{pmatrix} \dfrac{\alpha_1 b_1}{\gamma_1 + \rho} & 0 \\ 0 & 0 \end{pmatrix}$$

$$V^* = \begin{pmatrix} (\rho + r_1) & 0 \\ 0 & (\rho + \theta + r_2) \end{pmatrix}$$

Then,

$$(V^*)^{-1} = \begin{pmatrix} \dfrac{1}{(\rho + r_1)} & 0 \\ 0 & \dfrac{1}{(\rho + \theta + r_2)} \end{pmatrix}$$

Thus the Basic Reproduction Number R_0, is obtained as

$$R_0 = \frac{\alpha_1 b_1}{(\gamma_1 + \rho)(r_1 + \rho)} \tag{7}$$

3.4 Stability Analysis at Violence-free Equilibrium Point

Here, by proving the following theorem 1, we verify the stability at the disease free equilibrium point.

Theorem 1: The violence free equilibrium point of the model (1) is asymptotically stable if the eigen values of the Jacobian matrix are negative.

Proof: The Jacobian matrix of model (1) is given by

$$J = \begin{pmatrix} -\gamma_1 - \rho - V_1 \alpha_1 & -S_1 \alpha_1 & 0 & 0 & 0 & 0 \\ V_1 \alpha_1 & -\rho - r_1 & 0 & 0 & 0 & 0 \\ \gamma_1 & r_1 & -\rho & 0 & 0 & 0 \\ 0 & -S_2 \alpha_2 \omega & 0 & -\rho - V_1 \alpha_2 \omega & 0 & 0 \\ 0 & 0 & 0 & 0 & -r_2 - \rho - \theta & 0 \\ 0 & 0 & 0 & 0 & r_2 & -\gamma_2 - \rho \end{pmatrix} \tag{8}$$

In Echelon form the above matrix is written as

$$J(S_1^*, V_1^*, R_1^*, S_2^*, V_2^*, R_2^*) = \begin{pmatrix} -a_{11} & 0 & 0 & 0 & 0 & 0 \\ V_1^* \alpha_1 & -\rho - r_1 & 0 & 0 & 0 & 0 \\ \gamma_1 & r_1 & -\rho & 0 & 0 & 0 \\ 0 & -S_2^* \alpha_2 \omega & 0 & -\rho - V_1^* \alpha_2 \omega & 0 & 0 \\ 0 & 0 & 0 & 0 & -r_2 - \rho - \theta & 0 \\ 0 & 0 & 0 & 0 & r_2 & -\gamma_2 - \rho \end{pmatrix} \tag{9}$$

Where,

$$a_{11} = -\alpha_1^2(\gamma_1 + \rho + V_1^* \alpha_1)(-r_1 - \rho)$$

The following matrix is the Identity matrix of same dimension:

$$I=\begin{pmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{pmatrix} \tag{10}$$

The characteristic equation of equation (9) can be obtained by $\det(J-\lambda I)=0$.

$$\begin{pmatrix}-a_{11}-\lambda&0&0&0&0&0\\V_1^*\alpha_1&-\rho-r_1-\lambda&0&0&0&0\\\gamma_1&r_1&-\rho-\lambda&0&0&0\\0&-S_2^*\alpha_2\omega&0&-\rho-V_1^*\alpha_2\omega-\lambda&0&0\\0&0&0&0&-r_2-\rho-\theta-\lambda&0\\0&0&0&0&r_2&-\gamma_2-\rho-\lambda\end{pmatrix}=0 \tag{11}$$

Substituting $E_{vfe}(S_1^*,V_1^*,R_1^*,S_2^*,V_2^*,R_2^*)=\left(\frac{b_1}{(\rho+\gamma_1)},0,\frac{\gamma_1 b_1}{\rho(\rho+\gamma_1)},\frac{b_2}{\rho},0,0\right)$, we obtain

$$\begin{pmatrix}-\alpha_1^2(\gamma_1+\rho+V_1^*\alpha_1)(-r_1-\rho)-\lambda&0&0&0&0&0\\0&-r_1-\rho-\lambda&0&0&0&0\\\gamma_1&r_1&-\rho-\lambda&0&0&0\\0&-\frac{\alpha_2 b_2\omega}{\rho}&0&-\rho-\lambda&0&0\\0&0&0&0&-r_2-\rho-\theta-\lambda&0\\0&0&0&0&r_2&-\gamma_2-\rho-\lambda\end{pmatrix}=0 \tag{12}$$

By solving equation (12) the eigen values are,

$\lambda_1=-\rho$

$\lambda_2=-\rho$

$\lambda_3=-\alpha_1^2(\gamma_1+\rho)(-r_1-\rho)$

$\lambda_4=-\rho-\gamma_2$

$\lambda_5=-\rho-r_1$

$\lambda_6=-\rho-\theta-r_2$

Since $\lambda_1,\lambda_2,\lambda_3,\lambda_4$ and λ_5 are all negative, so the violence-free equilibrium point is asymptotically stable.
Hence, Theorem 1 is proved.

3.5 Stability Analysis at Endemic Equilibrium Point

In this section, we verify the stability at endemic equilibrium point by proving Theorem 2.

Theorem 2: The endemic equilibrium point of the model (1) is asymptotically stable if the eigenvalues of the Jacobian matrix are negative.
Proof: The Jacobian matrix of model (1) at the endemic equilibrium point is

$$J(S_1^*, V_1^*, R_1^*, S_2^*, V_2^*, R_2^*) = \begin{pmatrix} -a_{11} & 0 & 0 & 0 & 0 & 0 \\ V_1^*\alpha_1 & -\rho - r_1 & 0 & 0 & 0 & 0 \\ \gamma_1 & r_1 & -\rho & 0 & 0 & 0 \\ 0 & -S_2^*\alpha_2\omega & 0 & -\rho - V_1^*\alpha_2\omega & 0 & 0 \\ 0 & 0 & 0 & 0 & -r_2 - \rho - \theta & 0 \\ 0 & 0 & 0 & 0 & r_2 & -\gamma_2 - \rho \end{pmatrix} \quad (13)$$

Equation (13) is a 6×6 matrix and the characteristic equation is given by $\det(J - \lambda I) = 0$.

$$\begin{pmatrix} -a_{11} - \lambda & 0 & 0 & 0 & 0 & 0 \\ V_1^*\alpha_1 & -\rho - r_1 - \lambda & 0 & 0 & 0 & 0 \\ \gamma_1 & r_1 & -\rho - \lambda & 0 & 0 & 0 \\ 0 & -S_2^*\alpha_2\omega & 0 & -\rho - V_1^*\alpha_2\omega - \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & -r_2 - \rho - \theta - \lambda & 0 \\ 0 & 0 & 0 & 0 & r_2 & -\gamma_2 - \rho - \lambda \end{pmatrix} = 0$$

$$\Rightarrow (a_{11} - \lambda)(-\rho - r_1 - \lambda)(-\rho - \lambda)(-\rho - V_1^*\alpha_2\omega - \lambda)(-r_2 - \rho - \theta - \lambda)(-\gamma_2 - \rho - \lambda) = 0$$

Here the eigenvalues are given by

$\lambda_1 = -a_{11}$

$\lambda_2 = -\rho - r_1$

$\lambda_3 = -\rho$

$\lambda_4 = -\rho - V_1^*\alpha_2\omega$

$\lambda_5 = -\rho - \theta - r_2$

$\lambda_6 = -\rho - \gamma_2$

Where,

$a_{11} = -\alpha_1^2(\gamma_1 + \rho + V_1^*\alpha_1)(-r_1 - \rho)$

All the eigenvalues of the characteristic equation are real numbers. Since $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ and λ_6 are all negative, the endemic equilibrium point is asymptotically stable.
Hence, Theorem 2 is proved.

5. Numerical Simulation

We are conducting numerical simulations of the sexual violence model discussed in the previous sections. We used some hypothetical data to demonstrate the analytical results. The simulation is done using the MATLAB programming ODE45 solver. Our main objective is to illustrate the outcome through numerical simulations. These numerical results, together with the verification of our analytical observations, are very important from a practical point of view. The numerical result of our dynamical model shows different behaviors of our dynamical model. We have chosen the initial values as $S_1(0) = 800$, $V_1(t) = 350$, $R_1(t) = 80$, $S_2(t) = 1000$, $V_2(t) = 520$ and $R_2(t) = 200$ for solving the model numerically. Here we perform the numerical simulations for time t=0 to t=30. First, we have solved the model (1) representing all the parametric values considered for our model for the tabulated values in Table 1, and the simulated graph is presented in Figure 2. Then the result of the combined classes is presented in Figure 3.

Table 1. Parameter Specifications of model (1).

Parameters	Descriptions	Values
b_1	Recruitment rate of potentially violent population	0.29
b_2	Recruitment rate of susceptible victims' population	0.6
α_1	Rate of potentially violent becomes violent	0.004

α_2	Rate of susceptible victims becomes victims	0.0032
r_1	Rate of recovered violent by punishment	0.0166
r_2	Rate of recovered victims	0.043
γ_1	Rate of potentially violent becomes recovered violent by counseling	0.00066
γ_2	Rate of recovered victim becomes susceptible victim	0.0014
ω	Contact rate between violent individuals and susceptible victim	0.6
θ	Violence includes death rate	0.003
ρ	Natural death rate	0.0124

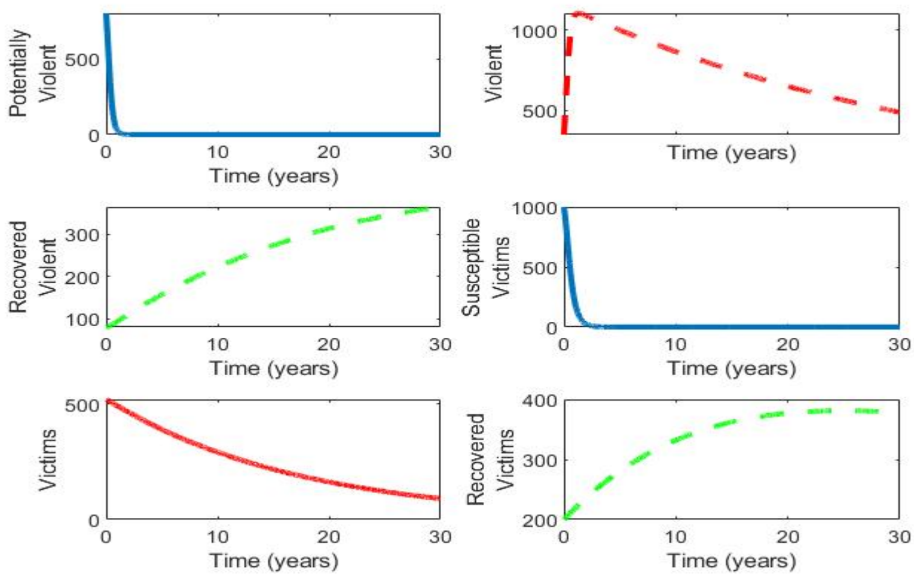

Figure 2. Trajectories of all the model compartments (1).

Figure 2 shows the dynamics of six compartments such as potentially violent, violent, recovered violent, susceptible victim, victims, and recovered victims. We have observed that the potentially violent population decreases from the initial state and it reaches to zero steadily. The violent population increases for first year and then it decreases quickly. The recovered violent populations increase from the initial state. Again the susceptible victims decrease from the initial state and it reaches to zero steadily. The victims increase for first year and then it decreases quickly and the recovered victims increase from the initial state.

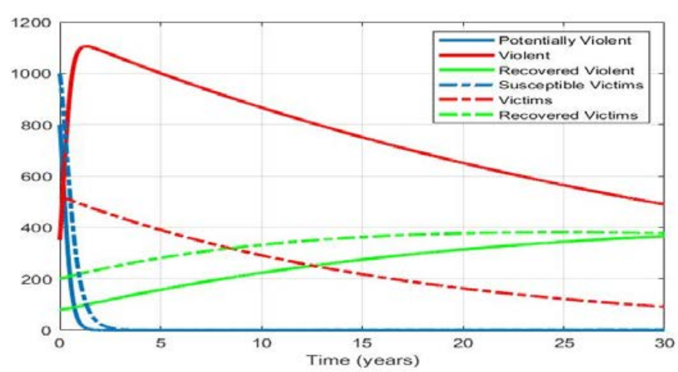

Figure 3. Combined figure for the model (1).

Figure 3 represents all the compartments together. It shows how potentially violent, violent, recovered violent, susceptible victim, victims, and recovered victims changes.

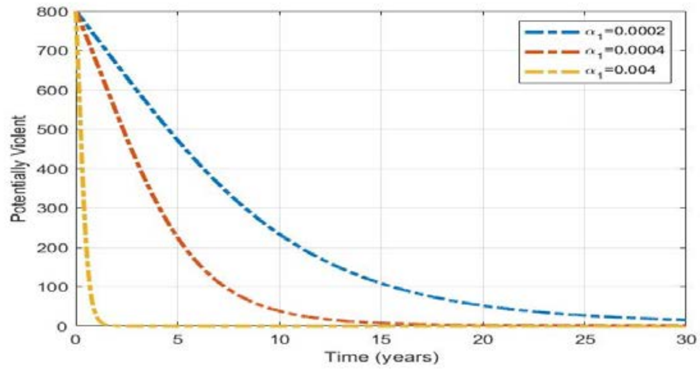

Figure 4. Variation of potentially violent populations with time (30 years) for different parameter values of α_1.

Figure 4 indicates the influence the parameter (α_1) on the potentially violent populations for 30 years period. In Figure 4, it has been noticed that the potentially violent populations decrease as the increase of the parameter α_1 i.e. the rate of potentially violent becomes violent (when $\alpha_1 \in (0.0002, 0.004)$) increases.

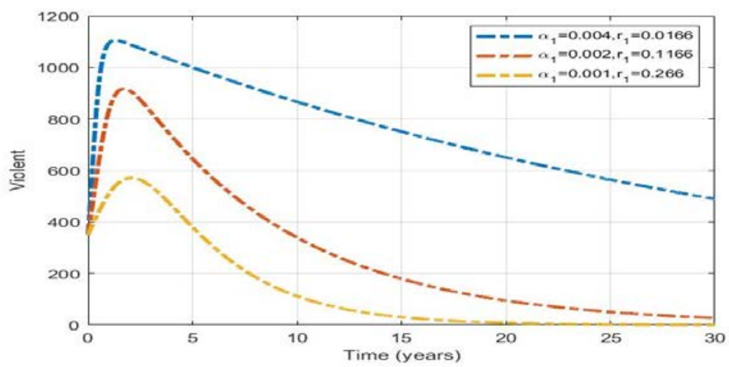

Figure 5. Variation of violent populations with time (30 years) for different parameter values of α_1 and r_1.

Figure 5 shows the graph for the different values of the parameter (α_1, r_1) on the violent populations for 30 years period. In Figure 5, it indicates that the violent populations decrease as the decrease of the parameter α_1 i.e. the rate of potentially violent becomes violent (when $\alpha_1 \in (0.001, 0.004)$) decreases and the increase of the parameter r_1 i.e. rate of recovered violent population (when $r_1 \in (0.0166, 0.266)$) increases. Then the sexual violence also reduced.

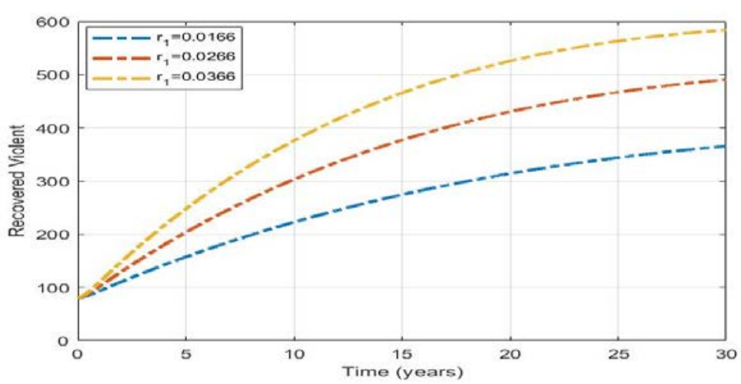

Figure 6. Variation of recovered violent populations with time (30 years) for different parameter values of r_1.

Figure 6 shows the graph for the different values of the parameter r_1 on the recovered violent populations for 30 years period. In Figure 6, it indicates that the recovered violent populations increase as the increase of the parameter r_1 i.e. rate of recovered violent population (when $r_1 \in (0.0166, 0.0366)$) increases.

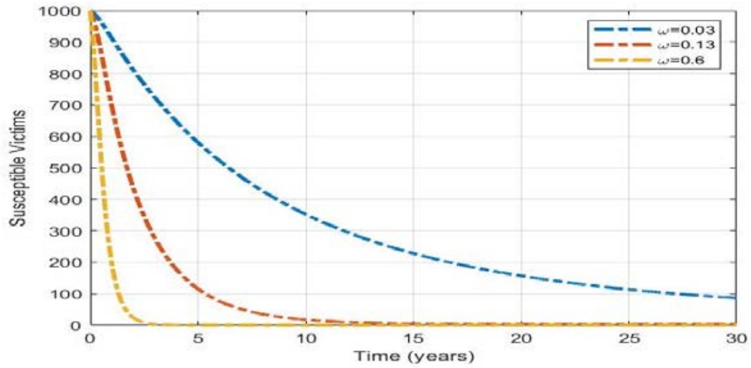

Figure 7. Variation of susceptible victims with time (30 years) for different parameter values of ω

Figure 7 indicates the influence the parameter (ω) on the susceptible victims for 30 years period. In Figure 7, it has been noticed that the susceptible victims decrease as the increase of the parameter ω i.e. Contact rate between violent individuals and susceptible victim (when $\omega \in (0.03, 0.6)$) increases.

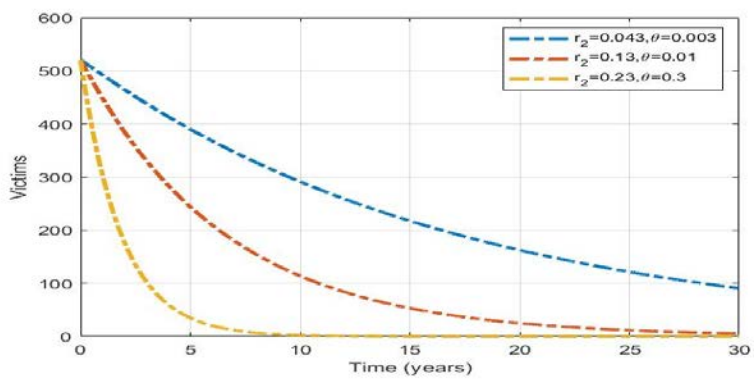

Figure 8. Variation of victims with time (30 years) for different parameter values of r_2 and θ.

Figure 8 shows the graph for the different values of the parameter (r_2, θ) on the victims for 30 years period. In Figure 8, it indicates that the victims decrease as the increase of the parameter (r_2) i.e rate of recovered victims (when $r_2 \in (0.043, 0.23)$) increases and also the increase of the parameter θ i.e violence includes death rate (when $\theta \in (0.003, 0.3)$) increases.

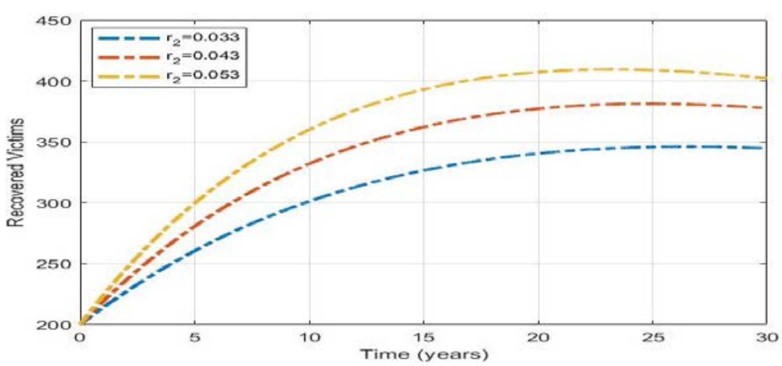

Figure 9. Variation of recovered victims with time (30 years) for different parameter values of r_2.

Figure 9 shows the graph for the different values of the parameter r_2 on the recovered victims for 30 years period. In Figure 9, it indicates that the recovered victims increase as the increase of the parameter r_2 i.e. rate of recovered victims (when $r_2 \in (0.033, 0.053)$) increases.

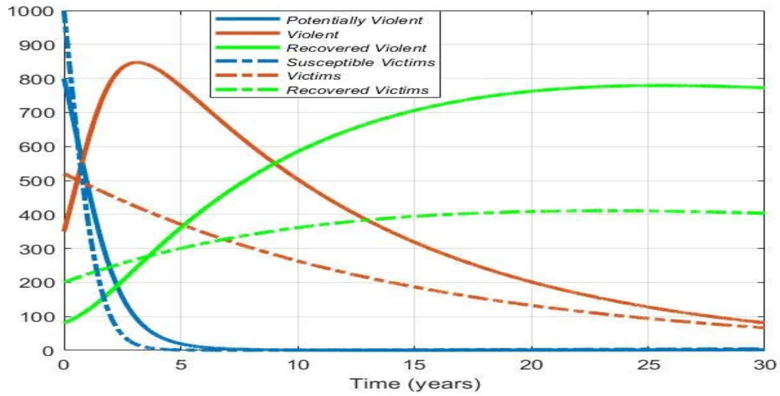

Figure 10. Attitude of the total population when all the parameters (Table 1) remained constant except α_1, r_1 and r_2.
Here $\alpha_1 = 0.001$ $r_1 = 0.08$ and $r_2 = 0.053$.

Figure 10 shows the graph for the values of the parameter $\alpha_1 = 0.001$ $r_1 = 0.08$ and $r_2 = 0.053$ on the total population. In Figure 10, it indicates that when the values of the parameter α_1 (rate of potentially violent becomes violent) decreases and r_1 (rate of recovered violent population) and r_2 (rate of recovered victims) increase then the recovery violent and recovery victims population increase significantly and also the violent population and victims reduced. Thus the violence can be reduced from our country.

6. Conclusion

In this paper, we have developed a mathematical model of sexual violence in terms of a set of nonlinear ordinary differential equations showing that susceptible victims are becoming victims gradually with the contact of violent population. So, women in our nation face this issue every day. With the determination of the basic reproductive ratio and stability analysis at the violence-free and endemic equilibrium points, we have analyzed the model. Finally, to illustrate the analytical results, numerical simulations were conducted. We have observed that the violence reduce when the violent populations and victims decreases. That is, the violent populations reduce with the decreases of transmission rate of potentially violent population decrease and recovery rate of violent population increase. Also, the victims will deduce when the recovery rate of victims will increase. Thus the outcome of the numerical experiment conducted that it can significantly eradicate sexual violence from society when proper measures are taken to change the attitudes of violent people.

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