Mathematical Approach to Assess the Severity of Road Accidents in Bangladesh Using a SEIR-type Model

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Abstract

Traffic safety is one of the most concerning issues worldwide now a day. Thousands of life are being lost every day only for road accidents in each and every country and geographical region. Road accidents have increased with rapid development of the transportation sector. Therefore every government has considered it as one of the vital problems of this time and started taking steps for reducing the road accident and damage caused by the road accidents. We analyze that massive amount of statistical data for investigating the contributory factors behind road accidents is impractical and time-consuming too. In this paper we have formulated a dynamical model for describing the road accident in dynamics faulty or misguided drivers and faulty vehicles are mainly liable for accidents. We express the dynamics in terms of a system of four ordinary differential equations, with four state variables and parameters we have formulated a mathematical model who is capable of determining the number of fatal road accidents. For validating the model we have investigated the positivity of the solutions, equilibrium points, stability at the equilibrium points and other necessary analytical analysis. For justifying our analysis we have performed numerical simulation with the parameter values obtained from authenticated sources visualized the results.

Keywords:
Basic reproductive ratio, Human and environmental factors, Mathematical model, Numerical simulation, Road accident.

1. Introduction
An accident is an unforeseen occurrence of physical harm to the inanimate structure of animation. In particular, road accidents are now recognized as a global phenomenon, with authorities worried that the number of people killed and seriously injured on their roads is increasing in virtually every country worldwide (Ahsan et al. 2011). Due to the simultaneous flow of different traffic, a mixed flow of traffic and pedestrians, accidents usually occur. In daily living, the nuisance of road accidents is inevitable. It cannot be prevented entirely. An accident in animating animated structures is an unexpected bodily damage event. In particular, road accidents are now recognized by the authorities of all countries around the world as a global phenomenon, with the authorities concerned about the increasing number of people killed and seriously injured on their roads. The World Road Traffic Injury Prevention Report (2004) (Ahsan et al. 2011). Every year, 1.2 million people are killed and around 50 million are injured in road accidents around the world. The story of the global road safety crisis reveals the enormous number of injuries and deaths due to road traffic accidents. Road crashes are the second leading cause of death for individuals aged between 5 and 29 and the third leading cause of death for individuals aged between 30 and 44. As the number of vehicles in developing countries has increased rapidly (WHO 2004), it is on the way to becoming the third leading cause of death and disability. In Bangladesh, road accidents are a growing concern because the country's problems with road safety on international roads are very serious. Traffic accidents have been increasing rapidly in Bangladesh over the last two decades, leading to enormous social and economic losses in terms of loss of life and property. A common cause of central significance in the management of road safety is the collection and use of accurate and comprehensive data relating to road accidents. The interpretation of this data can contribute to a better understanding of operational problems, to the preconditions for an accurate diagnosis of accident issues, A common cause of central significance in the management of road safety is the collection and use of accurate and comprehensive data relating to road accidents. The interpretation of this data can contribute to a better understanding

of operational problems, to the preconditions for an accurate diagnosis of accident issues, and the development of corrective measures and the evaluation of the effectiveness of our road safety programs. An extensive database is a basic prerequisite for any effective road safety initiative. Any organization involved in public or private road safety activities should have a clear idea of the nature, scale and distribution method of the road accident problem that they want to solve. The trend of accidents in Bangladesh in the six years from 1998 to 2003 is shown in **Figure 1.**

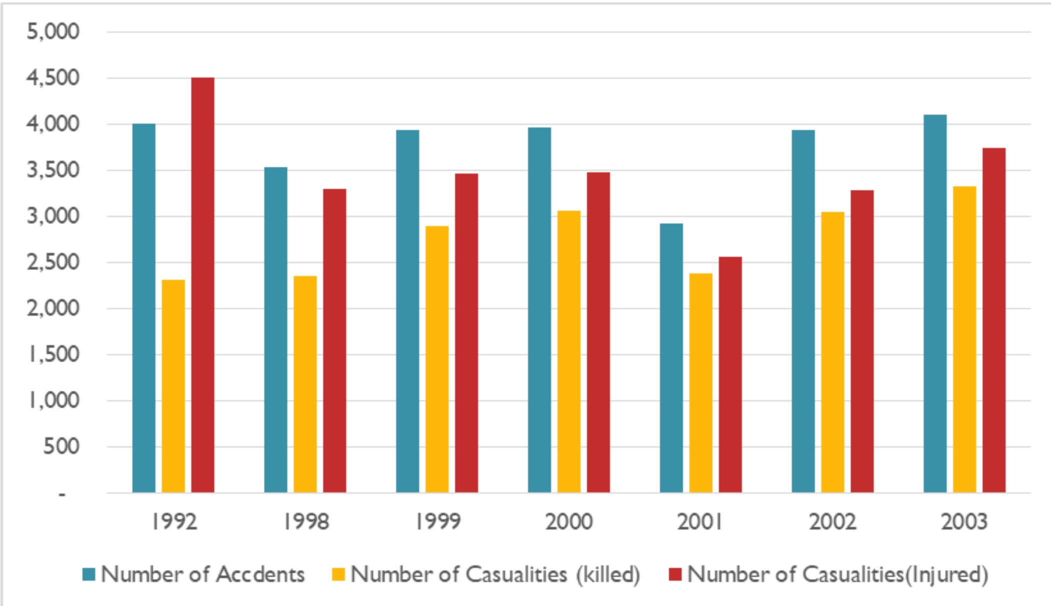

Figure 1: Road accident rates in Bangladesh (Anjuman et al. 2007).

The number of annual accidents and fatalities gives us some idea of the safety situation. In the history of the nation, the following table and figuere will show the road safety scenario.

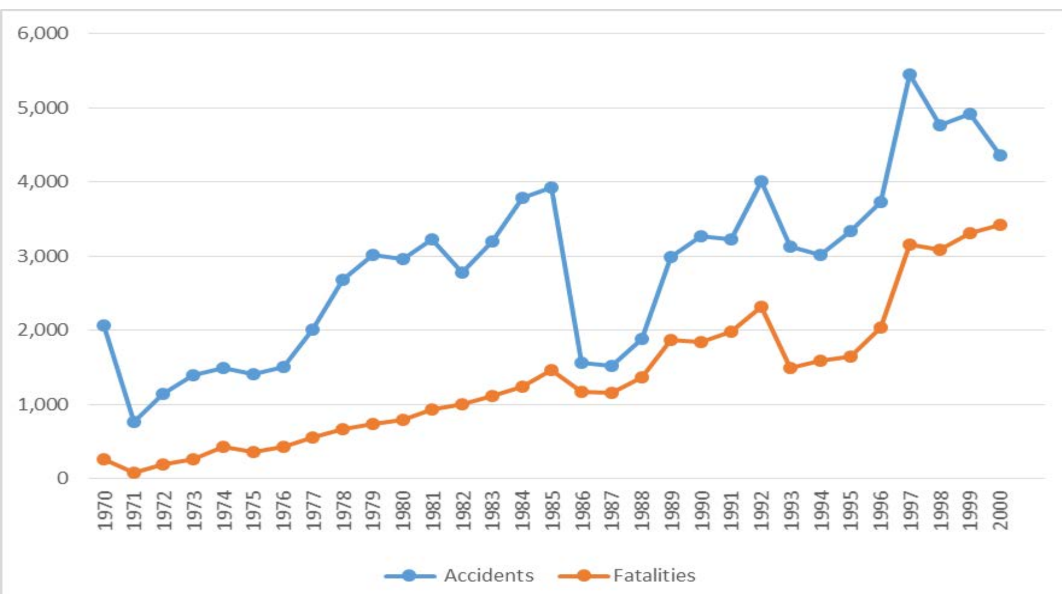

Figure 2: Trend of Road Accidents & Fatalities (1970-2000) (Anjuman et al. 2007).

In this paper, we analyze the equilibrium point and stability analysis for our model. We visualize the results and outcomes by numerically simulating our model and findings for better comprehension.

2. Model Formulation:

Road accidents are mainly caused by faulty vehicles and misguided drivers. Here we consider four compartmental model, where P(t) represents the pedestrian/ normal drivers, Dr(t) represents misguided drivers, V(t) represents people having faulty vehicles, and Ac(t) represent total road accident. We consider nine parameters, where φ describes the recruiting rate of deviated pedestrian for accident, α describes the contact rate of drivers and pedestrians for accident, β describes the contact rate of pedestrian and vehicles for accident, ρ describes the recruiting rate of faulty drivers for accident. And here we take ρ_1 , ρ_2, and ρ_3. The drivers who drive under the influence of alcohol are ρ_1, The drivers who talk on the phone while driving are ρ_2 and the drivers who are incompetent to drive are ρ_3, θ describes the contact rate of unexperienced drivers and faulty vehicles for accident, μ describes the recruiting rate of faulty vehicles for accident, σ describes the removal rate of faulty vehicles, ε describes the accident caused by faulty vehicles directly, and γ describes the accident caused by driver's fault directly.

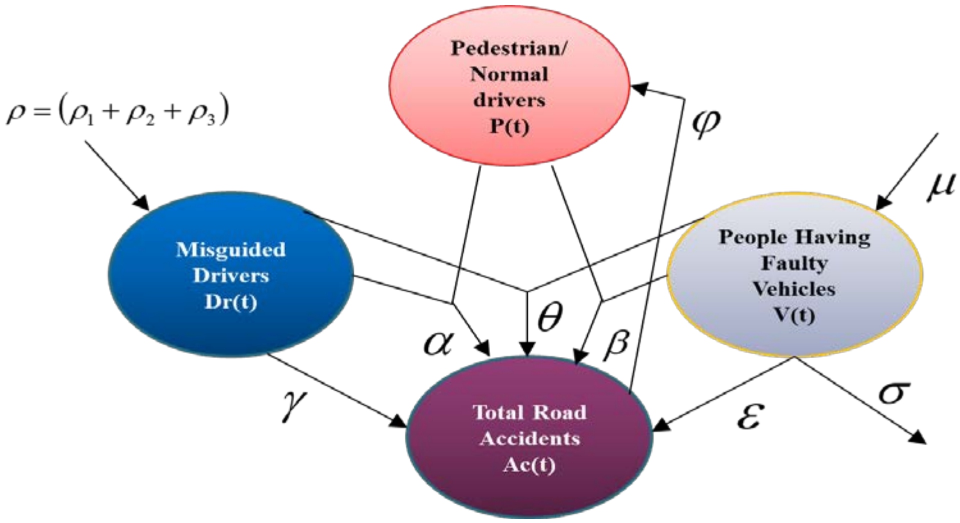

Figure 3: Schematic Diagram of the Road Accident Model

$$\frac{d}{dt}P(t) = \varphi \mathrm{Ac}(t) - \alpha \mathrm{Dr}(t)P(t) - \beta P(t)V(t)$$
$$\frac{d}{dt}\mathrm{D_r}(t) = (\rho_1 + \rho_2 + \rho_3)\mathrm{Dr}(t) - \gamma \mathrm{Dr}(t) - \theta \mathrm{Dr}(t)V(t)$$
$$\frac{d}{dt}V(t) = \mu V(t) - \epsilon V(t) - \sigma V(t)$$
$$\frac{d}{dt}\mathrm{A_c}(t) = \gamma \mathrm{Dr}(t) - \varphi \mathrm{Ac}(t) + \epsilon V(t) + \alpha \mathrm{Dr}(t)P(t) + \beta P(t)V(t) + \theta \mathrm{Dr}(t)V(t)$$
$$\rho = (\rho_1 + \rho_2 + \rho_3)$$
$$P(t) \geq 0, D_r(t) \geq 0, V(t) \geq 0, A_c(t) \geq 0$$

The solutions are all non-negative.

Table 1: Description of Parameters and Numerical Values

Variables	Descriptions	Values
φ	Recruiting rate of Deviated pedestrian for accident	0.7

α	Contact rate of drivers and pedestrians for accident	0.63
β	Contact rate of pedestrian and vehicles for accident	0.062
ρ	Recruiting rate of faulty drivers for accident	0.4
θ	Contact rate of unexperienced drivers and faulty vehicles for accident	0.152
μ	Recruiting rate of faulty vehicles for accident	0.2
σ	Removal rate of faulty vehicles	0.15
ò	Accident caused by faulty vehicles directly	0.002
γ	Accident caused by driver's fault directly	0.17

3. The Equilibrium Points:

Let, $E(P^*, Dr^*, V^*, Ac^*)$ be the equilibrium point of the model (1). In order to find the equilibrium point we need to solve $\frac{dP}{dt} == \frac{dD_r}{dt} = \frac{dV}{dt} = \frac{dA_C}{dt} = 0$ of the model (1). At the equilibrium point $E(P^*, Dr^*, V^*, Ac^*)$, the model (1) takes the following form:

$$\varphi \mathrm{Ac}(t) - \alpha \mathrm{Dr}(t) P(t) - \beta P(t) V(t) = 0$$
$$\rho \mathrm{Dr}(t) - \gamma \mathrm{Dr}(t) - \theta \mathrm{Dr}(t) V(t) = 0$$
$$\mu V(t) - \grave{o} V(t) - \sigma V(t) = 0$$
$$\gamma \mathrm{Dr}(t) - \varphi \mathrm{Ac}(t) + \grave{o} V(t) + \alpha \mathrm{Dr}(t) P(t) + \beta P(t) V(t) + \theta \mathrm{Dr}(t) V(t) = 0$$

Assuming $Ac^* = z$ and solving the above system, we get $E(P^*, Dr^*, V^*, Ac^*)$

$$P^* = -\frac{\varphi r \theta}{\beta\gamma - \beta\rho}$$
$$D_r^* = \frac{(\gamma - \rho)(\mu - \sigma)}{\rho\theta}$$
$$V^* = -\frac{\gamma - \rho}{\theta}$$
$$Ac^* = r$$

4. Stability Analysis:

Here, we perform stability analysis at equilibrium point by proving Theorem 1.

Theorem 1: The disease free equilibrium point of the model (1) is asymptotically stable if the eigenvalues of the Jacobian matrix are negative.

Proof: The Jacobian matrix of model (1) is given by

$$J = \begin{bmatrix} -\mathrm{Dr}\alpha - V\beta & -P\alpha & -P\beta & \varphi \\ 0 & \rho - \gamma - V\theta & -\mathrm{Dr}\theta & 0 \\ 0 & 0 & \mu - \grave{o} - \sigma & 0 \\ \mathrm{Dr}\alpha + V\beta & \gamma + P\alpha + V\theta & \grave{o} + P\beta + \mathrm{Dr}\theta & -\varphi \end{bmatrix}$$

Characteristic equation having eigenvalue λ is given by $\det(J - \lambda I) = 0$.

$$\det\left(J-\lambda I\right)=\begin{vmatrix} -\lambda-\text{Dr}\alpha-V\beta & -P\alpha & -P\beta & \varphi \\ 0 & \rho-\lambda-\gamma-V\theta & -\text{Dr}\theta & 0 \\ 0 & 0 & \mu-\lambda-\epsilon-\sigma & 0 \\ \text{Dr}\alpha+V\beta & \gamma+P\alpha+V\theta & \epsilon+P\beta+\text{Dr}\theta & -\lambda-\varphi \end{vmatrix}=0$$

Here the eigenvalues are given by:

$\lambda_1=0$

$\lambda_2=-\varphi-\alpha\text{Dr}-V\beta$

$\lambda_3=-\epsilon-\sigma+\mu$

$\lambda_4=-\gamma-\theta V+\rho$

Substituting the values of P^*, Dr^*, V^*, Ac^*, we get,

$\lambda_1=0$

$$\lambda_2=-\varphi-\frac{\alpha(\gamma-\rho)(\mu-\sigma)}{\rho\theta}+\frac{\beta(\gamma-\rho)}{\theta}$$

$\lambda_3=-\epsilon-\sigma+\mu$

$\lambda_4=0$

5. Numerical Simulation:

Numerical Simulation has been performed for validating the analytical findings.

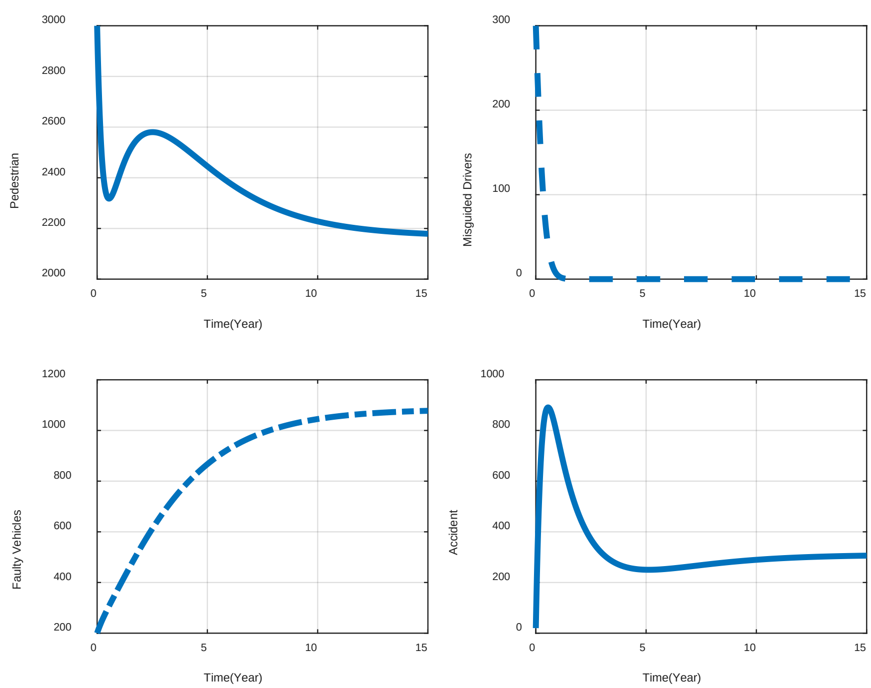

Figure 4: Graph of Pedestrian, Misguided Drivers, Faulty Vehicles, Accident for our model where time $t \in [0,15]$.

This is the graph of pedestrian, misguided driver, faulty vehicles and accident from the given value.

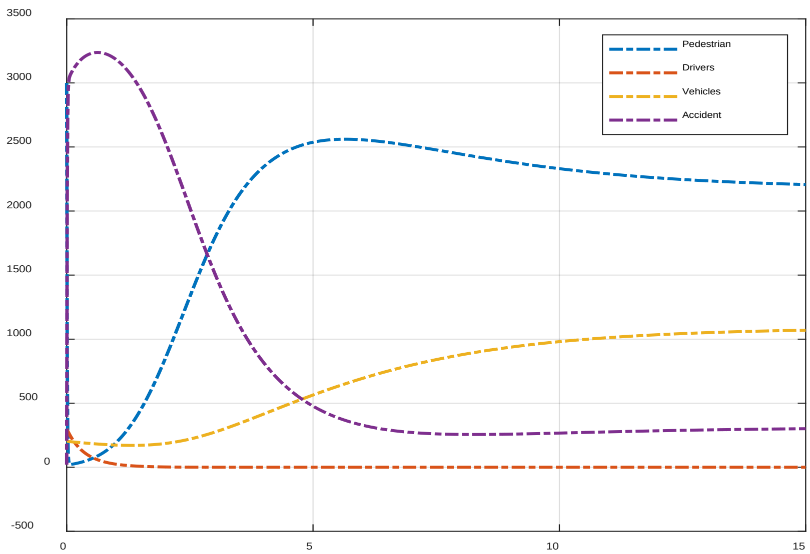

Figure 5: Combined graph of Pedestrian, Misguided Drivers, Faulty Vehicles, Accident for our model where time $t \in [0,15]$.

As time goes by, the rate of misguided driver decreases, the rate of faulty vehicles increases and the rate of misguided driver decreases, resulting in fewer accidents.

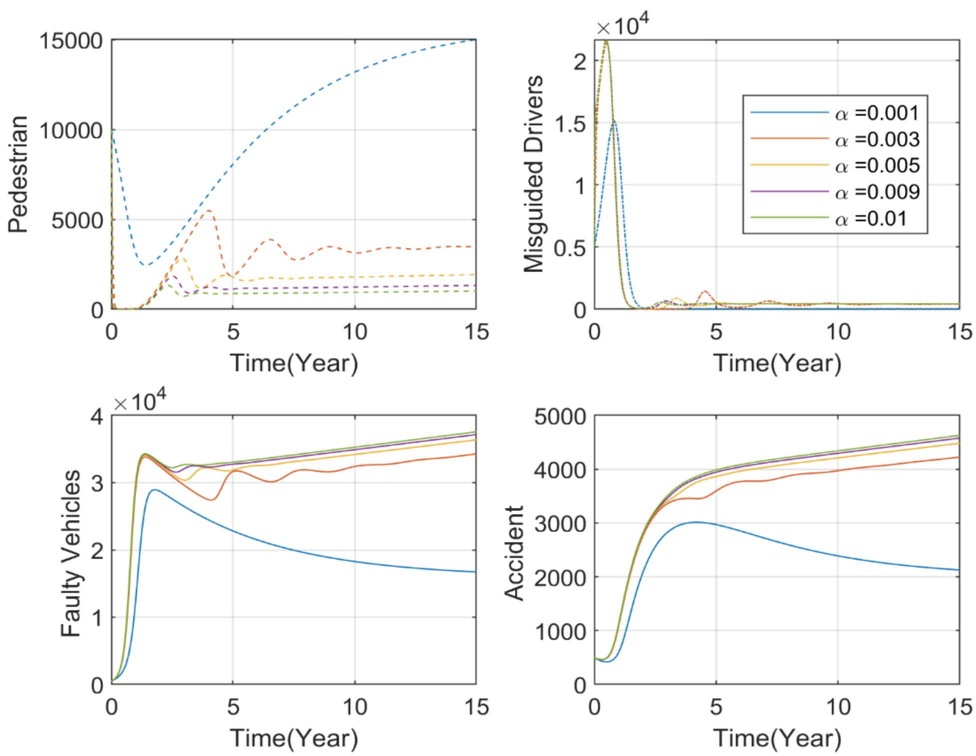

Figure 6: Graph of Pedestrian, Misguided Drivers, Faulty Vehicles, Accident for different values of α .

Inadvertent pedestrians who later find themselves as inexperienced drivers in many cases and increasing the total number of road accidents. And as a result of collisions by alpha rate between these two classes and population exchanges, the number of inexperienced drivers is increasing as well as the number of road accidents.

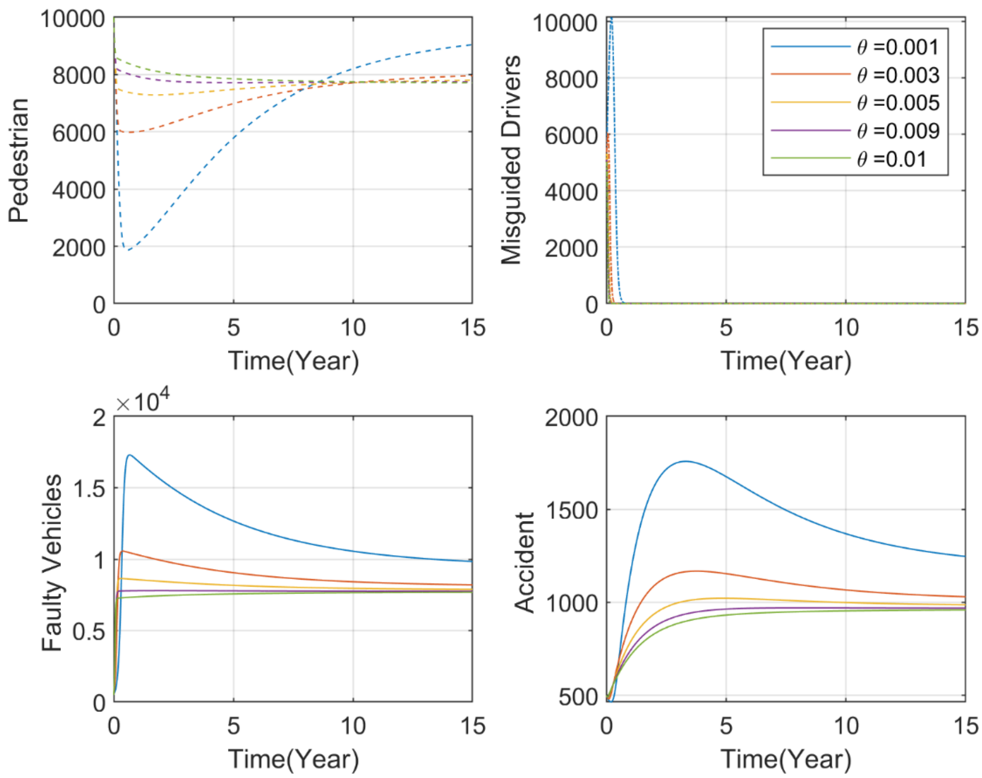

Figure 7: Graph of Pedestrian, Misguided Drivers, Faulty Vehicles, Accident for different values of θ.

The number of road accidents increases significantly when inexperienced drivers drive faulty vehicles and at the same time other faulty vehicles are parked on the road and pedestrians are present. Accidents are most likely to occur when someone is inexperienced as the driver of a faulty vehicle. For the same reason, if the drivers of faulty vehicles are inexperienced but the vehicles are faultless, the total number of road accidents decreases significantly. From Figure 4 we can clearly see that if the number of inexperienced drivers is significantly reduced, the number of road accidents as well as the number of accidents will be reduced automatically. Contact rate between Accused vehicles and inexperienced drivers θ have the greatest impact on road accidents.

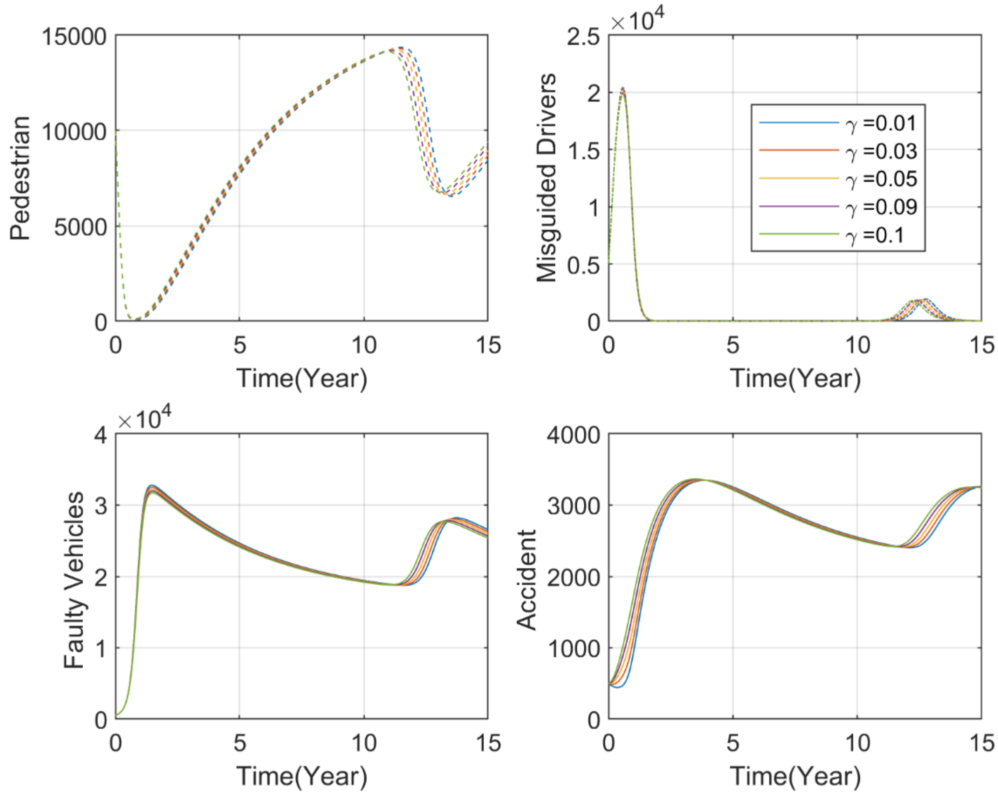

Figure 8: Graph of Pedestrian, Misguided Drivers, Faulty Vehicles, Accident for different values of γ .

From Figure 5 we can clearly see that pedestrians are involved in a large number of accidents only due to driver error or carelessness and if action is not taken against such inexperienced drivers, the number of road accidents continues to increase along with the accused vehicles. Clashes between pedestrians, faulty vehicles and inexperienced drivers result in a significant increase in road accidents.

6. Conclusion:

We can now describe and determine the solution from the above statistics and analysis. We have selected parameters (α, θ, γ) from them that have significantly reduced the number of inexperienced drivers and made a significant contribution to the reduction of road accidents. Again, pedestrians are only involved in a large number of accidents due to driver error or carelessness, and if no action is taken against such inexperienced drivers, along with the accused vehicles, the number of road accidents continues to increase.

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