Transportation Cost Minimization Using Transportation Method by Optimizing Job Assignment

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Abstract

Abstract Transportation is a non-value adding process but it's unavoidable for a company. It is essential that we minimize the transportation cost by optimizing the flow process. In this paper we optimized the transportation cost of a Bangladeshi LPG cylinder-based company. From three factories the goods must be delivered to thirteen different warehouses throughout the country to meet the demand. We applied Vogel's Approximation Method (VAM) to find the initial feasible solution and then used Modified Distribution (MoDi) method to find the optimized transportation cost. With this solution we can decide that from which warehouse we should distribute goods to a specific warehouse so that our transportation cost is as minimum as possible. The results were compared to the initial transportation solution the company used. The results proved that the modified distribution method generated the least cost. This will result in significant reduction of costs spent in transportation.

Keywords
Transportation Cost, Vogel's Approximation Method (VAM), Modified Distribution (MoDi) method, Optimized cost

Introduction
An important goal for any organization is to keep its expense as low as possible, especially when they are non-value adding expense. Unfortunately, not all non-value adding actions can be eliminated from a process such as transportation. Goods must be moved from one place to another even though this particular action does not increase the goods value like other actions. This is why we must keep this cost as low as possible, giving birth to a scope of research. Over the years many researches have been conducted to reduce this cost. The transportation problem is one of the earliest applications of linear programming problems. The basic transportation problem was originally developed by Hitchcock. Transportation method is a special linear programming method used to solve problems involving transporting products from several sources to several destinations. In this paper we emphasize on the importance of using mathematical techniques on daily transportation assignments and the enormous amount of money it saves.

Objectives
- To solve a transportation problem using Vogel's Approximation Method.
- To further optimize the initial solution using Modified Distribution Method.
- Compare results with traditional assignment practices.

Literature Review
G.M Appa (Donaldson, 1977) described a standard transportation problem in linear programming as a homogeneous product to be transported from m sources to n destinations. There is a cost of transporting from a location to another. This forces us to find an optimized decision that will keep this cost as low as possible. According to Appa, F.L Hitchcock (Hitchcock, 1941) was the first to work on such a problem. Lushu Li and K.K Lai (Lohgaonkar & Bajaj, 2009) have also shown a different view on the transportation problem as it helps not only in optimizing transportation but also in selecting a good location that would minimize costs. Similar study was conducted by Linda K. Nozick and Mark A. Turnquist (Nozick & Turnquist, 2001). However, it is observed that in most real-life cases, there are additional constraints that must be satisfied. Fortunately, many studies have been done which modify the algorithm in

order to satisfy these constraints. For example, D. Klingman and R. Russel (Klingman & Russell, 1975) have written in their paper on how to solve transportation problems with several constraints. Studies do not end here. There is more to be desired as in real scenarios there is time associated with for goods to move from one place to another. It is required to find a feasible distribution (of the supplies) which minimizes the maximum transportation time associated between a supply point and a demand point such that the distribution between the two points is positive. M.S Garfinkel and M.R Rao (Garfinkel & Rao, 1971) solved such bottleneck problems using their algorithm. There are many other papers that modify the basic algorithm to fit a specific constraint. Transportation problems can however be solved using a modified version of the simplex problem like G. A. Vignaux and Z. Michalewicz (Vignaux et al., n.d.) described the genetic algorithms for solving transportation problem. But L.R Ford Jr and D.R Fulkerson (Ford & Fulkerson, 1956) proved that Hitchcock's transportation problem solved with Kuhn's combinatorial algorithm is more efficient. And recently there has been much more development in finding more optimized transportation cost. Chandrasekhar Putcha, Aditya K. Putcha, MD Rohul Amin Bhuiyan3, and Nasima Farzana Hoque (Putcha et al., 2010) established a new method for optimizing transportation cost which is known as the Russel method**.** Then Abdul Quddoos, Shakeel Javaid and M. M. Khalid (Quddos et al., 2012) proposed a new method named ASM-method for optimizing transportation cost and established the numerical illustrations of the method. S. Ezhil Vannan and S. Rekha (Sudhakar et al., 2012) discussed about the exponential approach of optimizing transportation cost. Like these researchers are working to optimize the further optimize the transportation cost which is no doubt one of the most significant part of any company.

Methods

For our paper we used the general transportation problem solution algorithm to prove its superiority over intuitive assignment. We first collected data from a company that did not use algorithms to take decisions. We studied how their management used intuition to assign movements. Next, we applied Vogel's Approximation Method (VAM) on the taken data to find a initial feasible minimum solution.
Steps for Vogel's Approximation methods (Samuel & Venkatachalapathy, 2011) (Korukoğlu & Balli, 2011) are given below:

STEP 1: Represent the transportation problem in the standard tabular form.
STEP 2: Select the smallest element in each row and the next to the smallest element in that row. Here surrounding smallest one to be taken and difference is to be written as absolute. Find the difference. This is the penalty written on the right-hand side of each row. Repeat the same for each column. The penalty is written below each column.
STEP 3: Select the row or column with largest penalty. If there is a tie, the same can be broken arbitrarily.
STEP 4: Allocate the maximum feasible amount to the smallest cost cell in that row or column.
STEP 5: Allocate zero elsewhere in the row or column where the supply or demand is exhausted.
STEP 6: Remove all fully allocated rows or columns from further consideration. Then proceed with the remaining reduced matrix till no rows or columns remain.
STEP 7: Repeat steps 3-6 until all requirements have been meet
STEP 8: Compute total transportation cost for the feasible allocations using the original balanced-transportation cost matrix.

We could have also calculated the initial feasible minimum transportation cost using North West Corner (NWC) method or Least Cost Method (LCM) (Bit et al., 1992). But among these three VAM is believed to provide more appropriate results (Mishra, 2017). But from different research it has been found that these direct methods for finding optimal solution are not always reliable (Hasan, 2012), further optimization might be possible. So in our problem we also ran the optimality test and we further modified it using Modified Distribution (MoDi) method. Steps of MoDi method (Putcha et al., 2010) (Quddos et al., 2012) (Sudhakar et al., 2012) are the following:

STEP 1: Find out the basic feasible solution of the transportation problem using any one of the three
Methods discussed in Part one.
STEP 2: Introduce dual variables corresponding to the row constraints and the column constraints. If there are m origins and n destinations then there will be m+n dual variables. The dual variables corresponding to the row constraints are represented by ui, i=1,2,…..m where as the dual variables
corresponding to the column constraints are represented by vj, j=1,2,…..n. The values of the dual variables are calculated from the equation given below $u_i + v_j = c_{ij}$ if $x_{ij} > 0$ here c is the cost.
STEP 3: Any basic feasible solution has m + n -1 $x_{ij} > 0$. Thus, there will be m + n -1 equation to

determine m + n dual variables. One of the dual variables can be chosen arbitrarily. It is also to be noted that as the primal constraints are equations, the dual variables are unrestricted in sign.
STEP 4: If xij=0, the dual variables calculated in Step 3 are compared with the cij values of this allocation ui +vj -cij. If al ui +vj -cij <=0, then by the theorem of complementary slackness it can be shown that the corresponding solution of the transportation problem is optimum. If one or more ui +vj -cij is greater than zero, the we select the cell with the least positive value of ui +vj -cij and allocate as much as possible subject to the row and column constraints. The allocations of the number of adjacent cells are adjusted so that a basic variable becomes non-basic.
STEP 5: A fresh set of dual variables are calculated and repeat the entire procedure from Step 1 to Step 5.

- Penalty for unallocated sources is to be found using calculate ui +vj -cij .If all the penalties are zero or less than zero aka negative, then we have reached optimal solution. Otherwise, select the most positive penalty corresponding unallocated cell. If there is a tie, break the tie arbitrarily. This cell take some value and so another existing cell value will become zero. Exactly speaking on mathematical terms, The allocations of the number of adjacent cell are adjusted so that a basic variable becomes non-basic.
- So we run a thing called closed loop starting from that cell. Here the loop only ends and turns on the allocated sources. Now put + sign on our starting unallocated source and minus the next one and this way continues.
- The loop can only move horizontal and vertical direction.
- Closed loop thing. If there is more than two allocated cells are touched in a row or column or two allocated cells and finally starting point is touched, then only consider two, that is first and last, the other will be unchanged. In other words, only turning point containing allocated cells will get **(+),(-)** alternately.
- Closed loop would be shortest loop.
- Now we need to put some value to the unallocated cell selected. Here the Additional value will be the least among the minus things while you are working on closed loop thing.
- The cells at the turning points are called "Stepping Stones". So, this closed loop method is also known as stepping stones or stepping corner method.

Finally, we compared the results.

Data Collection

We collected data of the Bangladeshi LPG cylinder-based company over a span of 6 months. The different costs from each supply to demand zone were averaged and then used in our computation. We also had to make slight modifications and assumptions to fit the real data into the transportation problem model.
Final data table is given below,

Table 1: Main problem table

Production Zones/Warehouses	*Barisal*	*Bhairab*	*Chittagong*	*Comilla*	*Demra*	*Dohar*	*Gazipur*	*Jhenaidah*	*Madaripur*	*Manikganj*	*Rangpur*	*Sylhet*	*Tangail*	Supply
Mongla	12	32	45	39	26	25	33	12	10	28	49	40	31	142000
Bogura	37	21	47	31	21	23	15	21	33	18	8	42	13	153250
Comilla	24	20	14	3	13	18	17	22	19	19	48	29	24	62000
Demand	20210	27192	580	85880	22198	7676	48727	13029	19487	9981	25906	3598	28829	

Results and Discussion

Numerical Results

Intuitive System
Traditionally the above-mentioned LPG company use intuitions to assign movements rather than any algorithm-based system. Basically they used to distribute cylinders to a certain destination from the production zone which cost least to supply each cylinder to that particular destination. But this is obviously subject to the supply availability of that particular production zone. If the least costly supplier for any particular warehouse is out of stocks then this is automatically handed over to the next least cost suppliers. Thus, by adjusting corresponding demand and supply constraints the company used to meet the demand of each warehouse. Now lets observe the problem table,

For Barisal warehouse, least costly supplier is Mongla (12 taka per cylinder), Barisal's demand is 20210 cylinders which will be fulfilled completely by Mongla zone. SO, remaining products in Mongla zone = 142000-20210=121790.

For Bhairab warehouse, least costly supplier is Comilla production zone (20 taka per cylinder), Bhairab's demand is 27192 cylinders which will be fulfilled by Comilla zone. Remaining cylinders in comilla= 62000-27192= 34808.

For Chittagong warehouse, least costly supplier is Comilla (14 taka per cylinder), Chittagong's demand is 580 cylinders which will be met by Comilla zone. SO, remaining products in Mongla zone = 34808-580=34228

For Comilla warehouse, least costly supplier is Comilla (3 taka per cylinder), Comilla's demand is 85880 cylinders which cannot be fulfilled completely by Comilla zone because their remaining cylinder number is 34228 which is less than Comilla's demand. So, 34228 cylinders will be provided by Comilla zone and rest of the demand (51652 cylinders) will be fulfilled by Bogura zone (immediate next least cost supplier for comilla warehouse). After this, remaining products in Comilla zone is 0 and remaining cylinders in Bogura zone= 153250-51652=101598.

For Demra warehouse, least costly supplier is Comilla (13 taka per cylinder), But Comilla zone is already out of stocks, so this order will be automatically handed over to Bogura zone to meet Demra's demand (22198 cylinders) completely. SO, remaining products in Bogura zone = 101598-22198=79400.

For Dohar warehouse, its almost the same case as Demra. Comilla is the least costly supplier for Dohar but they have run out of stocks already, so this order will be automatically handed over to Bogura zone to meet Dohar's demand (7676 cylinders) completely. SO, remaining products in Bogura zone = 79400-7676=71724.

For Gazipur warehouse, least costly supplier is Bogura (15 taka per cylinder), Gazipur's demand is 48727 cylinders which will be fulfilled completely by Bogura zone. SO, remaining products in Bogura zone = 71724-48727=22997

For Jhenaidah warehouse, least costly supplier is Mongla (12 taka per cylinder), Jhenaidah's demand is 13029 cylinders which will be fulfilled completely by Mongla zone. SO, remaining products in Mongla zone = 121790-13029=108761

For Madaripur warehouse, least costly supplier is Mongla (10 taka per cylinder), Madaripur's demand is 19487 cylinders which will be fulfilled completely by Mongla zone. SO, remaining products in Mongla zone = 108761-19487=89274

For Manikganj warehouse, least costly supplier is Bogura (18 taka per cylinder), Manikganj's demand is 9981 cylinders which will be fulfilled completely by Bogura zone. SO, remaining products in Bogura zone = 22997-9981=13016

For Rangpur warehouse, least costly supplier is Bogura (08 taka per cylinder), Rangpur's demand is 25906 cylinders which outnumbers Bogura's remaining capacity (13016 cylinders). So order of meeting rest of Rangpur's demand is handed over to Comilla (48 taka/cylinder) but again they are also out of stock. Then its finally handed over to Mongla zone and Rangpur's rest of the demand (12890) is fulfilled completely. SO, remaining products in Mongla zone = 89274-12890=76384

For Sylhet warehouse, least costly supplier is Comilla (29 taka per cylinder), but they are out of stocks. So, Sylhet's demand (3598 cylinders) will be fulfilled completely by Mongla zone (immediate next least costly supplier for Sylhet zone). SO, remaining products in Mongla zone = 76384-3598=72786

For Tangail warehouse, as we can see both Comilla and Bogura zones are out of stock, there's only one option left for Tangail, which is Mongla zone. It will fulfill Tangail's demand of 28829 cylinders and remaining products in Mongla zone = 72786-28829= 43957

Table 2: Final table for traditional intuitive system

Production Zones/ Warehouses	*Barisal*	*Bhairab*	*Chittagong*	*Comilla*	*Demra*	*Dohar*	*Gazipur*	*Jhenaidah*	*Madaripur*	*Manikganj*	*Rangpur*	*Sylhet*	*Tangail*	Supply
Mongla	**12 (least costly)**	32	45	39	26	25	33	**12(least costly)**	**10(least costly)**	28	**49(only one available option)**	**40(2nd least costly & available)**	**31(only one available option)**	~~142000~~ 43957
Bogura	37	21	47	**31 (2nd least costly & stock available)**	**21(2nd least costly & available)**	**23 (2nd least costly & available)**	**15 (least costly)**	21	33	**18(least costly)**	**8(least costly but no stock)**	42	**13(least costly but no stock)**	~~153250~~
Comilla	24	**20 (least costly)**	**14 (least costly)**	**3 (least costly but out of stock)**	**13 (least costly but out of stock)**	**18 (least costly but out of stock)**	17	22	19	19	**48(2nd least costly but out of stock)**	**29(least costly but no stock)**	**24(2nd least costly but out of stock)**	~~62000~~
Demand	~~20210~~	~~27192~~	~~580~~	~~85880~~	~~22198~~	~~7676~~	~~48727~~	~~13029~~	~~19487~~	~~9981~~	~~25906~~	~~3598~~	~~28829~~	

In this way using intuitive assignment system company's estimated transportation cost = (20210*12)+(27192*20)+(580*14)+(34228*3)+(51652*31)+(22198*21)+(7676*23)+(48727*15)+(13029*12)+(19487*10)+(9981*18)+(13016*8)+(12890*49)+(3598*40)+(28829*31)= 6176220

Applying Transportation Cost Minimization Technique (VAM and MoDi method)
After observing the main problem table, we get
TOTAL number of supply constraints: 3
TOTAL number of demand constraints: 13

Here Total Demand = 313293 is less than Total Supply = 357250. So, we add a dummy demand constraint with 0 unit cost and with allocation 43957.

Now, the modified table is

Table 3: Problem table after balancing demand and supply constraints

	*D*1	*D*2	*D*3	*D*4	*D*5	*D*6	*D*7	*D*8	*D*9	*D*10	*D*11	*D*12	*D*13	D_{dummy}	Supply
*S*1	12	32	45	39	26	25	33	12	10	28	49	40	31	0	142000
*S*2	37	21	47	31	21	23	15	21	33	18	8	42	13	0	153250
*S*3	24	20	14	3	13	18	17	22	19	19	48	29	24	0	62000
Demand	20210	27192	580	85880	22198	7676	48727	13029	19487	9981	25906	3598	28829	43957	

Now we will apply **Vogel's Approximation Method** to find initial feasible cost,

Table 4: Iteration 1 for VAM

	*D*1	*D*2	*D*3	*D*4	*D*5	*D*6	*D*7	*D*8	*D*9	*D*10	*D*11	*D*12	*D*13	*Ddummy*	Supply	Row Penalty
*S*1	12	32	45	39	26	25	33	12	10	28	49	40	31	0	142000	10=10-0
*S*2	37	21	47	31	21	23	15	21	33	18	8	42	13	0	153250	8=8-0
*S*3	24	20	14	3	13	18	17	22	19	19	48	29	24	0	62000	3=3-0
Demand	20210	27192	580	85880	22198	7676	48727	13029	19487	9981	25906	3598	28829	43957		
Column Penalty	12=24-12	1=21-20	31=45-14	28=31-3	8=21-13	5=23-18	2=17-15	9=21-12	9=19-10	1=19-18	40=48-8	11=40-29	11=24-13	0=0-0		

The maximum penalty, 40, occurs in column *D*11.
The minimum c_{ij} in this column is $c_{211}=8$.

The maximum allocation in this cell is min (153250,25906) = **25906**.
It satisfy demand of *D*11 and adjust the supply of *S*2 from 153250 to 127344 (153250 - 25906=127344).

We repeated this similar iteration until all requirements are met. And after 16 iterations we finally get our final initial minimum solution using VAM method.

Initial feasible solution is

Table 5: Final solution table after applying VAM

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<tr><td>S1</td><td>12(20210)</td><td>32(21361)</td><td>45</td><td>39(24460)</td><td>26(22198)</td><td>25(7676)</td><td>33</td><td>12(13029)</td><td>10(19487)</td><td>28(9981)</td><td>49</td><td>40(3598)</td><td>31</td><td>0</td><td>142000</td><td>10 | 10 | 10 | 10 | 10 | 12 | 12 | 12 | 13 | 1 | 1 | 1 | 1 | 1 | 1 | 25 |</td></tr>
<tr><td>S2</td><td>37</td><td>21(5831)</td><td>47</td><td>31</td><td>21</td><td>23</td><td>15(48727)</td><td>21</td><td>33</td><td>18</td><td>8(25906)</td><td>42</td><td>13(28829)</td><td>0(43957)</td><td>153250</td><td>8 | 13 | 13 | 13 | 13 | 13 | 15 | 18 | 3 | 3 | -- | -- | -- | -- | -- | -- |</td></tr>
<tr><td>S3</td><td>24</td><td>20</td><td>14(580)</td><td>3(61420)</td><td>13</td><td>18</td><td>17</td><td>22</td><td>19</td><td>19</td><td>48</td><td>29</td><td>24</td><td>0</td><td>62000</td><td>3 | 3 | 3 | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- |</td></tr>
<tr><td>Demand</td><td>20210</td><td>27192</td><td>580</td><td>85880</td><td>22198</td><td>7676</td><td>48727</td><td>13029</td><td>19487</td><td>9981</td><td>25906</td><td>3598</td><td>28829</td><td>43957</td><td></td><td></td></tr>
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The initial minimum total transportation cost =12×20210+32×21361+39×24460+26×22198+25×7676+12×13029+10×19487+28×9981+40×3598+21×5831+15×48727+8×25906+13×28829+0×43957+14×580+3×61420=5051427
Here, the number of allocated cells = 16 is equal to m + n - 1 = 3 + 14 - 1 = 16
∴ This solution is **non-degenerate**

Optimality test using MoDi method...

Allocation Table is

Table 6: Allocation table for optimality test

	*D*1	*D*2	*D*3	*D*4	*D*5	*D*6	*D*7	*D*8	*D*9	*D*10	*D*11	*D*12	*D*13	*Ddummy*	Supply
*S*1	12 **(20210)**	32 **(21361)**	45	39 **(24460)**	26 **(22198)**	25 **(7676)**	33	12 **(13029)**	10 **(19487)**	28 **(9981)**	49	40 **(3598)**	31	0	142000
*S*2	37	21 **(5831)**	47	31	21	23	15 **(48727)**	21	33	18	8 **(25906)**	42	13 **(28829)**	0 **(43957)**	153250
*S*3	24	20	14 **(580)**	3 **(61420)**	13	18	17	22	19	19	48	29	24	0	62000
Demand	20210	27192	580	85880	22198	7676	48727	13029	19487	9981	25906	3598	28829	43957	

Iteration-1 of optimality test

1. Find u_i and v_j for all occupied cells(i,j), where $c_{ij}=u_i+v_j$

1. Substituting, $u_1=0$, we get

2.$c_{11}=u_1+v_1 \Rightarrow v_1=c_{11}-u_1 \Rightarrow v_1=12-0 \Rightarrow v_1=12$

3.$c_{12}=u_1+v_2 \Rightarrow v_2=c_{12}-u_1 \Rightarrow v_2=32-0 \Rightarrow v_2=32$

4.$c_{22}=u_2+v_2 \Rightarrow u_2=c_{22}-v_2 \Rightarrow u_2=21-32 \Rightarrow u_2=-11$

5.$c_{27}=u_2+v_7 \Rightarrow v_7=c_{27}-u_2 \Rightarrow v_7=15+11 \Rightarrow v_7=26$

6.$c_{211}=u_2+v_{11} \Rightarrow v_{11}=c_{211}-u_2 \Rightarrow v_{11}=8+11 \Rightarrow v_{11}=19$

7.$c_{213}=u_2+v_{13} \Rightarrow v_{13}=c_{213}-u_2 \Rightarrow v_{13}=13+11 \Rightarrow v_{13}=24$

8.$c_{214}=u_2+v_{14} \Rightarrow v_{14}=c_{214}-u_2 \Rightarrow v_{14}=0+11 \Rightarrow v_{14}=11$

9.$c_{14}=u_1+v_4 \Rightarrow v_4=c_{14}-u_1 \Rightarrow v_4=39-0 \Rightarrow v_4=39$

10.$c_{34}=u_3+v_4 \Rightarrow u_3=c_{34}-v_4 \Rightarrow u_3=3-39 \Rightarrow u_3=-36$

11.$c_{33}=u_3+v_3 \Rightarrow v_3=c_{33}-u_3 \Rightarrow v_3=14+36 \Rightarrow v_3=50$

12.$c_{15}=u_1+v_5 \Rightarrow v_5=c_{15}-u_1 \Rightarrow v_5=26-0 \Rightarrow v_5=26$

13.$c_{16}=u_1+v_6 \Rightarrow v_6=c_{16}-u_1 \Rightarrow v_6=25-0 \Rightarrow v_6=25$

14.$c_{18}=u_1+v_8 \Rightarrow v_8=c_{18}-u_1 \Rightarrow v_8=12-0 \Rightarrow v_8=12$

15.$c19=u1+v9 \Rightarrow v9=c19-u1 \Rightarrow v9=10-0 \Rightarrow v9=10$

16.$c110=u1+v10 \Rightarrow v10=c110-u1 \Rightarrow v10=28-0 \Rightarrow v10=28$

17.$c112=u1+v12 \Rightarrow v12=c112-u1 \Rightarrow v12=40-0 \Rightarrow v12=40$

Table 7: Iteration 1 for optimality test (Calculating u,v,c values for occupied cells)

	*D*1	*D*2	*D*3	*D*4	*D*5	*D*6	*D*7	*D*8	*D*9	*D*10	*D*11	*D*12	*D*13	*Ddummy*	Supply	*ui*
*S*1	12 **(20210)**	32 **(21361)**	45	39 **(24460)**	26 **(22198)**	25 **(7676)**	33	12 **(13029)**	10 **(19487)**	28 **(9981)**	49	40 **(3598)**	31	0	142000	*u*1 =0
*S*2	37	21 **(5831)**	47	31	21	23	15 **(48727)**	21	33	18	8 **(25906)**	42	13 **(28829)**	0 **(43957)**	153250	*u*2 =-11
*S*3	24	20	14 **(580)**	3 **(61420)**	13	18	17	22	19	19	48	29	24	0	62000	*u*3 =-36
Demand	20210	27192	580	85880	22198	7676	48727	13029	19487	9981	25906	3598	28829	43957		
vj	*v*1=12	*v*2=32	*v*3=50	*v*4=39	*v*5=26	*v*6=25	*v*7=26	*v*8=12	*v*9=10	*v*10 =28	*v*11 =19	*v*12 =40	*v*13=24	*v*14 =11		

2. Find dij for all unoccupied cells(i,j), where $dij=cij-(ui+vj)$

1.$d13=c13-(u1+v3)=45-(0+50)=-5$

2.$d17=c17-(u1+v7)=33-(0+26)=7$

3.$d111=c111-(u1+v11)=49-(0+19)=30$

4.$d113=c113-(u1+v13)=31-(0+24)=7$

5.$d114=c114-(u1+v14)=0-(0+11)=-11$

6.$d21=c21-(u2+v1)=37-(-11+12)=36$

7.$d23=c23-(u2+v3)=47-(-11+50)=8$

8.$d24=c24-(u2+v4)=31-(-11+39)=3$

9.$d25=c25-(u2+v5)=21-(-11+26)=6$

10.$d26=c26-(u2+v6)=23-(-11+25)=9$

11.$d28=c28-(u2+v8)=21-(-11+12)=20$

12.$d_{29}=c_{29}-(u_2+v_9)=33-(-11+10)=34$

13.$d_{210}=c_{210}-(u_2+v_{10})=18-(-11+28)=1$

14.$d_{212}=c_{212}-(u_2+v_{12})=42-(-11+40)=13$

15.$d_{31}=c_{31}-(u_3+v_1)=24-(-36+12)=48$

16.$d_{32}=c_{32}-(u_3+v_2)=20-(-36+32)=24$

17.$d_{35}=c_{35}-(u_3+v_5)=13-(-36+26)=23$

18.$d_{36}=c_{36}-(u_3+v_6)=18-(-36+25)=29$

19.$d_{37}=c_{37}-(u_3+v_7)=17-(-36+26)=27$

20.$d_{38}=c_{38}-(u_3+v_8)=22-(-36+12)=46$

21.$d_{39}=c_{39}-(u_3+v_9)=19-(-36+10)=45$

22.$d_{310}=c_{310}-(u_3+v_{10})=19-(-36+28)=27$

23.$d_{311}=c_{311}-(u_3+v_{11})=48-(-36+19)=65$

24.$d_{312}=c_{312}-(u_3+v_{12})=29-(-36+40)=25$

25.$d_{313}=c_{313}-(u_3+v_{13})=24-(-36+24)=36$

26.$d_{314}=c_{314}-(u_3+v_{14})=0-(-36+11)=25$

Table 8: Iteration 1 for optimality test (Finding d values for unoccupied cells)

	*D*1	*D*2	*D*3	*D*4	*D*5	*D*6	*D*7	*D*8	*D*9	*D*10	*D*11	*D*12	*D*13	*Ddummy*	Supply	*ui*
*S*1	12 **(20210)**	32 **(21361)**	45 [-5]	39 **(24460)**	26 **(22198)**	25 **(7676)**	33 [7]	12 **(13029)**	10 **(19487)**	28 **(9981)**	49 [30]	40 **(3598)**	31 [7]	0 [-11]	142000	*u*1 =0
*S*2	37 [36]	21 **(5831)**	47 [8]	31 [3]	21 [6]	23 [9]	15 **(48727)**	21 [20]	33 [34]	18 [1]	8 **(25906)**	42 [13]	13 **(28829)**	0 **(43957)**	153250	*u*2 =-11
*S*3	24 [48]	20 [24]	14 **(580)**	3 **(61420)**	13 [23]	18 [29]	17 [27]	22 [46]	19 [45]	19 [27]	48 [65]	29 [25]	24 [36]	0 [25]	62000	*u*3 =-36
Demand	20210	27192	580	85880	22198	7676	48727	13029	19487	9981	25906	3598	28829	43957		
vj	*v*1=12	*v*2=32	*v*3=50	*v*4=29	*v*5=26	*v*6=25	*v*7=26	*v*8=12	*v*9=10	*v*10 =28	*v*11 =19	*v*12 =40	*v*13=24	*v*14 =11		

3. Now choose the minimum negative value from all *dij* (opportunity cost) = *d*114 = [-11]
and draw a closed path from *S*1*Ddummy*.
Closed path is $S1D_{dummy} \rightarrow S1D2 \rightarrow S2D2 \rightarrow S2D_{dummy}$
Closed path and plus/minus sign allocation...

Table 9: Iteration 1 for optimality test (Closed loop)

	*D*1	*D*2	*D*3	*D*4	*D*5	*D*6	*D*7	*D*8	*D*9	*D*10	*D*11	*D*12	*D*13	*Ddummy*	Supply	*ui*
*S*1	12 **(20210)**	32 **(21361) (-)**	45 [-5]	39 **(24460)**	26 **(22198)**	25 **(7676)**	33 [7]	12 **(13029)**	10 **(19487)**	28 **(9981)**	49 [30]	40 **(3598)**	31 [7]	0 [-11] **(+)**	142000	*u*1 =0
*S*2	37 [36]	21 **(5831) (+)**	47 [8]	31 [3]	21 [6]	23 [9]	15 **(48727)**	21 [20]	33 [34]	18 [1]	8 **(25906)**	42 [13]	13 **(28829)**	0 **(43957) (-)**	153250	*u*2 =-11
*S*3	24 [48]	20 [24]	14 **(580)**	3 **(61420)**	13 [23]	18 [29]	17 [27]	22 [46]	19 [45]	19 [27]	48 [65]	29 [25]	24 [36]	0 [25]	62000	*u*3 =-36
Demand	20210	27192	580	85880	22198	7676	48727	13029	19487	9981	25906	3598	28829	43957		
vj	*v*1=12	*v*2=32	*v*3=50	*v*4=39	*v*5=26	*v*6=25	*v*7=26	*v*8=12	*v*9=10	*v*10 =28	*v*11 =19	*v*12 =40	*v*13=24	*v*14=11		

4. Minimum allocated value among all negative position **(-)** on closed path = 21361
Subtract 21361 from all **(-)** and Add it to all **(+)**

Table 10: Iteration 1 for optimality test (Subtracting minimum allocated value from all (-) and add to all (+))

	*D*1	*D*2	*D*3	*D*4	*D*5	*D*6	*D*7	*D*8	*D*9	*D*10	*D*11	*D*12	*D*13	*Ddummy*	Supply
*S*1	12 **(20210)**	32	45	39 **(24460)**	26 **(22198)**	25 **(7676)**	33	12 **(13029)**	10 **(19487)**	28 **(9981)**	49	40 **(3598)**	31	0 **(21361)**	142000
*S*2	37	21 **(27192)**	47	31	21	23	15 **(48727)**	21	33	18	8 **(25906)**	42	13 **(28829)**	0 **(22596)**	153250
*S*3	24	20	14 **(580)**	3 **(61420)**	13	18	17	22	19	19	48	29	24	0	62000
Demand	20210	27192	580	85880	22198	7676	48727	13029	19487	9981	25906	3598	28829	43957	

5. Repeat the step 1 to 4, until an optimal solution (all *dij*≥0.) is obtained.

After 5 iteration finally optimal solution is arrived.

Table 11: Final optimal solution table

	*D*1	*D*2	*D*3	*D*4	*D*5	*D*6	*D*7	*D*8	*D*9	*D*10	*D*11	*D*12	*D*13	*Ddummy*	Supply
*S*1	12 **(20210)**	32	45 **(580)**	39 **(11265)**	26 **(22198)**	25 **(7676)**	33	12 **(13029)**	10 **(19487)**	28	49	40 **(3598)**	31	0 **(43957)**	142000
*S*2	37	21 **(27192)**	47	31 **(12615)**	21	23	15 **(48727)**	21	33	18 **(9981)**	8 **(25906)**	42	13 **(28829)**	0	153250
*S*3	24	20	14	3 **(62000)**	13	18	17	22	19	19	48	29	24	0	62000
Demand	20210	27192	580	85880	22198	7676	48727	13029	19487	9981	25906	3598	28829	43957	

The minimum total transportation cost = 12×20210+45×580+39×11265+26×22198+25×7676+12×13029+10×19487+40×3598+0×43957+21×27192+31×12615+15×48727+18×9981+8×25906+13×28829+3×62000= 4612826

Graphical Results

As we can see from numerical results that after applying transportation cost minimization method upon the collected data overall transportation cost of the company has been reduced to a great extent. We can have better view of results from the graph below:

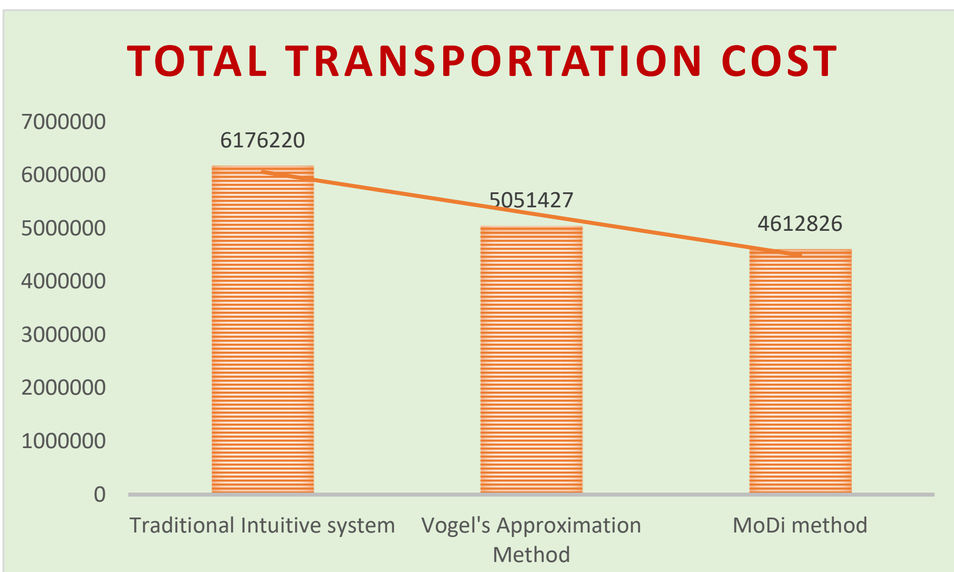

Figure 1: Comparison of minimized Transportation Cost using VAM and MoDi method with the Traditional intuitive system.

Proposed Improvements

From the calculations and results shown above we can propose that rather than supplying cylinders to the warehouses based on random intuitions, its way much cheaper to transport cylinders if we just apply some proved optimization techniques on the collected data.

We can just apply Vogel's approximation method (VAM) to find a basic initial feasible transportation method. Further, we ran the optimality test using Modified Distribution (MoDi) method and finally calculated the best optimized results and can also propose the company to follow that optimized transportation method which will reduce their overall transportation cost by almost 25% of the previous traditional transportation cost.

Validation

From calculation it is clear that there is a significant difference between company's traditional intuitive transportation cost and cost using VAM and MoDi method.
Usually, company's overall transportation cost using their intuitive assignment system us BDT 6176220.
If they can apply VAM method for transporting cylinders to warehouses that cost is reduced to BDT 5051427.

Saved amount after using VAM = 6176220-5051427= BDT 1124793

And if they can go further and apply Modified Distribution (MoDi) method on those data they can get the best optimized result and total transportation cost is reduced to BDT 4612826.

Saved amount after using MoDi method=6176220-4612826=BDT 1563394

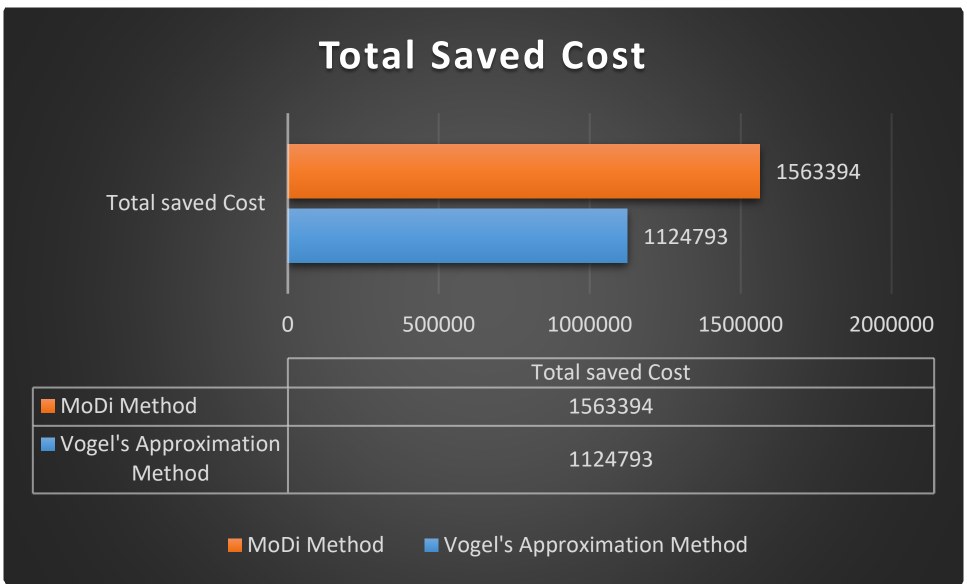

Figure 2: Total saved cost comparison using VAM and MoDi method

Conclusion

In this paper, our main objective was to apply transportation cost minimization technique using Vogel's Approximation Method (VAM) and Modified Distribution (MoDi method) upon a Bangladeshi LPG cylinder-based company's transportation data and by doing so we have showed that they can reduce their overall transportation cost upto 25% of their traditional cost by using our proposed algorithms. Finally, we can say that transportation being such a significant part of production industry, the study of transportation cost minimization techniques are very important for any company and further development in such research will help the concerned industries to a great extent.

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